NANDYAL BANKING COACHING CENTER

N.G.O's COLONY, NANDYAL, ANDHRA PRADESH

PH: 7416206879 | www.nandyalbankcoachingcenter.in

COMPLETE STUDY MATERIAL

ENGINEERING

— MECHANICAL —

PROFESSIONAL KNOWLEDGE for COMPETITIVE EXAMS

CWC AM (MECHANICAL) | SSC JE | RRB JE | PSU AE / AM | STATE AE / AEE

SOM & TOM • Machine Design • Thermodynamics • IC Engines • RAC • Heat Transfer
Fluid Machinery • Materials • Welding • Machining & CNC • Metrology • Industrial Engg







UNDER THE GUIDANCE OF

G. V. SATISH

SBI PO – 2005 Batch

INTERVIEW PANEL MEMBER

★ PROUD RECORD ★

3069+ STUDENTS GOT JOBS TILL NOW

NANDYAL BANKING COACHING CENTER

N.G.O's Colony, NANDYAL ☎ 7416206879

https://www.nandyalbankcoachingcenter.in/

CWC ASSISTANT MANAGER

Professional Knowledge – Engineering (Mechanical)

Central Warehousing Corporation · Post Code 06 · Advt. No. CWC/I-Manpower/DR/Rectt/2026/01

Prepared by

G V Satish

Chairman and Managing Director

Nandyal Banking Coaching Center, NGOs Colony, Nandyal – Ph: 7416206879

Nandyal Banking Coaching Center

About the Exam & How to Use This Book

Online examination – Assistant Manager (Engineering)-Mechanical

#	Test (not by sequence)	Questions	Marks	Time
1	**Professional / Subject Knowledge (this book)**	60	60	55 min
2	General Awareness	40	40	20 min
3	English Language	40	40	25 min
4	Reasoning & Computer Aptitude	30	30	25 min
5	Data Analysis & Interpretation and Quantitative Aptitude	30	30	25 min
6	Language & Comprehension Abilities – Descriptive	2	30	30 min
	Total	**202**	**230**	**3 hours**

- **Negative marking**: ¼ of the marks of the question for each wrong answer; no penalty for a blank.
- **Qualifying marks** in Professional Knowledge: **40%** (UR), **35%** (OBC/EWS/SC/ST/PwBD/ExSM). Same for the Descriptive test.
- Shortlisting for interview / DV up to **1:5**. Eligibility: **B.E./B.Tech in Mechanical Engineering**; age 18–28 years (as on 15.11.2026). Pay scale **₹40,000–1,40,000 (E-1, IDA)**.
- Roughly **1 minute per question**: these notes are built for recall. Each section ends with a **Previously Asked / Likely Questions** box; stars show weightage (★★★ = must-do).

Priority order of subjects (by usual weightage)

#	Subject	Syllabus part
1	Thermodynamics, IC Engines, RAC & Heat Transfer	B
2	Strength of Materials, Theory of Machines & Machine Design	A
3	Machining, Metal Cutting, NTM, CNC, CAD/CAM & Additive Manufacturing	C, D
4	Fluid Mechanics, Pumps & Compressors	B
5	Industrial Engineering & Operations Research	E
6	Engineering Materials & Heat Treatment	C
7	Casting, Welding, Brazing & Fabrication	C
8	Metrology, Inspection & Maintenance	C
9	Last-Day Revision (formula & value sheet)	All
10	500 Most Expected Questions & Answers	All

Nandyal Banking Coaching Center

Index

Index 3

1. Engineering Mechanics & Strength of Materials 6
1.1 Statics, Friction & Dynamics ★★★ 6
1.2 Stress, Strain & Elastic Constants ★★★ 7
1.3 SFD, BMD & Bending Stresses ★★★ 9
1.4 Deflection of Beams ★★★ 10
1.5 Torsion of Shafts & Springs ★★★ 10
1.6 Columns & Struts ★★★ 11
1.7 Principal Stresses, Mohr's Circle & Theories of Failure ★★★ 12
1.8 Thin/Thick Cylinders & Strain Energy Theorems ★★☆ 13

2. Theory of Machines & Vibrations 15
2.1 Mechanisms & Kinematics ★★★ 15
2.2 Cams & Followers ★★☆ 16
2.3 Gears & Gear Trains ★★★ 17
2.4 Belts, Ropes & Chains ★★★ 19
2.5 Flywheel & Governors ★★★ 19
2.6 Balancing ★★★ 20
2.7 Gyroscope ★★☆ 21
2.8 Mechanical Vibrations ★★★ 22

3. Machine Design 24
3.1 Design Fundamentals ★★★ 24
3.2 Fatigue & Endurance ★★★ 25
3.3 Joints – Riveted, Welded & Threaded ★★★ 26
3.4 Shafts, Keys & Couplings ★★★ 27
3.5 Bearings ★★★ 28
3.6 Brakes & Clutches ★★★ 29
3.7 Springs, Belts & Gears Design ★★☆ 30
3.8 Pressure Vessels & Flywheel Design ★☆☆ 31

4. Fluid Mechanics & Fluid Machinery 33
4.1 Fluid Properties & Statics ★★★ 33
4.2 Fluid Kinematics, Bernoulli & Momentum ★★★ 34
4.3 Flow Through Pipes, Boundary Layer, Drag & Lift ★★★ 35
4.4 Dimensional Analysis & Similitude ★★☆ 36
4.5 Hydraulic Turbines ★★★ 37
4.6 Pumps ★★★ 38
4.7 Hydraulic Systems & Fluid Power ★★☆ 40

5. Thermodynamics & Power Cycles 42
5.1 Basic Concepts & Zeroth Law ★★★ 42
5.2 First Law & Applications ★★★ 43
5.3 Second Law, Entropy & Availability ★★★ 44
5.4 Properties of Pure Substances & Steam ★★★ 45
5.5 Gas Power Cycles ★★★ 46
5.6 Vapour Power Cycles & Steam Generators ★★★ 47
5.7 Steam & Gas Turbines, Nozzles ★★☆ 49

6. I.C. Engines, Air Compressors & Compressed Air Systems 51
6.1 IC Engine Basics & Classification ★★★ 51
6.2 Engine Performance & Testing ★★★ 52
6.3 Combustion in SI & CI Engines ★★★ 53
6.4 Fuel, Ignition, Cooling & Lubrication Systems ★★☆ 55
6.5 Emissions & Alternative Fuels ★★☆ 56
6.6 Reciprocating Air Compressors ★★★ 57
6.7 Rotary Compressors & Compressed Air Systems ★★☆ 58

7. Refrigeration, Air Conditioning & Psychrometry 61
7.1 Refrigeration Basics & Air Refrigeration ★★★ 61
7.2 Vapour Compression Refrigeration (VCR) ★★★ 62
7.3 Refrigerants ★★★ 63
7.4 Vapour Absorption & Other Refrigeration Systems ★★☆ 65
7.5 Refrigeration Equipment & Cold Storage Practice ★★☆ 66
7.6 Psychrometry ★★★ 67
7.7 Air Conditioning Systems & Cooling Load ★★★ 69

Nandyal Banking Coaching Center

8. Heat Transfer 72
8.1 Conduction – Fundamentals ★★★ 72
8.2 Critical Insulation & Fins ★★★ 73
8.3 Transient (Unsteady) Conduction ★★☆ 74
8.4 Convection ★★★ 74
8.5 Boiling & Condensation ★★☆ 76
8.6 Radiation ★★★ 77
8.7 Heat Exchangers ★★★ 78

9. Engineering Materials & Heat Treatment 81
9.1 Crystal Structure & Imperfections ★★★ 81
9.2 Mechanical Properties & Testing ★★★ 82
9.3 Phase Diagrams & Iron–Carbon Diagram ★★★ 84
9.4 Heat Treatment ★★★ 86
9.5 Steels & Alloying Elements ★★★ 88
9.6 Non-Ferrous Metals, Polymers, Ceramics & Composites ★★☆ 90

10. Casting, Welding, Brazing & Fabrication 93
10.1 Casting – Patterns, Moulds & Cores ★★★ 93
10.2 Gating, Risering & Solidification ★★★ 94
10.3 Casting Processes & Defects ★★★ 95
10.4 Welding Fundamentals & Arc Welding ★★★ 97
10.5 Gas, Resistance & Solid-State Welding ★★★ 98
10.6 Weldability, Defects & Inspection ★★☆ 99
10.7 Brazing, Soldering & Adhesive Bonding ★★☆ 100
10.8 Metal Forming & Sheet Metal Fabrication ★★★ 101

11. Machining, Metal Cutting & Computer-Aided Manufacturing 104
11.1 Metal Cutting Mechanics ★★★ 104
11.2 Tool Geometry, Tool Materials & Tool Wear ★★★ 105
11.3 Machine Tools & Operations ★★★ 107
11.4 Abrasive Machining & Finishing ★★★ 109
11.5 Non-Traditional Machining ★★★ 111
11.6 Jigs, Fixtures & Work Holding ★★☆ 113
11.7 CNC, CAD/CAM & CIM ★★★ 114
11.8 Additive Manufacturing ★★★ 117

12. Metrology, Inspection & Maintenance Engineering 120
12.1 Limits, Fits & Tolerances ★★★ 120
12.2 Linear & Angular Measurement ★★★ 121
12.3 Surface Finish & Form Measurement ★★☆ 123
12.4 Errors, Calibration & Quality Control ★★☆ 124
12.5 Non-Destructive Testing & Inspection Techniques ★★★ 125
12.6 Maintenance Engineering & Tools ★★★ 127

13. Production Planning, Industrial Engineering & Operations Research 130
13.1 Production Planning & Control ★★★ 130
13.2 Forecasting ★★★ 131
13.3 Inventory Control ★★★ 132
13.4 MRP, ERP & Lean Manufacturing ★★★ 134
13.5 Scheduling & Sequencing ★★★ 135
13.6 Work Study & Ergonomics ★★☆ 136
13.7 Linear Programming, Transportation & Assignment ★★★ 138
13.8 Queuing Theory, Network Models, PERT/CPM & Decision Theory ★★★ 139

14. Last-Day Revision (Formula & Value Sheet) 142

15. 500 Most Expected Questions & Answers 144
15.1 Engineering Mechanics & Strength of Materials (Q 1–45) 144
15.2 Theory of Machines & Vibrations (Q 46–85) 145
15.3 Machine Design (Q 86–125) 145
15.4 Fluid Mechanics & Fluid Machinery (Q 126–165) 146
15.5 Thermodynamics & Power Cycles (Q 166–210) 147
15.6 I.C. Engines & Air Compressors (Q 211–245) 148
15.7 Refrigeration, Air Conditioning & Psychrometry (Q 246–280) 149
15.8 Heat Transfer (Q 281–315) 149
15.9 Engineering Materials & Heat Treatment (Q 316–350) 150
15.10 Casting, Welding & Fabrication (Q 351–385) 151
15.11 Machining, Metal Cutting & CAM (Q 386–430) 152
15.12 Metrology, Inspection & Maintenance (Q 431–455) 153

Nandyal Banking Coaching Center

15.13 Production Planning, Industrial Engineering & OR (Q 456–500) 153

Nandyal Banking Coaching Center

1. Engineering Mechanics & Strength of Materials

> **NOTE**
>
> Applied mechanics and SOM are the base of the Mechanical paper and usually give **6–10 of the 60 questions**. Most questions are one-step: a formula, a ratio or a standard value. Typical items are belt friction $e^{\mu\theta}$, self-locking ($\eta < 50\%$), $E = 2G(1+\nu)$, the impact factor 2, standard BM and deflection coefficients, the 1.5 and 4/3 shear-stress ratios, Euler effective lengths, Mohr's circle, the Tresca and von Mises constants (0.5 and 0.577), thin-cylinder stresses (pd/2t, pd/4t) and the helical spring formula $8WD^3n/Gd^4$. Learn the tables by heart.

1.1 Statics, Friction & Dynamics ★★★

- **Force** is a **vector**. It is fixed by magnitude, direction, sense and point of application. 1 N = 1 kg·m/s²; 1 kgf = **9.81 N**.
- **Principle of transmissibility**: a force may be slid along its **line of action** without changing its external effect on a **rigid body**.
- **Resultant** of two forces P, Q at angle θ: $R = \sqrt{(P^2 + Q^2 + 2PQ\cos\theta)}$, with $\tan\alpha = Q\sin\theta/(P + Q\cos\theta)$. R is **maximum (P + Q) at θ = 0°** and **minimum (P − Q) at θ = 180°**. At θ = 90°, $R = \sqrt{(P^2 + Q^2)}$. Two equal forces P at angle θ give **R = 2P cos(θ/2)**. At θ = 120° the resultant equals P.
- **Equilibrant** = equal to the resultant, opposite in sense and on the same line.
- **Lami's theorem**: if **three coplanar concurrent forces** keep a body in equilibrium, $P/\sin\alpha = Q/\sin\beta = R/\sin\gamma$. Each angle is the one **between the other two forces**. It applies to **exactly three** forces.
- **Varignon's theorem**: the moment of the resultant about any point = the **algebraic sum of the moments of its components** about that point.
- **Couple** = two equal, opposite, parallel, non-collinear forces. Its moment is **F × arm** and is the **same about every point** (a free vector). A couple has **zero resultant force** and can be balanced **only by another couple**. A force can be replaced by an equal force at another point plus a couple.
- **Equilibrium conditions (coplanar)**: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$, which gives **3 equations**. In space there are **6 equations**. Concurrent coplanar forces need only **2**.
- **Free-body diagram (FBD)**: the body is isolated and **all external forces and reactions** are drawn on it. A **two-force member** in equilibrium carries equal, opposite, collinear forces along the line joining the two points. Three non-parallel forces in equilibrium must be **concurrent**.
- **Support reactions**: roller = **1** (normal to the surface); hinge/pin = **2**; fixed = **3** (H, V, M).
- **Truss**: perfect (statically determinate) frame when **m = 2j − 3**. If m < 2j − 3 it is **deficient** (a mechanism); if m > 2j − 3 it is **redundant**. The method of joints solves at most 2 unknowns per joint; the method of sections solves at most **3 unknowns** per cut.

Friction

- **Laws of dry (Coulomb) friction**: friction **opposes relative motion**. Limiting friction F = μR is **proportional to the normal reaction** and **independent of the apparent area of contact**. Kinetic friction is roughly **independent of sliding velocity**, and $\mu_k < \mu_s$. Friction depends on the nature of the surfaces.
- **Order of magnitude**: static > dynamic (sliding) > **rolling** (least). Limiting friction is the **maximum** static friction.
- **Angle of friction** φ: $\tan\varphi = \mu$ (the angle between the resultant reaction and the normal at limiting friction).
- **Angle of repose** α = the maximum incline on which a body rests without sliding; **α = φ**, so $\tan\alpha = \mu$. **Cone of friction** has a semi-vertex angle of φ.
- **Least pull** to drag a body of weight W on a horizontal plane: P_{min} = **W sin φ**, applied at **φ to the horizontal**. Up an incline α: $P_{min} = W\sin(\alpha + \varphi)$, applied at φ to the plane.
- **Ladder** problems: reactions at the wall and the floor; with a smooth wall, the floor friction alone resists slipping.
- **Belt/rope friction**: $T_1/T_2 = e^{\mu\theta}$, with θ (angle of lap) in **radians**. For a **V-belt or rope** with groove half-angle β: $T_1/T_2 = e^{\mu\theta\,\mathrm{cosec}\,\beta}$, so the grip is greater. Power = $(T_1 - T_2)v$. Centrifugal tension $T_c = mv^2$. Maximum power is transmitted when $\mathbf{T_c = T_{max}/3}$, i.e. $v = \sqrt{(T_{max}/3m)}$.

Centroid & moment of inertia

Shape	Centroid (from base / diameter)	I about centroidal axis	I about base / diameter
Rectangle b × d	d/2	**bd³/12**	**bd³/3**
Triangle b × h	**h/3** from base (2h/3 from apex)	**bh³/36**	**bh³/12** (base); bh³/4 (through apex ∥ base)
Circle, dia d	centre	**πd⁴/64**; polar J = **πd⁴/32**	5πd⁴/64 (tangent)
Semicircle, radius r	**4r/3π** (≈ 0.424r)	**0.11r⁴** (≈ 0.1098r⁴)	**πr⁴/8** = πd⁴/128
Quarter circle	4r/3π from each straight edge	0.055r⁴	πr⁴/16
Hollow circle D, d	centre	π(D⁴ − d⁴)/64	—
Semicircular arc (wire)	**2r/π**	—	—
Solid cone / hemisphere	**h/4** / **3r/8** from base	—	—
Hollow (thin) hemisphere / cone shell	**r/2** / h/3	—	—

- **Parallel axis theorem**: $I = I_G + Ah^2$ (any axis parallel to a centroidal one). **Perpendicular axis theorem**: $I_z = I_x + I_y$, valid for **plane laminae** only.
- **Radius of gyration** $k = \sqrt{(I/A)}$. Rectangle: $k = d/\sqrt{12}$ = **0.289d**; circle: k = **d/4**.
- **Mass MI**: thin rod about its centre **ML²/12**, about one end **ML²/3**; disc/solid cylinder about its axis **MR²/2**; thin ring **MR²**; solid sphere **2MR²/5**; thin spherical shell **2MR²/3**.
- The **product of inertia** is **zero** if either axis is an **axis of symmetry**. Principal axes are the axes about which $I_{xy} = 0$.

Nandyal Banking Coaching Center

Dynamics

- **Equations of motion**: $v = u + at$; $s = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2as$; distance in the n-th second = $u + a(2n - 1)/2$.
- **Projectile** (speed u, angle θ, level ground): time of flight **$T = 2u \sin\theta/g$**; maximum height **$H = u^2 \sin^2\theta/2g$**; range **$R = u^2 \sin 2\theta/g$**. R is maximum at **$\theta = 45°$**, where $R_{max} = u^2/g = 4H$. **Complementary angles** (θ, 90° − θ) give the **same range**. The path is a **parabola**, and the velocity at the top is u cos θ (horizontal).
- On an inclined plane of angle α, the maximum range up the plane is $u^2/[g(1 + \sin\alpha)]$, reached at $\theta = 45° + \alpha/2$ from the horizontal.
- **D'Alembert's principle**: adding the **inertia force (−ma)** turns a dynamic problem into one of **static equilibrium**: ΣF − ma = 0. Lift: R = m(g + a) going up, m(g − a) going down, and **zero in free fall**.
- **Work–energy principle**: net work = **change in KE**. **Impulse–momentum**: FΔt = m(v − u). Impulse has units of **N·s**. Momentum is conserved when no external impulse acts.
- **Circular motion**: centripetal acceleration $v^2/r = \omega^2 r$. **Banking**: $\tan\theta = v^2/(rg)$. Simple pendulum: $T = 2\pi\sqrt{(L/g)}$. The **seconds pendulum** (T = 2 s) has L ≈ **0.994 m**.
- **Rolling without slipping**: KE = $\frac{1}{2}mv^2(1 + k^2/r^2)$. Acceleration down an incline = $g \sin\alpha/(1 + k^2/r^2)$: **solid sphere 5/7**, **disc/solid cylinder 2/3**, **ring 1/2** of g sin α. The solid sphere reaches the bottom **first**.
- **Coefficient of restitution** $e = (v_2 - v_1)/(u_1 - u_2)$ = relative velocity of separation ÷ relative velocity of approach. **Perfectly elastic: e = 1** (KE conserved). **Perfectly plastic: e = 0** (the bodies move together; maximum KE loss). 0 < e < 1 for real bodies.
- Loss of KE in direct impact = **$m_1m_2(1 - e^2)(u_1 - u_2)^2/[2(m_1 + m_2)]$**.
- A ball dropped from height h rebounds to **e^2h**. $e = \sqrt{(h_{rebound}/h_{drop})}$. The n-th rebound height is $e^{2n}h$. In an elastic collision between equal masses, the two bodies **exchange velocities**.

Simple machines

- **Mechanical advantage** MA = W/P (load ÷ effort). **Velocity ratio** VR = distance moved by the effort ÷ distance moved by the load. **Efficiency** η = MA/VR. In an **ideal machine** η = 1 and MA = VR.
- **Law of machine**: P = mW + C. Maximum MA = **1/m**; maximum efficiency = **1/(m × VR)**.
- **Reversible machine**: η > **50%**, so the load comes down when the effort is removed. **Self-locking (irreversible)**: η < **50%**, e.g. screw jacks and worm reduction gears.
- **Screw jack**: VR = 2πL/p (L = handle length, p = lead). Self-locking if **φ > α** (friction angle > helix angle). Maximum efficiency = **$(1 - \sin\varphi)/(1 + \sin\varphi)$**.

Machine	Velocity ratio
Simple wheel and axle	D/d
Differential wheel and axle	$2D/(d_1 - d_2)$
Weston differential pulley block	**2D/(D − d)**
1st / 2nd / 3rd system of pulleys (n pulleys)	**2^n / n / $2^n - 1$**
Simple screw jack	2πL/p
Worm and worm wheel	(effort wheel dia × teeth on worm wheel) / (load drum dia × number of worm starts)

KEY FORMULAE

- $R^2 = P^2 + Q^2 + 2PQ\cos\theta$; Lami: $P/\sin\alpha = Q/\sin\beta = R/\sin\gamma$
- $\mu = \tan\varphi = \tan$(angle of repose); $T_1/T_2 = e^{\mu\theta}$ (flat), $e^{\mu\theta\ \mathrm{cosec}\ \beta}$ (V)
- $R = u^2 \sin 2\theta/g$; $H = u^2 \sin^2\theta/2g$; $T = 2u \sin\theta/g$
- $e = (v_2 - v_1)/(u_1 - u_2)$; rebound height = e^2h
- $\eta = MA/VR$; $P = mW + C$; $\eta_{max} = 1/(m \cdot VR)$

MEMORY TRICK

"Half or less, it'll rest": if η ≤ 50%, the load **stays put** when the effort is removed (self-locking). Pulley systems: **"Double, Number, Double-minus-one"** = 2^n, n, $2^n - 1$. Rolling race: **Sphere beats Disc beats Ring** (5/7 > 2/3 > 1/2).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- A machine is self-locking if its efficiency is → **less than 50%**.
- Angle of repose vs angle of friction → **equal**.
- T_1/T_2 for a flat belt → **$e^{\mu\theta}$** (θ in radians).
- For maximum horizontal range a projectile is fired at → **45°**.
- Coefficient of restitution for a perfectly plastic impact → **0**.
- MI of a triangle about its base → **$bh^3/12$**; about its centroidal axis → **$bh^3/36$**.

1.2 Stress, Strain & Elastic Constants ★★★

- **Stress** = internal resistance per unit area, measured in **N/m^2 = Pa**; 1 MPa = 1 N/mm^2. **Strain** is dimensionless.
- **Hooke's law**: stress ∝ strain up to the **limit of proportionality**, so σ = Eε. **E** = Young's modulus, a measure of **stiffness**. **Steel E ≈ 200–210 GPa**, cast iron ≈ 100–120, copper ≈ 110–120, brass ≈ 100, **aluminium ≈ 70**, concrete ≈ 25–30, rubber ≈ 0.01–0.1 GPa.
- **Poisson's ratio ν** = −lateral strain/longitudinal strain. Theoretical range for isotropic materials: **$-1 < \nu \le 0.5$**; practical range 0 to 0.5. **Steel ≈ 0.25–0.30** (0.3 usual), CI ≈ 0.23–0.27, concrete ≈ 0.1–0.2, **cork ≈ 0**, **rubber ≈ 0.5** (incompressible). ν = 0.5 means **no volume change**.
- **Shear modulus G** = τ/γ (modulus of rigidity); for steel ≈ **80 GPa** (0.4E). **Bulk modulus K** = p/ε_v; K → ∞ as ν → 0.5.
- An isotropic, homogeneous material has **2 independent elastic constants**. An orthotropic material has **9** and a general anisotropic material **21**.
- **Volumetric strain**: for a bar in axial load, $\varepsilon_v = \varepsilon(1 - 2\nu)$. Under triaxial stress, $\varepsilon_v = (\sigma_x + \sigma_y + \sigma_z)(1 - 2\nu)/E$. Rectangular block: $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$. Cylinder: $\varepsilon_v = \varepsilon_L + 2\varepsilon_d$. Sphere: $\varepsilon_v = 3\varepsilon_d$.

KEY FORMULAE — ELASTIC CONSTANTS

- $E = 2G(1 + \nu) = 3K(1 - 2\nu)$
- $E = 9KG/(3K + G)$; $\nu = (3K - 2G)/(6K + 2G)$
- With ν = 0.25: E = 2.5G, K = 2E/3. With ν = 0.5: E = 3G and K = ∞. With ν = 0: E = 2G = 3K.
- Generalised Hooke: $\varepsilon_x = [\sigma_x - \nu(\sigma_y + \sigma_z)]/E$

Stress–strain curve (mild steel, tension)

- Sequence: **limit of proportionality → elastic limit → upper yield point → lower yield point → strain hardening → ultimate**

Nandyal Banking Coaching Center

tensile stress → fracture.

- **Necking** starts at the **ultimate stress** point. Engineering stress at fracture is **less than** the UTS, but the **true stress at fracture is higher** because the area has reduced. The true stress–strain curve keeps rising to fracture.
- The **lower yield point** is taken as the design yield stress because it is more stable; the **upper yield** value depends on the test speed and specimen. MS (Fe 250): $\sigma_y \approx$ **250 MPa**, UTS ≈ 410 MPa, elongation ≈ **23–25%**.
- **Proof stress**: non-ferrous metals (Al, Cu), alloy/high-tensile steels and other materials with **no clear yield point** use the **0.2% offset** method (offset strain 0.002).
- **Brittle** materials (CI, concrete, glass) have no yield point, fracture at small strain (< 5% elongation), and are much stronger in compression than in tension.
- **True strain** $\varepsilon_T = \ln(1+\varepsilon)$; **true stress** $\sigma_T = \sigma(1+\varepsilon)$, assuming constant volume. Gauge length of a proportional test piece = $\mathbf{5.65\sqrt{A_0}}$ (= 5d for round bars).
- **Factor of safety** = failure stress ÷ working stress. It is based on **yield** for ductile materials and on **ultimate** for brittle ones.

Axial deformation

Case	Elongation δ
Uniform bar	**PL/AE**
Stepped bar	$(P/E)\,\Sigma\, L_i/A_i$
Tapered circular bar ($d_1 \to d_2$)	$\mathbf{4PL/(\pi E d_1 d_2)}$
Tapered flat plate (width $b_1 \to b_2$, thickness t)	$PL\ln(b_1/b_2)/[Et(b_1 - b_2)]$
Prismatic bar hanging under its own weight (γ = weight density)	$\mathbf{\gamma L^2/2E}$ = WL/2AE (half that of the same load applied at the end)
Conical bar under its own weight	$\mathbf{\gamma L^2/6E}$ (one-third of the prismatic bar)
Bar of uniform strength (self-weight + P)	Area $A_x = A_0\, e^{\gamma x/\sigma}$

Thermal stress & composite bars

- Free thermal expansion $\delta = \mathbf{\alpha L \Delta T}$, which causes **no stress**. If the bar is **fully restrained**, $\sigma = \mathbf{E\alpha\Delta T}$, independent of length and area. If supports yield by Δ: $\sigma = E(\alpha L\Delta T - \Delta)/L$.
- α steel ≈ $\mathbf{12 \times 10^{-6}}$ **/°C**; copper ≈ 17×10^{-6}; aluminium ≈ 23×10^{-6}; brass ≈ 19×10^{-6}.
- **Composite bar (parallel)**: both parts have the **same strain**. $P = \sigma_1A_1 + \sigma_2A_2$ and $\sigma_1/E_1 = \sigma_2/E_2$, so $\sigma_1 = m\sigma_2$, where **modular ratio** $\mathbf{m = E_1/E_2}$. The **stiffer material carries more stress**.
- **Composite bar heated** (e.g. steel rod inside a copper tube): the material with the **higher α (copper) goes into compression** and the lower-α one (steel) into tension. Force equilibrium: $\sigma_sA_s = \sigma_cA_c$; compatibility: $\sigma_s/E_s + \sigma_c/E_c = (\alpha_c - \alpha_s)\Delta T$.
- **Bars in series** carry the **same force** and their deformations add.

Strain energy & impact

- **Strain energy** (axial) $U = \tfrac{1}{2}P\delta = \mathbf{(\sigma^2/2E) \times V}$. In shear, $U = (\tau^2/2G) \times V$.
- **Resilience** = strain energy stored up to the elastic limit. **Proof resilience** = maximum strain energy stored at the elastic limit = $(\sigma_e^2/2E)V$. **Modulus of resilience** = proof resilience per unit volume = $\mathbf{\sigma_y^2/2E}$. Unit: $\mathbf{J/m^3 = N/m^2}$.
- **Modulus of toughness** = **total area** under the stress–strain curve up to fracture (energy per unit volume to fracture).

Type of loading	Maximum stress
Gradually applied P	$\sigma = P/A$
Suddenly applied P (h = 0)	$\boldsymbol{\sigma = 2P/A}$ (twice the gradual value)
Impact: P falls through height h	$\boldsymbol{\sigma = (P/A)[1 + \sqrt{(1 + 2AEh/(PL))}]}$
Impact with $h \gg \delta$	$\sigma \approx \sqrt{(2EPh/(AL))}$

Material properties (definitions)

Property	Meaning / note
Elasticity	Regains its original shape after the load is removed
Plasticity	Permanent deformation without fracture (used in forging and rolling)
Ductility	Can be drawn into **wire under tension**; measured by % elongation and % reduction in area. Most ductile: **gold**, then silver and platinum
Malleability	Can be rolled or hammered into **sheets under compression**. Most malleable: **gold**; lead is malleable but not ductile
Brittleness	Fractures with little deformation (CI, glass)
Toughness	Energy absorbed **up to fracture**. Measured by **Izod/Charpy** impact tests
Resilience	Energy absorbed within the **elastic** range (springs)
Stiffness	Resistance to deformation, measured by **E**
Hardness	Resistance to indentation, scratching and wear. Tests: **Brinell** (steel ball), **Vickers** (136° diamond pyramid), **Rockwell** (cone/ball, depth-based), Mohs (scratch), Shore scleroscope (rebound)
Creep	Slow permanent strain under **constant load over time**; significant at high temperature (> 0.4 T_m). Stages: primary (decreasing rate), **secondary (constant, minimum rate)**, tertiary (accelerating to rupture)
Fatigue	Failure under **cyclic/fluctuating** stress below the UTS (and often below yield). The **S–N curve** gives the endurance limit. Steel has an endurance limit at ~10^6–10^7 cycles; **Al has none**
Stress relaxation	Stress falls with time at **constant strain** (bolts, springs)
Bauschinger effect	After plastic deformation in tension, the yield strength in compression is **lowered**

MEMORY TRICK

"2-G, 3-K": $E = \mathbf{2G}(1+\nu) = \mathbf{3K}(1-2\nu)$, where the 2 goes with G and the 3 with K. **"Sudden doubles"**: a suddenly applied load gives **2×** the stress and 2× the deflection of the same load applied gradually.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Relation between E, G and ν → $\mathbf{E = 2G(1+\nu)}$.
- The maximum value of Poisson's ratio for an elastic material → **0.5**.
- Stress due to a suddenly applied load compared with a gradual one → **twice**.
- Elongation of a bar under its own weight → $\mathbf{\gamma L^2/2E}$ (= WL/2AE).
- Thermal stress in a fully restrained bar → $\mathbf{E\alpha\Delta T}$ (independent of L and A).
- Proof stress for Al/Cu is found by the → **0.2% offset method**.

Nandyal Banking Coaching Center

1.3 SFD, BMD & Bending Stresses ★★★

- **Shear force** at a section = algebraic sum of the vertical forces on **one side** of it. **Bending moment** = algebraic sum of the moments on one side. Sagging BM is +ve; hogging BM is −ve.
- **Relations**: **dM/dx = V**, so the **BM is maximum where the SF is zero or changes sign**. **dV/dx = −w** (load intensity). The change in BM between two sections = **area of the SFD** between them.
- **Degree of curves**: no load → SF constant, BM linear. Point load → SF jumps, BM changes slope (kink). **UDL** → SF linear, **BM parabolic (2nd degree)**. **UVL** → SF parabolic, **BM cubic**. A **pure couple** gives no change in SF but a **sudden jump in BM**.
- **Point of contraflexure** (inflection) = where the BM **changes sign** (M = 0 inside the span). It occurs in **overhanging, fixed and propped beams**. **Simply supported and cantilever beams under ordinary downward loads have none.**
- **Pure bending**: constant BM and **zero SF**, e.g. the middle part of a SS beam carrying two equal symmetric point loads (four-point bending).

Beam & load (span L)	Max SF	Max BM	Location
Cantilever, point load W at the free end	W	**WL**	fixed end
Cantilever, UDL w	wL	**wL²/2**	fixed end
Cantilever, UVL 0 at free end → w at fixed end	wL/2	**wL²/6**	fixed end
SS, central point load W	W/2	**WL/4**	mid-span
SS, eccentric W (a, b)	Wb/L (larger)	**Wab/L**	under the load
SS, UDL w	wL/2	**wL²/8**	mid-span
SS, UVL 0 → w (triangular)	wL/3 (at the larger end)	**wL²/(9√3)** ≈ 0.064wL²	**L/√3** from the zero end
SS, symmetric triangular load (peak w at centre)	wL/4	wL²/12	mid-span
SS, couple M at one end	M/L	M	at the couple
Fixed–fixed, UDL	wL/2	**wL²/12** (support, hogging); wL²/24 (centre)	ends; contraflexure at 0.211L from each end
Fixed–fixed, central W	W/2	**WL/8** (ends and centre)	contraflexure at L/4 from each end
Propped cantilever, UDL	5wL/8 (fixed end); prop R = **3wL/8**	**wL²/8** (fixed end); +9wL²/128 at 3L/8 from the prop	contraflexure at L/4 from the fixed end
Propped cantilever, central W	prop R = **5W/16**	**3WL/16** (fixed end); +5WL/32 under the load	—

Bending stress (theory of simple bending)

- **Assumptions**: plane sections remain plane (Bernoulli–Euler); material homogeneous, isotropic and linearly elastic; E the same in tension and compression; radius of curvature large compared with the depth; pure bending.
- **Flexure formula**: **M/I = σ/y = E/R**. Stress varies **linearly** with distance from the **neutral axis (NA)**. It is **zero at the NA** and maximum at the extreme fibres.
- For pure bending the NA passes through the **centroid**.
- **Section modulus** Z = I/y_{max}, so σ_{max} = M/Z. Z measures **bending strength**. **Flexural rigidity** = EI; **moment of resistance** = σZ.

Section	Section modulus Z
Rectangle b × d	**bd²/6**
Square side a (edge horizontal)	**a³/6**
Square with its diagonal horizontal	**a³/(6√2)** ≈ 0.118a³ (weaker; ratio 1/√2)
Solid circle, dia d	**πd³/32**
Hollow circle D, d	π(D⁴ − d⁴)/(32D)
Triangle (b, h), NA at h/3	bh²/24 (to apex) ; bh²/12 (to base)

- For equal area, Z of a **square > circle** (ratio ≈ 1.18). An **I-section** is the most economical in bending because its material is far from the NA. The flanges resist most of the **BM**; the web resists most of the **shear**.
- Strongest rectangular beam cut from a round log of diameter D: **b = D/√3**, d = √(2/3)D, so **d/b = √2**. Stiffest beam (max I): b = D/2, d = (√3/2)D, so **d/b = √3**.
- Unsymmetrical (T, channel) sections in CI are used with the **flange on the tension side**, because CI is weak in tension.

Shear stress distribution

- **τ = VAȳ/(Ib)** (V = SF; Aȳ = first moment of area above the level; b = width at that level).
- It is **zero at the extreme fibres**. It is maximum at the NA for rectangles and circles, but **not for a triangle**.

Section	τ_{max}/τ_{avg}	Location of τ_{max}	Shape
Rectangle	**1.5**	NA	parabolic
Solid circle	**4/3** (1.33)	NA	parabolic
Triangle	**1.5** (at the NA it is 4/3)	**h/2** from the apex	parabolic
Thin-walled circular tube	**2**	NA	—
Square with diagonal horizontal	9/8 (1.125)	d/8 above and below the NA	—
I-section	—	NA (web)	web parabolic; sudden jump at the web–flange junction

Beams of uniform strength & composite beams

- **Beam of uniform strength**: σ_{max} is the same at every section (M/Z constant), which saves material. Examples: leaf springs, tapered cantilevers.
- SS beam with a central point load: at **constant depth**, the **width varies linearly** (rhombus/diamond plan); at **constant width**, the **depth varies as √x** (parabolic).

Nandyal Banking Coaching Center

- Cantilever with an end load: at constant depth, the width varies linearly (triangular plan). Cantilever with a UDL: at constant depth, the width $\propto x^2$.
- **Composite (flitched) beams**: strain is the same at the interface. Use the **equivalent section**: the steel width is multiplied by the **modular ratio $m = E_s/E_w$**. At the same level, $\sigma_s = m\sigma_w$. Total M = $M_1 + M_2$, with both parts sharing the same radius of curvature.

KEY FORMULAE

- $dM/dx = V$; $dV/dx = -w$
- $M/I = \sigma/y = E/R$; $\sigma = M/Z$
- $\tau = VA\bar{y}/(Ib)$; τ_{max} (rect) = $3V/(2bd)$; τ_{max} (circle) = $4V/(3A) = 16V/(3\pi d^2)$

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Maximum BM in a SS beam with UDL → **$wL^2/8$** at mid-span.
- BM diagram shape for a UDL → **parabola**; for a UVL → **cubic**.
- Maximum shear stress in a rectangular section relative to the average → **1.5 times**.
- Bending stress at the neutral axis → **zero**.
- Points of contraflexure occur in → **overhanging/fixed/propped beams** (not in a SS beam or a cantilever).
- Section modulus of a circular section → **$\pi d^3/32$**.

1.4 Deflection of Beams ★★★

- **Differential equation of the elastic curve**: **$EI\,d^2y/dx^2 = M$**. Slope = dy/dx; $EI\,d^3y/dx^3 = V$; $EI\,d^4y/dx^4 = -w$.
- **Methods**: (1) **double integration**; (2) **Macaulay's method**, which uses a single BM equation with bracket terms and suits **several point loads**; (3) **moment-area method (Mohr's theorems)**, best for **cantilevers** and symmetric beams; (4) **conjugate beam method**; (5) strain energy (Castigliano); (6) the unit-load (virtual work) method.
- **Mohr's theorem I**: the change in slope between A and B = **area of the M/EI diagram** between A and B. **Mohr's theorem II**: the deflection of B relative to the tangent at A = **moment of the M/EI area** between A and B, taken about B.
- **Conjugate beam**: it is loaded with M/EI. The **SF of the conjugate beam = slope** and the **BM of the conjugate beam = deflection** of the real beam. Support conversions: **fixed ↔ free**; simple (end) support stays simple; internal hinge ↔ internal roller support.
- Deflection $\propto$ **L^3** (point load) or **L^4** (UDL), and $\propto$ **$1/EI$**. Doubling the span under a UDL increases the deflection **16 times**. Doubling the depth of a rectangular beam reduces the deflection to **1/8**.

Beam & load (span L)	Max slope	Max deflection
Cantilever, end load W	**$WL^2/2EI$**	**$WL^3/3EI$**
Cantilever, UDL w (total wL)	**$wL^3/6EI$**	**$wL^4/8EI$**
Cantilever, couple M at the free end	ML/EI	**$ML^2/2EI$** (circular arc)
Cantilever, UVL (zero at free end, w at fixed end)	$wL^3/24EI$	**$wL^4/30EI$**
Cantilever, load W at distance a from the fixed end	$Wa^2/2EI$	at free end: $Wa^3/3EI + (Wa^2/2EI)(L - a)$
SS, central W	**$WL^2/16EI$**	**$WL^3/48EI$**
SS, UDL w	**$wL^3/24EI$**	**$5wL^4/384EI$**
SS, eccentric W (a, b)	—	under the load: **$Wa^2b^2/(3EIL)$**
SS, equal end couples M (pure bending)	ML/2EI	**$ML^2/8EI$**
Fixed–fixed, central W	0 at the ends	**$WL^3/192EI$** (¼ of SS)
Fixed–fixed, UDL	0 at the ends	**$wL^4/384EI$** (⅕ of SS)
Propped cantilever, UDL	—	**$wL^4/185EI$** at 0.4215L from the prop
Propped cantilever, central W	—	$WL^3/(48\sqrt{5}\,EI) \approx 0.00932WL^3/EI$ at 0.447L from the prop; $7WL^3/768EI$ under the load

- **Propped cantilever** (one degree indeterminate). Find the prop reaction by equating the cantilever deflection to the upward deflection due to the prop: $wL^4/8EI = RL^3/3EI$ gives **$R = 3wL/8$**. For a central point load, R = **5W/16**.
- **Fixed beam** (two degrees indeterminate). Fixed-end moments: UDL **$wL^2/12$**; central W **$WL/8$**; eccentric W: M_A = **Wab^2/L^2**, M_B = **Wa^2b/L^2**. Sinking of one support by δ induces end moments **$6EI\delta/L^2$**. A moment applied at one end carries **half** over to the far fixed end (carry-over factor ½).
- Fixing the ends reduces the maximum BM ($wL^2/8 \rightarrow wL^2/12$) and the deflection (to ⅛ for a UDL and ¼ for a central load).
- A cantilever end load deflects **16 times** as much as the same load at the centre of a SS beam of equal span: $(WL^3/3)/(WL^3/48) = 16$.

MEMORY TRICK

Cantilever **"2-3, 6-8"**: point load → $WL^2/2$, $WL^3/3$; UDL → $wL^3/6$, $wL^4/8$. SS **"16-48, 24-5/384"**. Fixed beam deflections are SS ÷ 4 (point load: 48 → 192) and SS ÷ 5 (UDL: 5/384 → 1/384).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Maximum deflection of a SS beam with a UDL → **$5wL^4/384EI$**.
- Deflection at the free end of a cantilever with an end load → **$WL^3/3EI$**.
- Ratio of the maximum deflections of a SS beam and a fixed beam under a UDL → **5 : 1**.
- In the conjugate beam method, a fixed end of the real beam becomes → **a free end**.
- Prop reaction of a propped cantilever with a UDL → **3wL/8**.
- Best method for a beam with several point loads → **Macaulay's method**.

1.5 Torsion of Shafts & Springs ★★★

- **Torsion equation**: **$T/J = \tau/r = G\theta/L$** (θ in radians). Shear stress varies **linearly** from **zero at the centre** to a maximum at the **outer surface**.
- **Assumptions**: circular sections **remain plane** (no warping) and radii stay straight; material homogeneous and elastic; twist uniform along the length. Non-circular sections **warp**.

Nandyal Banking Coaching Center

- **Polar MI**: solid $J = \pi d^4/32$; hollow $J = \pi(D^4 - d^4)/32$. **Polar section modulus** $Z_p = J/r$: solid $\pi d^3/16$; hollow $\pi(D^4 - d^4)/(16D)$.
- Strength of a solid shaft: $\mathbf{T = (\pi/16)\tau d^3}$. Torque capacity $\propto d^3$; angle of twist $\propto 1/d^4$.
- **Torsional rigidity** = GJ. **Torsional stiffness** $k_t = T/\theta = \mathbf{GJ/L}$.
- **Power**: $\mathbf{P = 2\pi NT/60}$ W (T in N·m, N in rpm). Where the torque fluctuates, use T_{max} for design.
- **Hollow vs solid (same material)**: a hollow shaft is **stronger and stiffer for the same weight**, because the material near the axis carries little stress. With $k = d_i/D$:
  - Same **weight** (same area): $T_{hollow}/T_{solid} = (1 + k^2)/\sqrt{(1 - k^2)}$. For k = 0.5 this is **≈ 1.44**.
  - Same **outer diameter**: $T_h/T_s = 1 - k^4$. For k = 0.5 the hollow shaft has **15/16** of the strength with **3/4** of the weight.
  - Same **strength**: the hollow shaft is lighter. For k = 0.5 it needs D = 1.022 × solid dia and weighs about 78% as much.
- **Shafts in series** (stepped or two materials): the **same torque** T in each part; $\theta = \theta_1 + \theta_2$; $1/k = 1/k_1 + 1/k_2$.
- **Shafts in parallel** (composite shaft, or a torque applied between two fixed ends): the **same angle of twist**; $T = T_1 + T_2$; $k = k_1 + k_2$.
- Torsional strain energy: solid shaft $U = \mathbf{(\tau^2/4G) \times V}$; hollow shaft $U = (\tau^2/4G)[(D^2 + d^2)/D^2]V$.
- In **pure torsion**, the principal stresses are **±τ at 45°** to the axis. A **brittle** shaft (chalk, CI) fails on a **45° helical** surface (tension). A **ductile** shaft fails on a **plane ⊥ axis** (shear).

Combined bending & torsion

- Bending stress $\sigma = 32M/(\pi d^3)$; torsional shear stress $\tau = 16T/(\pi d^3)$.
- **Equivalent twisting moment** $T_e = \mathbf{\sqrt{(M^2 + T^2)}}$, used with the **maximum shear stress theory** (ductile): $\tau_{max} = 16T_e/(\pi d^3)$.
- **Equivalent bending moment** $M_e = \mathbf{\frac{1}{2}[M + \sqrt{(M^2 + T^2)}]} = \frac{1}{2}(M + T_e)$, used with the **maximum principal stress theory** (brittle): $\sigma_{max} = 32M_e/(\pi d^3)$.
- The ASME code adds shock and fatigue factors: $T_e = \sqrt{[(K_m M)^2 + (K_t T)^2]}$.

Springs

- **Close-coiled helical spring** under an axial load W: the wire is mainly in **torsion** (helix angle < 10°). The **open-coiled** spring has combined **bending and torsion**.
- Spring index **C = D/d** (mean coil dia ÷ wire dia), usually **4–12** (6–9 preferred). A small C gives high curvature stress; a large C makes the spring prone to buckling and tangling.
- **Wahl's factor** $K_w = \mathbf{(4C - 1)/(4C - 4) + 0.615/C}$. It accounts for **direct shear + curvature**, and the maximum stress is at the **inside** of the coil. The shear stress factor alone is $K_s = 1 + 1/(2C) = 1 + 0.5/C$.
- **Stiffness** $k = W/\delta = \mathbf{Gd^4/(8D^3n)}$. Cutting a spring into two halves **doubles** the stiffness of each half. Spring energy $U = \frac{1}{2}W\delta$.
- **Springs in series** carry the **same load**: $1/k = 1/k_1 + 1/k_2$, and the deflections add. **Springs in parallel** share the **same deflection**: $k = k_1 + k_2$. Concentric (nested) springs act in parallel.
- **Solid length** = n′d (n′ = total coils). Squared-and-ground ends add **2 inactive coils** (n′ = n + 2).
- **Leaf (laminated) spring**: it is a **beam of uniform strength**, with n leaves of width b and thickness t. The leaves are pre-stressed by **nipping** (giving them different radii of curvature) so that the full-length leaves are not over-stressed. The **longest (master) leaf** has eyes for the shackle. Leaf springs are used in automobile **suspensions** (rear axles of trucks).

Spring	Maximum stress	Deflection
Close-coiled helical (W, R = D/2, n active coils)	$\tau = \mathbf{16WR/(\pi d^3) = 8WD/(\pi d^3)}$ (× K_w)	$\mathbf{\delta = 64WR^3n/(Gd^4) = 8WD^3n/(Gd^4)}$
Semi-elliptic leaf (span L, central load W)	$\mathbf{\sigma = 3WL/(2nbt^2)}$	$\mathbf{\delta = 3WL^3/(8Enbt^3)}$
Quarter-elliptic leaf (cantilever L, end load W)	$\mathbf{\sigma = 6WL/(nbt^2)}$	$\mathbf{\delta = 6WL^3/(Enbt^3)}$

KEY FORMULAE

- $T/J = \tau/r = G\theta/L$; $T = (\pi/16)\tau d^3$; $P = 2\pi NT/60$
- $T_e = \sqrt{(M^2 + T^2)}$; $M_e = \frac{1}{2}[M + \sqrt{(M^2 + T^2)}]$
- $\delta = 8WD^3n/(Gd^4)$; $k = Gd^4/(8D^3n)$; $K_w = (4C - 1)/(4C - 4) + 0.615/C$

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Shear stress in a solid circular shaft is maximum at → **the outer surface** (zero at the axis).
- Equivalent twisting moment for combined M and T → $\mathbf{\sqrt{(M^2 + T^2)}}$.
- If the wire diameter of a helical spring is doubled, its stiffness becomes → **16 times** ($k \propto d^4$).
- Stiffness of two equal springs in series → **k/2**; in parallel → **2k**.
- Wahl's factor accounts for → **direct shear and curvature effects**.
- A shaft running at N rpm transmits power → $\mathbf{2\pi NT/60}$.

1.6 Columns & Struts ★★★

- **Column** = a vertical compression member; **strut** = a compression member in any direction (e.g. in trusses). A **short column** fails by **crushing**; a **long column** fails by **buckling**; an intermediate column fails by a combination of the two.
- **Slenderness ratio** $\lambda = L_e/k_{min}$, where $k_{min} = \sqrt{(I_{min}/A)}$. Buckling occurs about the axis of **least I**.
- **Euler's formula**: $\mathbf{P_{cr} = \pi^2 E I_{min}/L_e^2}$, so the Euler stress is $\sigma_{cr} = \pi^2 E/\lambda^2$. It depends only on E and the geometry, **not on the material strength**.
- **Euler assumptions**: the column is initially **perfectly straight**; the load is **exactly axial**; the material is homogeneous and elastic (below the proportional limit); the column is long, its self-weight is ignored, and it fails by buckling alone.
- **Limitation**: Euler is valid only for long columns. For mild steel, the limiting slenderness is $\lambda = \pi\sqrt{(E/\sigma_y)} \approx$ **89** (E = 200 GPa, σ_y = 250 MPa); **~80** is the textbook value. As $\lambda \to 0$, Euler gives an infinite load, which is absurd for short columns.

End conditions	Effective length L_e	P_{cr}	Relative to both-hinged
Both ends hinged	**L**	π^2EI/L^2	1
One end fixed, other free	**2L**	$\pi^2EI/(4L^2)$	**1/4**
Both ends fixed	**L/2**	$4\pi^2EI/L^2$	**4**

Nandyal Banking Coaching Center

One end fixed, other hinged	**L/√2** (≈ 0.7L)	$2\pi^2EI/L^2$	**2**

- Ratio of the crippling loads in order free : hinged : fixed–hinged : fixed = **1 : 4 : 8 : 16** (the same as 1/4 : 1 : 2 : 4).
- IS 800 practical effective lengths (because ends are not perfectly fixed): both fixed **0.65L**; fixed–hinged **0.8L**; both hinged **1.0L**; fixed–free **2.0L**.
- **Rankine–Gordon formula** (empirical, for all lengths): $\mathbf{1/P_R = 1/P_c + 1/P_E}$, giving $\mathbf{P = \sigma_c A/[1 + a(L_e/k)^2]}$, where $\mathbf{a = \sigma_c/(\pi^2 E)}$ is the Rankine constant. For short columns it gives P_c (crushing); for long ones it gives P_E.

Material	σ_c (MPa)	Rankine constant a
Mild steel	**320**	**1/7500**
Wrought iron	250	1/9000
Cast iron	550	**1/1600**
Timber	40	1/750

- Other formulae: **Johnson's parabolic** (intermediate columns); **straight-line** formula; the **Perry–Robertson** formula, which allows for initial curvature and is the basis of IS 800; the **secant formula** for eccentric loads.
- **Eccentric loading (short column)**: σ = P/A ± M/Z = (P/A)(1 ± 6e/b) for a rectangle. There is **no tension** anywhere if **e ≤ b/6**.
- **Secant formula** (long column, eccentricity e): $\sigma_{max} = (P/A)[1 + (ec/k^2)\sec((L_e/2)\sqrt{(P/EI)})]$.

Section	Core (kernel) — no tension if the load lies within
Rectangle b × d	**Rhombus** with diagonals **b/3 and d/3**; e ≤ b/6 on each axis (**middle-third rule**)
Square side a	Rhombus with diagonal a/3
Solid circle, dia d	Concentric circle of **diameter d/4** (radius d/8, i.e. e ≤ d/8) — **middle-quarter rule**
Hollow circle D, d	Circle of radius $(D^2 + d^2)/(8D)$

MEMORY TRICK

Effective length **"Free doubles, Fixed halves, Fixed-Hinge root-halves"**: 2L, L/2, L/√2. Since P ∝ $1/L_e^2$, the loads go **¼, 4, 2**. Core: **rectangle = one-third (middle third), circle = one-quarter (d/4)**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Effective length of a column fixed at both ends → **L/2**.
- Crippling load of a column fixed at both ends compared with both ends hinged → **4 times**.
- Euler's formula is valid for → **long columns** (high slenderness ratio).
- Rankine constant for mild steel → **1/7500**.
- Core of a circular section of diameter d → **a circle of diameter d/4**.
- Maximum eccentricity for no tension in a rectangular section → **b/6**.

1.7 Principal Stresses, Mohr's Circle & Theories of Failure ★★★

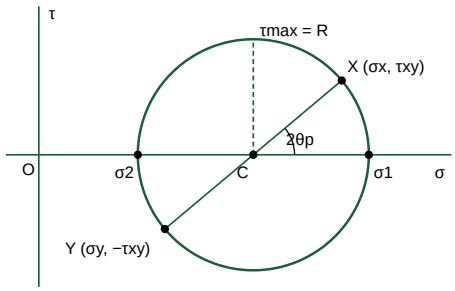
- **Principal planes** are planes with **zero shear stress**. The normal stresses on them are the **principal stresses** (maximum and minimum normal stress). In 2-D the two principal planes are **90° apart**.
- The planes of **maximum shear** are at **45°** to the principal planes. The normal stress on them is $\mathbf{(\sigma_x + \sigma_y)/2}$, which is not zero in general.
- **Invariant**: $\sigma_x + \sigma_y = \sigma_1 + \sigma_2 = \sigma_{x'} + \sigma_{y'}$ (the first stress invariant).
- **Uniaxial σ**: $\tau_{max} = \sigma/2$ on 45° planes. This is why ductile specimens show 45° (Lüders) lines in tension.
- **Pure shear τ**: $\sigma_1 = +\tau$, $\sigma_2 = -\tau$ on planes at **45°**.
- **Equal biaxial or hydrostatic** stress: every plane is a principal plane and **τ = 0** on all planes (Mohr's circle becomes a **point**).
- **Absolute maximum shear (3-D)** = $\mathbf{(\sigma_{max} - \sigma_{min})/2}$. If σ_1 and σ_2 have the same sign and $\sigma_3 = 0$, then $\tau_{abs} = \sigma_1/2$, which is larger than the in-plane $(\sigma_1 - \sigma_2)/2$.

KEY FORMULAE

- $\sigma_n = (\sigma_x + \sigma_y)/2 + [(\sigma_x - \sigma_y)/2]\cos 2\theta + \tau_{xy}\sin 2\theta$
- $\tau_\theta = -[(\sigma_x - \sigma_y)/2]\sin 2\theta + \tau_{xy}\cos 2\theta$
- $\sigma_{1,2} = (\sigma_x + \sigma_y)/2 \pm \sqrt{\{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2\}}$
- $\tan 2\theta_p = 2\tau_{xy}/(\sigma_x - \sigma_y)$; $\tau_{max} = (\sigma_1 - \sigma_2)/2 = \sqrt{\{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2\}}$
- Principal strains: same form, with τ_{xy} replaced by $\gamma_{xy}/2$

Mohr's circle facts

- **Centre** at $((\sigma_x + \sigma_y)/2, 0)$; **radius** = $\tau_{max} = \sqrt{\{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2\}}$.
- An angle θ on the element appears as **2θ** on the circle. The points representing the x and y faces are **diametrically opposite**.
- Where the circle cuts the σ-axis gives the principal stresses; the top and bottom points give $\pm\tau_{max}$.
- The circle is **centred at the origin** for **pure shear**, **passes through the origin** for **uniaxial stress**, and **reduces to a point** for equal biaxial or hydrostatic stress.
- Mohr's circle for strain has its centre at $(\varepsilon_x + \varepsilon_y)/2$ and a vertical axis of **γ/2**.



Mohr's circle: centre C = (σx + σy)/2, radius R = τmax; X and Y are diametrically opposite; the principal stresses lie on the σ-axis at C ± R.

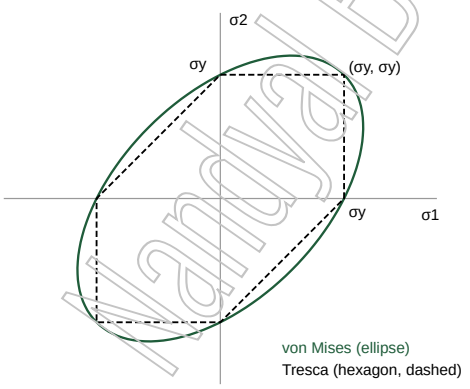
Theories of failure

Theory	Failure criterion ($\sigma_3 = 0$)	Suitable for	2-D envelope	Shear yield τ_y (pure shear)

Nandyal Banking Coaching Center

Max principal stress (Rankine)	$\sigma_1 = \sigma_y$ (σ_{ut})	**Brittle** materials	**Square**	$\tau_y = \sigma_y$
Max shear stress (Guest–Tresca)	$\sigma_1 - \sigma_2 = \sigma_y$ (max − min = σ_y)	**Ductile**; most **conservative/safe**	**Hexagon**	**$\tau_y = 0.5\sigma_y$**
Max principal strain (St. Venant)	$\sigma_1 - \nu\sigma_2 = \sigma_y$	Not generally recommended	**Rhombus**	$\sigma_y/(1+\nu) \approx 0.77\sigma_y$
Total strain energy (Haigh)	$\sigma_1^2 + \sigma_2^2 - 2\nu\sigma_1\sigma_2 = \sigma_y^2$	Ductile	Ellipse	$\sigma_y/\sqrt{(2(1+\nu))} \approx 0.62\sigma_y$
Distortion energy (von Mises–Hencky; max octahedral shear)	**$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2$**	**Ductile**; **best fit** with experiments	**Ellipse**	**$\tau_y = \sigma_y/\sqrt{3} = 0.577\sigma_y$**

- The Tresca **hexagon is inscribed in** the von Mises **ellipse**. They meet at six points (uniaxial and equal-biaxial states). The largest difference (≈ 15.5%) is in **pure shear**.
- The von Mises ellipse has its major axis along **$\sigma_1 = \sigma_2$**: semi-major axis **$\sqrt{2}\sigma_y$**, semi-minor axis $\sqrt{(2/3)}\sigma_y$.
- **Hydrostatic stress causes no yielding** under Tresca or von Mises, because only shear or distortion produces yielding. The **maximum strain energy theory wrongly predicts** failure under hydrostatic pressure.
- Von Mises equivalent stress in 3-D: $\sigma_e = \sqrt{[\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]]}$. With combined σ and τ (shafts): σ_e = **$\sqrt{(\sigma^2 + 3\tau^2)}$** (von Mises) and $\sigma_e = \sqrt{(\sigma^2 + 4\tau^2)}$ (Tresca).
- For brittle materials with unequal tensile and compressive strength, use the **Coulomb–Mohr** (internal friction) or modified Mohr theory.
- All theories agree for **uniaxial stress**. When σ_1 and σ_2 are like and equal, the **Rankine, Tresca and von Mises** theories agree (failure at $\sigma = \sigma_y$); the strain-based theories do not. Tresca and Rankine coincide in the 1st and 3rd quadrants.



Yield envelopes for plane stress: the Tresca hexagon is inscribed in the von Mises ellipse; they touch at (±σy, 0), (0, ±σy) and ±(σy, σy).

MEMORY TRICK

"Brittle → Principal; Ductile → Shear/Distortion". Envelope shapes by theory: **Rankine Square, Strain Rhombus, Tresca Hexagon, Energy Ellipse**. Shear yield: **Tresca half (0.5), von Mises one-over-root-three (0.577)**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Shear stress on principal planes → **zero**.
- Planes of maximum shear are inclined to the principal planes at → **45°**.
- Radius of Mohr's circle represents → **maximum shear stress**.
- Failure theory best suited to brittle materials → **maximum principal stress theory**.
- Shear yield stress by the von Mises theory → **$0.577\sigma_y$**; by Tresca → **$0.5\sigma_y$**.
- Yield locus of the maximum shear stress theory → **hexagon**.

1.8 Thin/Thick Cylinders & Strain Energy Theorems ★★☆

- **Thin cylinder**: **t < d/20** (d/t > 20). Stresses are taken as uniform across the thickness, and radial stress is neglected. Thick cylinder: d/t < 20, analysed with **Lamé's equations**.
- **Hoop (circumferential) stress** σ_h = **pd/2t**. It causes **longitudinal** bursting (along the length) and is resisted by the **longitudinal joint**.
- **Longitudinal stress** σ_L = **pd/4t** = $\sigma_h/2$. It causes **circumferential** bursting and is resisted by the **circumferential joint**.
- With joint efficiencies: $\sigma_h = pd/(2t\eta_L)$ and $\sigma_L = pd/(4t\eta_c)$. The **longitudinal joint is the more critical**, so it is made stronger (e.g. a butt joint).
- In-plane maximum shear = $(\sigma_h - \sigma_L)/2$ = **pd/8t**. Including the radial stress ≈ 0, the absolute maximum shear = $\sigma_h/2$ = **pd/4t**.
- **Thin sphere**: σ = **pd/4t** in every direction. For the same p, d and allowable stress, a sphere needs **half the thickness** of a cylinder.
- **Wire-wound cylinders**: initial tension in the wire puts the tube in compression, which raises its pressure capacity.

Quantity	Thin cylinder	Thin sphere
Stress	$\sigma_h = pd/2t$; $\sigma_L = pd/4t$	pd/4t
Hoop (diametral) strain	**(pd/4tE)(2 − ν)**	(pd/4tE)(1 − ν)
Longitudinal strain	**(pd/4tE)(1 − 2ν)**	—
Volumetric strain	**(pd/4tE)(5 − 4ν)** = $2\varepsilon_h + \varepsilon_L$	**(3pd/4tE)(1 − ν)** = 3ε
Change in volume	$\delta V = \varepsilon_v \times V$	$\delta V = \varepsilon_v \times (\pi/6)d^3$

- With ν = 0.5, the longitudinal strain of a thin cylinder is **zero**. Ratio of hoop to longitudinal strain = (2 − ν)/(1 − 2ν).

Thick cylinders — Lamé's theory

- **Lamé's equations** (tension +ve): **$\sigma_r = A - B/r^2$** (radial), **$\sigma_\theta = A + B/r^2$** (hoop). $\sigma_\theta + \sigma_r = 2A$ is **constant** through the wall, and the longitudinal stress is uniform.
- **Assumptions**: plane sections remain plane (ε_L constant); material homogeneous and isotropic; σ_L uniform.
- Internal pressure p only (inner radius r_i, outer r_o): $\sigma_r = -p$ at the inner surface and 0 at the outer. The **maximum hoop stress is at the inner surface** = **$p(r_o^2 + r_i^2)/(r_o^2 - r_i^2)$**. The minimum hoop stress (outer surface) = $2pr_i^2/(r_o^2 - r_i^2)$. σ_L (closed ends) = $pr_i^2/(r_o^2 - r_i^2)$.
- The hoop stress variation is **hyperbolic**. However thick the wall, the maximum hoop stress is always > p, so there is a limiting

Nandyal Banking Coaching Center

pressure ($p = \sigma_{allow}$ as $t \to \infty$). Thicker walls give diminishing returns.

- **Compound cylinders**: an outer tube is **shrink-fitted** on an inner tube. The shrinkage (contact) pressure puts **initial compressive hoop stress in the inner tube** and tension in the outer tube, so the hoop stress becomes **more uniform** and the pressure capacity rises. Used in gun barrels and high-pressure vessels. Other methods: **autofrettage** (pre-yielding the bore) and wire winding.
- Shrink allowance (same material): $\delta_r/r = (\sigma_{\theta,outer} - \sigma_{\theta,inner})/E$ at the junction.

Energy theorems

- **Clapeyron's theorem**: strain energy $U = \frac{1}{2}\Sigma P\delta$ for loads applied gradually to a linear elastic body.
- **Strain energy in bending**: $U = \int M^2dx/(2EI)$. In torsion: $\int T^2dx/(2GJ)$. Axial: $\int P^2dx/(2AE)$. Shear: $\int (kV^2)dx/(2AG)$.
- **Castigliano's first theorem**: $\partial U/\partial\delta_i = P_i$. **Castigliano's second theorem** (the one usually asked): $\boldsymbol{\delta_i = \partial U/\partial P_i}$ (deflection in the direction of the load) and $\theta_i = \partial U/\partial M_i$. For deflection at a point with no load, apply a **fictitious load** Q and set Q = 0 after differentiating. It applies to **linear elastic** structures.
- **Principle of least work** (corollary of Castigliano): in an indeterminate structure, the redundants make U **minimum**: $\partial U/\partial R = 0$.
- **Maxwell's reciprocal theorem**: the deflection at A due to a unit load at B = the **deflection at B due to a unit load at A** ($\delta_{AB} = \delta_{BA}$). It follows that the **flexibility matrix is symmetric**.
- **Betti's theorem** (generalised): the work done by load system P on the displacements due to system Q = the work done by Q on the displacements due to P. Maxwell's theorem is the special case of Betti's.
- **Unit load method**: $\delta = \int Mm\,dx/(EI)$, where m = BM due to a unit load at the point.

KEY FORMULAE

- $\sigma_h = pd/2t$; $\sigma_L = pd/4t$; sphere $\sigma = pd/4t$
- ε_v (cyl) $= (pd/4tE)(5 - 4\nu)$; ε_v (sphere) $= (3pd/4tE)(1 - \nu)$
- Lamé: $\sigma_r = A - B/r^2$; $\sigma_\theta = A + B/r^2$
- $\delta = \partial U/\partial P$; $U = \int M^2dx/2EI$; $\delta_{AB} = \delta_{BA}$

MEMORY TRICK

"Hoop is Half-t, Long is Quarter-t": pd/2t vs pd/4t. Volumetric strain **"5 − 4v for the can, 3(1 − v) for the ball"**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Ratio of hoop stress to longitudinal stress in a thin cylinder → **2 : 1**.
- Hoop stress in a thin spherical shell → **pd/4t**.
- Volumetric strain of a thin cylinder → **(pd/4tE)(5 − 4v)**.
- In a thick cylinder under internal pressure, the maximum hoop stress occurs at → **the inner surface**.
- Deflection = partial derivative of strain energy with respect to load → **Castigliano's (second) theorem**.
- $\delta_{AB} = \delta_{BA}$ is stated by → **Maxwell's reciprocal theorem**.

Nandyal Banking Coaching Center

2. Theory of Machines & Vibrations

NOTE

Theory of Machines (TOM) typically gives **6–8 questions** in a 60-MCQ mechanical paper. Most-asked areas: DOF (Kutzbach/Grubler), Grashof's law and inversions, number of instantaneous centres, Coriolis component, Hooke's joint, belt tension ratio and maximum power, governors (Watt height, isochronism), balancing of reciprocating masses, gyroscopic effect on ship, and vibration formulas (ω_n, ζ, logarithmic decrement, transmissibility). Learn the direction rules and the "which one is balanced" facts – they are favourite one-liners.

2.1 Mechanisms & Kinematics ★★★

- **Kinematic link (element)** = resistant body (rigid or **resistant**, e.g. belt in tension, fluid in compression) having relative motion with others. Binary link (2 joints), ternary (3), quaternary (4).
- **Kinematic pair** = two links in contact permitting constrained relative motion.
- **Lower pair** = **surface (area) contact** – e.g. revolute, prismatic, screw, spherical. **Higher pair** = **point or line contact** – e.g. cam & follower, gear teeth, ball/roller bearing, wheel on rail.
- **Self-closed pair** (held by geometry, e.g. pin joint) vs **force-closed pair** (held by external force – spring/gravity, e.g. cam & follower).
- Constrained motion: **completely** (piston in cylinder, square bar in square hole), **incompletely** (circular shaft in round hole – can rotate & slide), **successfully constrained** (foot-step bearing; shaft prevented from upward motion by its own load; cam–follower with spring).
- **Kinematic chain** = closed assembly of links with definite relative motion. **Mechanism** = kinematic chain with **one link fixed**. **Machine** = mechanism that transmits force/does useful work. **Structure** – no relative motion (DOF = 0).
- **Inversion** = fixing different links of the same chain; number of inversions = **number of links** (four-bar → 4 inversions). Relative motion between links does not change with inversion; absolute motions do.

Pair	Relative motion	DOF	Type / example
Turning (revolute, R)	Rotation about an axis	1	Lower; crank pin, shaft in journal bearing with collars
Sliding (prismatic, P)	Translation	1	Lower; piston–cylinder, ram in guide
Rolling	Rolling without slip	1	Ball/roller bearing, wheel on rail (classed higher pair in most texts)
Screw (helical, H)	Rotation + coupled translation	1	Lower; lead screw & nut, bolt–nut
Cylindrical (C)	Independent rotation + translation	2	Lower; shaft free to rotate and slide in a hole
Spherical (globular, S)	Rotation about a point	3	Lower; ball & socket joint, car steering ball joint
Planar (flat)	2 translations + 1 rotation in a plane	3	Lower; block on a flat surface

KEY FORMULAE – DEGREES OF FREEDOM

- Kutzbach (planar): **$F = 3(n - 1) - 2j - h$**; n = links (incl. fixed), j = lower pairs (1-DOF joints), h = higher pairs.
- Grubler's criterion (F = 1, lower pairs only): **$3n - 2j - 4 = 0$**. A four-bar satisfies it (12 – 8 – 4 = 0).
- Spatial (Kutzbach 3-D): $F = 6(n - 1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5$.
- A joint where k links meet counts as **(k – 1)** binary joints.
- Chain check (Klein): $l = 2p - 4$ and $j = (3/2)l - 2$ (l links, p pairs). LHS > RHS → locked (structure); LHS < RHS → unconstrained.
- F = 0 → **structure**; F < 0 → **superstructure** (statically indeterminate); F = 1 → constrained mechanism; F = 2 → two inputs needed (e.g. five-bar).

Assembly	n	j	h	F
Three links pinned (triangle)	3	3	0	**0** (structure)
Four-bar / slider-crank	4	4	0	**1**
Five-bar	5	5	0	**2**
Cam + knife-edge follower + frame	3	2	1	**1**
Cam + roller follower (roller as link)	4	3	1	2 → **1 redundant** (roller spin)

Grashof's law and four-bar inversions

- **Grashof's law**: for continuous relative rotation of at least one link, **$s + l \le p + q$** (s = shortest, l = longest, p, q = other two links).
- Grashof chain ($s + l < p + q$): fix link **adjacent to shortest** → **crank–rocker** (crank–lever); fix the **shortest link** → **double crank** (drag link); fix link **opposite the shortest** → **double rocker**.
- Non-Grashof ($s + l > p + q$): every inversion is a **double rocker (triple rocker)**.
- $s + l = p + q$ → **change-point** mechanism (links become collinear; parallelogram linkage is an example).
- Four-bar examples: **beam engine** (crank–lever), **coupling rod of locomotive** (double crank, parallelogram), **Watt's indicator mechanism** (double lever), pantograph (copying, parallelogram linkage), Ackermann steering (double rocker).
- **Transmission angle** (μ, between coupler and output link): ideal 90°; keep **≥ 40–45°**. At the **toggle (dead-centre) position** (input link and coupler collinear) the mechanical advantage becomes **infinite** – used in stone crushers, toggle clamps, riveting presses.

Slider-crank and double slider-crank inversions

Chain	Link fixed	Inversion / application
Single slider-crank (4 links: frame/cylinder, crank, connecting rod, slider)	Cylinder (frame)	**Reciprocating engine & compressor**
	Crank	**Whitworth quick-return** (shaper/slotter), **rotary (Gnome) engine**
	Connecting rod	**Oscillating cylinder engine**, **crank & slotted lever quick-return** (shaper)
	Slider (piston)	**Pendulum pump (bull engine)**, hand pump

Nandyal Banking Coaching Center

Double slider-crank (2 sliding + 2 turning pairs)	Slotted plate (frame with two guides)	**Elliptical trammel** – any point on the link traces an ellipse; **midpoint traces a circle**
	One slider	**Scotch yoke** – converts rotation into **SHM** of the yoke
	Link joining the two sliders (crank)	**Oldham's coupling** – connects **parallel shafts with small lateral offset**; max sliding speed of intermediate disc = **ω × offset**

- Numbering of "1st/2nd/3rd inversion" differs between Khurmi and Rattan – learn by **which link is fixed**.
- **Quick-return ratio** = time of cutting ÷ time of return = **β/α = (360° − α)/α**, α = crank angle of return stroke.
- Crank & slotted lever: **cos(α/2) = r/c** (r = crank radius, c = distance between crank centre and lever pivot); stroke length = 2L·(r/c), L = slotted-lever length.
- Quick-return mechanisms are used in **shaper, slotter**; Whitworth gives a smoother cutting stroke, crank & slotted lever is more common.

Instantaneous centre (I-centre) and velocity/acceleration

- Number of I-centres: **N = n(n − 1)/2** → four-bar or slider-crank (n = 4): **6**; six-link: **15**; five-link: 10.
- **Kennedy's (three-centres-in-line) theorem**: the three I-centres of three bodies in plane motion lie on a **straight line**.
- Types: **fixed (permanent & fixed)**, **permanent but not fixed**, **neither fixed nor permanent**. For a slider on a straight guide, the I-centre lies at **infinity** perpendicular to the path.
- For a wheel rolling without slip, I-centre is the **contact point**; top point velocity = **2v**.
- **Centrode** – locus of I-centre; space (fixed) centrode and body (moving) centrode roll on each other.
- Velocity of a point = ω × (distance from I-centre); velocity image is similar to the link (velocity polygon).
- Rubbing velocity at a pin joint = $r_{pin}(\omega_1 - \omega_2)$ if links rotate in the **same** sense; $r_{pin}(\omega_1 + \omega_2)$ if **opposite**.
- Acceleration of a point on a link relative to another: **centripetal $\omega^2 r = v^2/r$** (towards centre) + **tangential αr** (perpendicular).
- **Coriolis component = 2vω** (v = sliding velocity of slider on the rotating link, ω = angular velocity of the link). Direction: rotate v by **90° in the sense of ω**. Present in **crank & slotted lever, Whitworth, oscillating cylinder**; **absent** in ordinary slider-crank with fixed straight guide.
- Graphical acceleration of slider-crank: **Klein's construction** (also Ritterhaus, Bennett).

KEY FORMULAE – SLIDER-CRANK (N = L/R)

- Piston velocity $v_p = \omega r (\sin\theta + \sin 2\theta/2n)$
- Piston acceleration $a_p = \omega^2 r (\cos\theta + \cos 2\theta/n)$
- Angular velocity of connecting rod $\omega_c = \omega \cos\theta/\sqrt{(n^2 - \sin^2\theta)} \approx \omega \cos\theta/n$
- Coriolis acceleration = 2vω (m/s²)

Straight-line mechanisms, Hooke's joint, steering

Exact straight line	Approximate straight line
Peaucellier (8 links), **Hart** (6 links), **Scott-Russell** (sliding pair)	**Watt**, Modified Scott-Russell, **Grasshopper**, **Tchebicheff**, **Robert's**

- **Pantograph** – parallelogram linkage to **copy/enlarge/reduce** drawings and guide cutting tools.
- **Hooke's (universal) joint** connects two **intersecting shafts** at a small angle α; driven shaft speed fluctuates though mean speed is equal.
- Speed ratio $\omega_2/\omega_1 = \cos\alpha/(1 - \cos^2\theta \sin^2\alpha)$; **maximum = 1/cosα** (θ = 0°, 180°); **minimum = cosα** (θ = 90°, 270°).
- Speeds equal **4 times per revolution**, at $\tan\theta = \pm\sqrt{\cos\alpha}$. Max fluctuation of speed (÷ ω_1) = **tanα·sinα**.
- **Double Hooke's joint** gives **constant velocity ratio** if (i) the intermediate shaft's forks lie in the same plane and (ii) input and output shafts make **equal angles** with it. Used in automobile propeller shafts.
- **Correct steering condition** (all wheels rolling about one centre): **cotφ − cotθ = c/b**; φ = outer wheel angle, θ = inner wheel angle, c = distance between front stub-axle pivots, b = wheelbase.
- **Davis steering**: **sliding pairs**, theoretically correct in **all positions**, but wears and becomes inaccurate; tanα = c/2b.
- **Ackermann steering**: only **turning pairs**, correct in only **3 positions** (straight + one each side); preferred in practice for simplicity.

MEMORY TRICK

Grashof: "**Fix the Shortest → both Spin** (double crank); fix its Neighbour → crank–rocker; fix the Opposite → both rOck." Hooke's joint: max ratio "**1 over cos**", min "**cos**" – product of max and min = 1.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Number of instantaneous centres in a six-link mechanism? → **15**.
- Coriolis component of acceleration occurs in? → **Crank & slotted lever quick-return mechanism** (2vω).
- Elliptical trammel and Oldham's coupling are inversions of? → **Double slider-crank chain**.
- Ball and socket joint is which pair, with how many DOF? → **Spherical (lower) pair, 3 DOF**.
- Exact straight-line mechanism with only turning pairs and 8 links? → **Peaucellier**.
- Steering gear with sliding pairs? → **Davis** (Ackermann has turning pairs).

2.2 Cams & Followers ★★☆

- Cam–follower is a **higher pair**; converts rotary motion into reciprocating/oscillating motion. Used in IC-engine valve gear, automatic machines, textile machinery.
- Cam types: **radial (disc/plate)** – follower moves perpendicular to cam axis (most common); **cylindrical (barrel)** – follower moves parallel to cam axis; **wedge/translating**, **spiral**, **conjugate**, **globoidal**, **tangent cam** and **circular-arc cam** (specified-contour cams in IC engines).

Follower (by surface)	Key feature
Knife-edge	Simplest; high wear due to sliding and point contact; any cam profile can be followed; rarely used

Nandyal Banking Coaching Center

Roller	**Rolling** contact, low wear; most widely used (stationary gas/oil engines, aircraft engines)
Flat-faced (mushroom)	Low side thrust; used in **automobile valve gear**; offset to make it rotate (even wear); **cannot follow a concave cam**
Spherical-faced	Avoids stress concentration of flat follower; used in automobile engines

- By motion: **reciprocating (translating)** or **oscillating**. By path: **radial** (line of motion through cam centre) or **offset** (reduces side thrust / pressure angle).
- **Base circle** = **smallest circle** drawn from cam centre tangent to the **cam profile**; its size governs cam size.
- **Trace point** = reference point on follower (knife-edge tip / roller centre). **Pitch curve** = locus of trace point.
- **Prime circle** = smallest circle from cam centre tangent to **pitch curve** = base circle radius + roller radius.
- **Pressure angle** = angle between **direction of follower motion** and the **common normal** to the pitch curve. Should be small: max **≈ 30°** for reciprocating and **≈ 45°** for oscillating followers. Large pressure angle → side thrust, jamming.
- **Pitch point** = point on pitch curve with **maximum pressure angle**; **pitch circle** passes through it.
- **Lift (stroke)** = maximum displacement of follower. **Dwell** = follower at rest while cam rotates.
- Avoid **undercutting**: radius of curvature of pitch curve must exceed roller radius → use larger base circle. At high speed, follower may lose contact (**jump**) – stiffer spring required.

KEY FORMULAE – FOLLOWER MOTION (LIFT S, CAM SPEED Ω RAD/S, OUTSTROKE ANGLE Θ_0 RAD)

- Uniform velocity: $v = \omega S/\theta_o$; acceleration **infinite** at start & end (modified by parabolic/circular arcs).
- SHM: $v_{max} = \pi\omega S/(2\theta_o)$; $a_{max} = \pi^2\omega^2 S/(2\theta_o^2)$ (at start & end of stroke).
- Uniform acceleration & retardation (parabolic): $v_{max} = 2\omega S/\theta_o$ (mid-stroke); $a = 4\omega^2 S/\theta_o^2$ (constant).
- Cycloidal: $y = S[\theta/\theta_o - (1/2\pi)\sin(2\pi\theta/\theta_o)]$; $v_{max} = 2\omega S/\theta_o$; $a_{max} = 2\pi\omega^2 S/\theta_o^2$ (at $\theta = \theta_o/4$).

Motion	v_{max} ($\times \omega S/\theta_o$)	a_{max} ($\times \omega^2 S/\theta_o^2$)	Jerk / remarks
Uniform velocity	1	∞ at ends	Worst; only for low speed
SHM	$\pi/2$ = 1.57	$\pi^2/2$ ≈ 4.93	Acceleration jumps at start/end (with dwell) → **infinite jerk** there; moderate speeds
Uniform acc./retardation	2	4 **(minimum a_{max})**	Abrupt change of acceleration at start, mid, end → infinite jerk
Cycloidal	2	2π ≈ 6.28	Acceleration **zero at start & end**, continuous → **finite jerk**; best for **high-speed** cams

MEMORY TRICK

Acceleration coefficients rise as **4 < 4.93 < 6.28** (Parabolic < SHM < Cycloidal) but smoothness rises the same way – "**Pay Smooth Cost**": cycloidal costs the most acceleration but gives no shock.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Follower motion preferred for high-speed cams? → **Cycloidal**.
- Follower motion giving minimum peak acceleration for a given lift? → **Uniform acceleration & retardation**.
- Smallest circle tangent to the cam profile? → **Base circle**.
- Point on pitch curve having maximum pressure angle? → **Pitch point**.
- Follower generally used in automobile engine valves? → **Flat-faced (mushroom)**.

2.3 Gears & Gear Trains ★★★

- Gears give a **positive drive** (no slip), constant velocity ratio, compact for small centre distances; costlier than belts, need accurate manufacture and lubrication.
- **Pitch circle** – imaginary circle that rolls without slip; **pitch point** – point of tangency of pitch circles.
- **Addendum** = radial height above pitch circle; **dedendum** = depth below pitch circle; **clearance** = dedendum – addendum; **working depth** = sum of addenda of mating gears; **backlash** = space width – tooth thickness (on pitch circle).
- **Pressure angle (angle of obliquity) φ** = angle between common normal (line of action) and common tangent at pitch point. Standard: **14½°** (old composite/full depth) and **20°** (most common); 25° also used.
- **Base circle** radius $r_b = r\cos\varphi$; involute is generated from the base circle. **Base pitch** $p_b = p_c\cos\varphi$.

Proportion (20° full depth, IS/standard)	Value
Module m	**m = d/T** (mm)
Circular pitch p_c	**$\pi d/T = \pi m$**
Diametral pitch P_d	**T/d** (teeth per inch of PCD); **$p_c \times P_d = \pi$**; $m = 25.4/P_d$
Addendum	**1 m**
Dedendum	**1.25 m** (older 14½° composite: 1.157 m)
Clearance	**0.25 m**
Whole depth / working depth	**2.25 m / 2 m**
Tooth thickness (on pitch circle)	**$\pi m/2 = 1.5708$ m**
Fillet radius	0.4 m
Outside (addendum) diameter	**m(T + 2)**
20° stub tooth	Addendum 0.8 m, dedendum 1 m (stronger tooth)

- **Law of gearing**: for constant angular velocity ratio, the **common normal at the point of contact must always pass through a fixed point (pitch point)** on the line of centres. $\omega_1/\omega_2 = O_2P/O_1P = T_2/T_1 = d_2/d_1$.
- Velocity of sliding between teeth = $(\omega_1 + \omega_2)$ × distance of contact point from pitch point (external gears); **zero at the pitch**

Nandyal Banking Coaching Center

point (pure rolling).

Involute profile	Cycloidal profile
Single curve – easy to manufacture (straight-sided rack cutter)	Double curve (epicycloid face + hypocycloid flank) – difficult
Pressure angle constant throughout engagement	Pressure angle varies (zero at pitch point)
Velocity ratio unaffected by small change of **centre distance**	Exact centre distance essential
Interference can occur	**No interference**
Convex-to-convex contact – more wear	Convex-to-concave – less wear, stronger tooth
Almost universal in power transmission	**Watches, clocks**, instruments

Interference, minimum teeth and contact ratio

- **Interference** = tip of one tooth contacts the **non-involute flank** (inside base circle) of the mating tooth. Occurs when contact extends beyond the **interference points** (tangency points of line of action with base circles). Cutter removes the interfering flank → **undercutting** (weakens tooth).
- Remedies: increase number of teeth on pinion, increase pressure angle (20°/25°), use **stub teeth** (reduced addendum), **profile shift / addendum modification**, undercut the flank.
- **Minimum teeth on pinion meshing with a rack** (addendum 1m): $T_{min} = \mathbf{2/\sin^2\varphi}$ → **14½°: ≈ 32**; **20°: ≈ 18** (17.1; 17 accepted with slight undercut); 25°: ≈ 12.
- For two equal gears (G = 1): 20° → **13**; 14½° → **23**. Minimum teeth increase as gear ratio increases.
- **Contact ratio** = arc of contact ÷ circular pitch = path of contact ÷ base pitch. Must be **> 1** for continuous drive; desirable **≥ 1.2–1.4**; spur gears typically 1.4–1.8. Higher contact ratio → quieter, smoother.

KEY FORMULAE – CONTACT (R, R = PITCH RADII OF GEAR & PINION; R_A, R_A = ADDENDUM RADII)

- Path of approach = $\sqrt{(R_A^2 - R^2\cos^2\varphi)} - R\sin\varphi$
- Path of recess = $\sqrt{(r_A^2 - r^2\cos^2\varphi)} - r\sin\varphi$
- Arc of contact = path of contact / $\cos\varphi$
- Max possible path of contact (no interference) = $(r + R)\sin\varphi$
- Contact ratio = path of contact / $(p_c\cos\varphi)$

Types of gears

Gear	Shaft arrangement	Key facts
Spur	Parallel	Straight teeth parallel to axis; **no axial thrust**; line contact all at once → noisy at high speed
Helical	Parallel (or crossed)	Helix angle **15–30°**; gradual engagement, quiet; produces **axial thrust**; $m_n = m\cos\psi$; virtual teeth $T_v = \mathbf{T/\cos^3\psi}$
Herringbone (double helical)	Parallel	Two opposite helices → **axial thrusts cancel**; helix 30–45°; heavy power (turbines, rolling mills)
Bevel (straight / spiral)	**Intersecting** (usually 90°)	Cone frustum; **mitre gears** = equal bevels at 90°; $T_v = T/\cos\delta$ (δ = pitch angle)
Hypoid	Non-parallel, **non-intersecting** (offset)	Automobile rear-axle differential (lower propeller shaft)
Crossed helical (spiral)	Non-parallel, non-intersecting	**Point contact**, light loads
Worm & worm wheel	Non-intersecting, **perpendicular**	High VR (**up to ~100**) in one stage; VR = T_{wheel}/**number of starts**; can be **self-locking**
Rack & pinion	Rotary ↔ linear	Rack = gear of infinite radius; straight-sided teeth for involute

- Worm efficiency $\eta = \tan\lambda/\tan(\lambda + \varphi)$ (λ = lead angle, φ = friction angle). **Self-locking** when $\lambda \le \varphi$ ($\tan\lambda \le \mu$); a self-locking worm drive has **η < 50%**. Used in hoists, lifts.
- Internal (annular) gear: centres on same side, both rotate in the **same direction**; more teeth in contact, compact.

Gear trains

Train	Feature	Use
Simple	One gear per shaft; $N_1/N_n = T_n/T_1$; **idlers do not change ratio**, only direction (odd idlers → same direction)	Bridging large centre distance
Compound	Two or more gears on an intermediate shaft; speed ratio = product of driven teeth ÷ product of driving teeth	Large VR in small space
Reverted	First and last gear **co-axial**; $r_1 + r_2 = r_3 + r_4$ (same module: $T_1 + T_2 = T_3 + T_4$)	**Lathe back gear**, clocks (hour & minute hands), automobile gearbox
Epicyclic (planetary)	Axis of at least one gear moves (planet on an arm); very high VR, compact	**Differential**, hoists, Humpage's speed reducer, automatic transmissions, pulley blocks

- **Tabular method**: (1) fix arm, give +1 rev to one gear, write others; (2) multiply by x; (3) add y to all (arm motion); solve with given conditions.
- Relative-velocity method: $(N_{last} - N_{arm})/(N_{first} - N_{arm}) = \pm$ train value with arm fixed.
- Sun–planet–annulus (same module): $\mathbf{T_A = T_S + 2T_P}$; $d_A = d_S + 2d_P$.
- Automobile **differential**: on a straight road both rear wheels same speed; on turning, outer wheel faster; speed of crown (cage) = **average** of wheel speeds.

KEY FORMULAE – TORQUES IN EPICYCLIC TRAIN

- Equilibrium: $\mathbf{T_{input} + T_{output} + T_{fixing} = 0}$
- Energy (no friction): $\mathbf{T_{input}\omega_{input} + T_{output}\omega_{output} = 0}$ (fixed member does no work)
- Holding torque $T_{fix} = -(T_{in} + T_{out}) = T_{in}(N_{in}/N_{out} - 1)$ (algebraic, using signed speeds)

MEMORY TRICK

"**32 – 18 – 12**" for 14½°, 20°, 25° (pinion with rack). Dedendum = "**one and a quarter**", clearance "**a quarter**", addendum "**one**".

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Module of a gear with 40 teeth and 200 mm PCD? → **5 mm**.
- Gears connecting non-parallel, non-intersecting shafts in automobile differential? → **Hypoid**.
- Condition for self-locking worm? → Lead angle **≤ friction angle** (η < 50%).
- Gear train in which input and output shafts are co-axial? → **Reverted** (also epicyclic).
- Profile used in watches? → **Cycloidal**.
- Herringbone gears are used to? → **Eliminate axial thrust**.

2.4 Belts, Ropes & Chains ★★★

- Belts/ropes = **friction drives** (slip possible) for large centre distances; chains = **positive drive**. Flat belt: up to ~10 m centre distance (approximate ranges vary by text); V-belt: short distances, multiple belts; ropes: large distances (> 8 m), heavy power.
- Flat-belt materials: leather (oak-tanned), cotton/fabric, rubber, balata (not above ~40 °C). V-belt: rubberised fabric with cord; groove angle **30°–40°** (β = half angle).
- **Velocity ratio** $N_2/N_1 = d_1/d_2$; with belt thickness t: $(d_1 + t)/(d_2 + t)$; with total slip s %: $N_2/N_1 = (d_1/d_2)(1 - s/100)$.
- **Slip** = insufficient friction; **creep** = relative motion due to **unequal stretching** of belt on tight and slack sides (elastic). With creep: $N_2/N_1 = (d_1/d_2)(E + \sqrt{\sigma_2})/(E + \sqrt{\sigma_1})$. Both reduce VR slightly.
- **Open belt**: pulleys rotate in the **same** direction; **cross belt**: **opposite** direction, larger angle of contact, more wear (rubbing at crossing).
- Angle of contact (lap) is governed by the **smaller pulley** for open drive; for cross belt it is **same on both pulleys**.
- **Crowning** of pulley face (slightly convex rim) keeps the belt **centred** – belt tends to run towards the larger diameter. Crowning is not done for V-belt pulleys.
- **Idler/jockey pulley** increases angle of contact and maintains tension.
- **Centrifugal tension does not affect power transmitted** (it adds equally to both sides) but increases total tension in the belt.

KEY FORMULAE – BELT DRIVE (R_1, R_2 PULLEY RADII, X CENTRE DISTANCE)

- Open belt length $L = \pi(r_1 + r_2) + 2x + (r_1 - r_2)^2/x$; angle of contact $\theta = (180° - 2\alpha)$, $\sin\alpha = (r_1 - r_2)/x$
- Cross belt length $L = \pi(r_1 + r_2) + 2x + (r_1 + r_2)^2/x$; $\theta = (180° + 2\alpha)$, $\sin\alpha = (r_1 + r_2)/x$ → depends only on $(r_1 + r_2)$ (stepped cone pulleys: sum of radii constant)
- Flat belt: $T_1/T_2 = e^{\mu\theta}$ (θ in radians); $2.3 \log(T_1/T_2) = \mu\theta$
- V-belt / rope: $T_1/T_2 = e^{\mu\theta \operatorname{cosec}\beta}$; effective $\mu = \mu/\sin\beta$
- Power $P = (T_1 - T_2)v$ W (T in N, v in m/s)
- Centrifugal tension $T_c = mv^2$ (m = mass per metre length)
- Maximum tension $T_{max} = \sigma_{allow} \times b \times t = T_1 + T_c$
- Condition for maximum power: $T_c = T_{max}/3$; $T_1 = 2T_{max}/3$; $v = \sqrt{(T_{max}/3m)}$
- Initial tension: $T_0 = (T_1 + T_2)/2$ (neglecting T_c); $T_0 = (T_1 + T_2 + 2T_c)/2$ (with T_c); for elastic belt $\sqrt{T_1} + \sqrt{T_2} = 2\sqrt{T_0}$

Item	Flat belt	V-belt	Chain
Contact	Rim face	**Sides of groove** (not bottom)	Sprocket teeth
Slip	Yes	Less (wedging, higher effective μ)	**None** (positive)
Velocity ratio	~ up to 1:4 typical	Up to ~1:7	Up to ~1:8 (constant mean)
Centre distance	Large	Short	Moderate (30–50 × pitch preferred)
Efficiency	~95–98%	~90–96%	~96–98%
Lubrication	Not needed	Not needed	Required

Ropes

- Fibre ropes – manila, hemp, cotton; groove angle ~45°. Wire ropes for hoists, cranes, elevators, mine haulage.
- Designation **6 × 19 = 6 strands, 19 wires per strand**. 6 × 7 – haulage (stiff, abrasion resistant); **6 × 19 – general hoisting (most common)**; 6 × 37 – cranes (more flexible); 8 × 19 – extra flexible.
- **Regular (ordinary/cross) lay**: wires twisted **opposite** to strands – does not kink/untwist easily, most common. **Lang lay**: wires and strands twisted in the **same** direction – more flexible, better wear resistance, tends to **untwist**.
- Sheave diameter should be large relative to rope (D/d ≈ 40–60 or more) to limit bending stress.

Chain drives

- Types: **hoisting** (oval/square link), **conveyor** (detachable/hook-joint, closed-joint), **power transmission** (block, **roller** – most common, **inverted-tooth/silent** – quiet, high speed).
- Velocity ratio $N_1/N_2 = T_2/T_1$. Pitch circle diameter D = **p/sin(180°/T)**.
- **Polygon (chordal) effect**: chain wraps as a polygon → periodic speed variation; v_{max}/v_{min} = **1/cos(180°/T)**. Reduced by **more teeth** on the smaller sprocket (≥ 17–19 recommended; min ~ 9–12 for low speed).
- Use an **even number of links** to avoid an offset (cranked) link; odd teeth on sprocket for even wear.

MEMORY TRICK

Max power belt: "**One-third goes in spinning**" – $T_c = T_{max}/3$, so two-thirds (T_1) is left for work. Cross belt: "**Cross adds**" – $(r_1 + r_2)^2$ and $180° + 2\alpha$.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Condition for maximum power transmission by a belt? → $T_c = T_{max}/3$.
- Velocity of belt for max power? → $\sqrt{(T_{max}/3m)}$.
- Relative motion due to unequal stretching of belt? → **Creep**.
- Pulley faces are crowned to? → **Prevent the belt from slipping off / keep it centred**.
- Rope "6 × 19" means? → **6 strands, 19 wires each**.
- Speed fluctuation in chain drives is due to? → **Polygonal (chordal) action**.

2.5 Flywheel & Governors ★★★

- **Turning moment (crank effort) diagram**: torque vs crank angle; area under curve = work done. Fluctuation is **maximum for a**

Nandyal Banking Coaching Center

single-cylinder four-stroke engine, least for multi-cylinder engines.

- **Flywheel** stores energy when supply > demand and releases it when demand > supply → controls speed fluctuation **within a cycle**. It acts as a reservoir; does not control mean speed.
- **Governor** regulates **mean speed over several cycles** by changing fuel/steam supply as load changes.
- **Maximum fluctuation of energy** $\Delta E = E_{max} - E_{min}$ (from the largest and smallest cumulative energy points on the diagram).
- Coefficient of fluctuation of energy $C_E = \Delta E \div$ work done per cycle.
- Rim provides **~90%** of flywheel moment of inertia (arms/hub ~10%). Rim speed limit: cast iron **≈ 25–30 m/s**, steel **≈ 40–50 m/s**.
- Punching press: energy needed only during punching; flywheel supplies it; motor rating based on average power. Energy from flywheel $\Delta E = E_1(1 - t/T)$ (t = punching time, T = time per cycle).

KEY FORMULAE – FLYWHEEL

- Coefficient of fluctuation of speed $C_s = (\omega_1 - \omega_2)/\omega = 2(N_1 - N_2)/(N_1 + N_2)$; ω = mean
- Coefficient of steadiness = $1/C_s$
- $\Delta E = I\omega^2 C_s = mk^2\omega^2 C_s = 2E \cdot C_s$ ($E = \frac{1}{2}I\omega^2$, mean KE); for a rim $I \approx mR^2 \rightarrow \Delta E = mv^2 C_s$
- Hoop (tensile) stress in rotating thin rim $\sigma = \rho v^2$ (independent of cross-section) → $v_{max} = \sqrt{(\sigma/\rho)}$

Machine	Typical C_s (approx.)
Crushing machines, hammering, punching	0.1–0.2 (large permissible)
Machine tools	~0.03
Textile / paper machinery	0.01–0.03
Electric generators (AC)	0.002–0.005 (very small)

Governors

Type	Key feature
Watt (simple conical pendulum)	No central load; $h = g/\omega^2 = 895/N^2$ m (N in rpm); only for **slow speeds** (height falls rapidly at high speed)
Porter	Watt + **heavy central dead load M** on sleeve; can run at higher speed; equal arms: $h = (m + M)/m \times 895/N^2$
Proell	Porter with balls fixed on **extension of lower (coupler) arm**; $N^2 = (FM/BM)\cdot(m + M)/m\cdot(895/h)$; **more sensitive** – smaller balls needed for same speed
Hartnell	**Spring-loaded**, bell-crank levers; compact, high speed, any position; spring stiffness $s = 2(F_2 - F_1)/(r_2 - r_1) \times (x/y)^2$
Wilson–Hartnell, Hartung, **Pickering**	Spring-controlled; Pickering uses leaf springs (gramophones, small instruments)
Inertia governor	Responds to **rate of change** of speed – quicker than centrifugal

- Controlling force $F_c = m\omega^2 r$ (inward force needed on each ball at equilibrium).
- **Sensitiveness** = $(N_2 - N_1)/N$, N = mean speed; smaller range → more sensitive.
- **Stability**: governor is stable if radius of rotation **increases with speed** (one radius for each speed).
- Controlling-force curve $F_c = ar \pm b$: **$F_c = ar - b$ → stable**; **$F_c = ar$ (line through origin) → isochronous**; **$F_c = ar + b$ → unstable**.
- **Isochronous governor** – same equilibrium speed at all radii (range zero, **infinitely sensitive**); **Porter/Watt cannot be isochronous**; Hartnell can be made isochronous by suitable spring. Isochronous governor tends to **hunt**.
- **Hunting** – continuous fluctuation of engine speed above and below mean due to an **over-sensitive** governor.
- **Effort** = mean force on sleeve for a given (usually 1%) change of speed; **Power** = effort × sleeve lift. Porter (equal arms): effort = $c(m + M)g$; power = $4c^2(m + M)gh/(1 + 2c)$.
- **Coefficient of insensitiveness** = $(N'' - N')/N$ – caused by **friction** at sleeve and joints.

MEMORY TRICK

Watt height: "**895 over N-squared**" metres – at 60 rpm, h ≈ 0.25 m; at 120 rpm, h ≈ 0.06 m, hence Watt is useless at high speed. Stability: "**minus b is Better**" (ar – b stable).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Height of Watt governor at 60 rpm? → **≈ 0.248 m**.
- Governor with same equilibrium speed for all radii of rotation? → **Isochronous**.
- Function of flywheel vs governor? → Flywheel – **cyclic fluctuations**; governor – **mean speed with load**.
- Hoop stress in flywheel rim? → **$\sigma = \rho v^2$**.
- Hartnell governor is which type? → **Spring-loaded centrifugal**.
- Fluctuation of energy in terms of mean KE? → **$\Delta E = 2E \cdot C_s$**.

2.6 Balancing ★★★

- Unbalance causes vibration, noise, bearing loads, fatigue. Balancing = adding/removing mass so that unbalanced forces and couples are eliminated or minimised.
- **Static balancing**: **$\Sigma mr = 0$** – centre of mass on the axis of rotation; shaft stays in any position on knife edges.
- **Dynamic balancing**: **$\Sigma mr = 0$ and $\Sigma mrl = 0$** – no resultant force and no resultant couple. A dynamically balanced system is always statically balanced, **not vice versa**.
- Single rotating mass can be balanced by (a) one mass **in the same plane**, diametrically opposite ($m_1r_1 = m_2r_2$), or (b) **two masses in two different planes**.
- Several masses in **one plane**: draw force polygon (mr vectors); closing side gives balancing mr.
- Several masses in **different planes**: choose a **reference plane** (one balancing plane), draw **couple polygon** (mrl) to find the other balancing mass, then force polygon. Minimum **two balancing masses in two planes** are required (Dalby's method).
- Long rotors (turbines, motor armatures) are balanced on **dynamic balancing machines**; field balancing done on site.

Reciprocating masses

- Inertia force of reciprocating mass m: $F = m\omega^2 r(\cos\theta + \cos 2\theta/n)$, $n = l/r$.

Nandyal Banking Coaching Center

- **Primary unbalanced force = $m\omega^2 r \cos\theta$** → frequency = engine speed; max $m\omega^2 r$ at θ = 0°, 180°.
- **Secondary unbalanced force = $m\omega^2 r \cos 2\theta/n$** → frequency = **twice** engine speed; max $m\omega^2 r/n$ at θ = 0°, 90°, 180°, 270°. Secondary is n times smaller than primary.
- Secondary force is treated as an imaginary crank of radius **r/4n** rotating at **2ω** (at angle 2θ).
- Reciprocating mass **can never be completely balanced** by a rotating mass: balancing the along-stroke component introduces an equal perpendicular component.
- **Partial balancing**: a fraction c of the reciprocating mass is balanced; B·b = c·m·r. Unbalanced force along line of stroke = **$(1 - c)m\omega^2 r \cos\theta$**; perpendicular = **$c \cdot m\omega^2 r \sin\theta$**. Practice: c = **2/3 to 3/4**; c = ½ minimises the maximum resultant.

Partial balancing of locomotives (two cylinders, cranks at 90°)

Effect	Cause / definition	Max value
Variation of tractive effort	Unbalanced primary force **along the line of stroke** (horizontal)	**$\pm\sqrt{2}(1 - c)m\omega^2 r$**
Swaying couple	Unbalanced primary forces along the line of stroke acting at distance a apart → couple about vertical axis	**$\pm(a/\sqrt{2})(1 - c)m\omega^2 r$**
Hammer blow	Max **vertical** unbalanced force due to the balance mass B (added for reciprocating parts) – varies rail pressure	**$B\omega^2 b$**; wheel lifts if $B\omega^2 b$ > dead load P → limiting ω = $\sqrt{(P/Bb)}$

- Locomotives are classed as **inside-** or **outside-cylinder** and as **uncoupled** or **coupled**; in coupled locomotives the balance mass is shared among the coupled wheels → less hammer blow per wheel.

Multi-cylinder and V-engines

- Conditions for complete primary balance (inline): primary **force polygon closes** and primary **couple polygon closes**. Similarly for secondary with secondary cranks at 2θ.
- For a **V-twin of 90°** (common crank): resultant primary force = **$m\omega^2 r$, constant**, along the crank → can be **fully balanced by a rotating counter-mass**; resultant secondary force = **$\sqrt{2} \cdot m\omega^2 r \sin 2\theta/n$**, horizontal (unbalanced).
- General V-twin (angle 2α): primary vertical $2m\omega^2 r \cos^2\alpha \cos\theta$, horizontal $2m\omega^2 r \sin^2\alpha \sin\theta$.
- Radial engines: **direct and reverse crank** method – primary forces can be balanced fully by a single counter-mass for 3 or more cylinders equally spaced.

Engine	Primary force	Primary couple	Secondary force	Secondary couple
Single cylinder	Unbalanced	–	Unbalanced	–
2-cyl inline, cranks 180°	**Balanced**	Unbalanced	Unbalanced	Balanced
4-cyl inline (cranks 0°, 180°, 180°, 0°)	**Balanced**	**Balanced**	**Unbalanced ($4m\omega^2 r/n$)**	Balanced
6-cyl inline (120° pairs)	Balanced	Balanced	Balanced	Balanced → **completely balanced**
V-twin 90°	Resultant constant (balanceable by counterweight)	–	Unbalanced ($\sqrt{2}\, m\omega^2 r/n$)	–

MEMORY TRICK

"**Hammer hits Vertically, Tractive pulls Horizontally, Sway twists.**"
For engines: "**Four is Primary-perfect, Six is all-perfect.**"

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Frequency of secondary unbalanced force? → **Twice the crank speed**.
- Maximum vertical unbalanced force due to balance weight in a locomotive? → **Hammer blow**.
- Minimum number of balancing masses for several masses in different planes? → **Two**.
- Inline four-cylinder four-stroke engine is unbalanced in? → **Secondary forces**.
- Fraction of reciprocating mass usually balanced in locomotives? → **2/3 to 3/4**.
- Condition for dynamic balance? → **$\Sigma mr = 0$ and $\Sigma mrl = 0$**.

2.7 Gyroscope ★★☆

- A spinning rotor (spin ω about spin axis) whose axis is turned (**precession**, ω_p) needs an **active gyroscopic couple** about the third mutually perpendicular axis. The body exerts an equal and opposite **reactive gyroscopic couple** on its frame/bearings – this is the effect felt by ships, aircraft and vehicles.
- Spin axis, precession axis and couple axis are **mutually perpendicular**. Angular momentum vector by right-hand rule; active couple vector = **$\omega_p \times (I\omega)$** (direction in which the spin vector tip moves).
- Applications: gyro-compass, gyro-stabilisers (ships), aircraft instruments (turn indicator, artificial horizon), inertial navigation.

KEY FORMULAE – GYROSCOPE

- **$C = I\omega\omega_p$** (N·m); $I = mk^2$
- Ship/aircraft on a turn of radius R at speed v: $\omega_p = v/R$
- Pitching (SHM, amplitude φ, period t_p): $\omega_{p,max} = \varphi(2\pi/t_p)$ → $C_{max} = I\omega \cdot \varphi \cdot 2\pi/t_p$ (at mean position)
- Two-wheeler (tilt θ from vertical): gyroscopic couple $C_g = (v^2/(R\, r_w))(2I_w + G\, I_e)\cos\theta$; centrifugal couple $C_c = (mv^2/R)\, h \cos\theta$; balancing couple = $mgh \sin\theta$
- Four-wheeler: $C_g = (v^2/(R\, r_w))(4I_w \pm G\, I_e)$ (+ if engine rotates same sense as wheels); $C_c = (mv^2/R)h$

Effect on a ship (rotor rotating clockwise when viewed from stern)

Motion of ship	Reactive gyroscopic effect

Nandyal Banking Coaching Center

Steering to left (port)	**Raises bow, lowers stern**
Steering to right (starboard)	Lowers bow, raises stern
Pitching – bow rising	Turns ship towards **starboard (right)**
Pitching – bow falling	Turns ship towards **port (left)**
Rolling	**No gyroscopic effect** – axis of rolling is parallel to spin axis (no precession of spin axis)

- Terms: **bow** = front, **stern (aft)** = rear, **starboard** = right side, **port** = left side (looking from stern to bow).
- If the rotor rotates **anticlockwise** from stern, all effects **reverse**.
- **Aeroplane** with propeller rotating clockwise viewed from the rear (pilot's seat): turning **left → nose rises, tail dips**; turning right → nose dips, tail rises.
- **Two-wheeler** taking a turn: both gyroscopic and centrifugal couples act **outwards** (overturning); rider **leans inward** so that mgh sinθ balances them.
- **Four-wheeler** on a curve: gyroscopic and centrifugal couples **increase reaction on outer wheels** and reduce it on inner wheels; if inner-wheel reaction becomes negative, inner wheels **lift off** (vehicle tends to overturn outward). Reverse-rotating engine reduces the gyroscopic part.
- Ship stabiliser rotor precessed deliberately to oppose rolling; gyroscopic effect on ship's rotor is felt as extra **bearing loads**.

MEMORY TRICK

Clockwise rotor from stern: "**Left Lifts the bow**" (port turn raises bow), and "**Bow Up → go Starboard**". Rolling: "**Rolling = no gyro**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Gyroscopic couple formula? → **$I\omega\omega_p$**.
- Gyroscopic effect during rolling of ship? → **Nil**.
- Ship rotor clockwise from stern, ship turns left – effect? → **Bow rises, stern dips**.
- Angle between spin axis and axis of precession? → **90°**.
- For a two-wheeler on a turn, gyroscopic couple tends to? → **Overturn it outward** (rider leans inward).

2.8 Mechanical Vibrations ★★★

- **Free vibration** – after initial disturbance, no external force; body vibrates at **natural frequency**. **Forced vibration** – under a periodic external force; at steady state, vibrates at **forcing frequency**.
- Types by motion: **longitudinal** (particles parallel to shaft axis), **transverse** (perpendicular to axis), **torsional** (twisting about axis).
- **Number of natural frequencies = number of DOF**. Lowest = fundamental.
- **Resonance**: forcing frequency = natural frequency → amplitude very large (limited only by damping).

KEY FORMULAE – FREE UNDAMPED (SDOF)

- Equation: $m\ddot{x} + kx = 0 \rightarrow \omega_n = \sqrt{(k/m)} = \sqrt{(g/\delta)}$ rad/s (δ = static deflection = mg/k)
- $f_n = (1/2\pi)\sqrt{(g/\delta)} \approx 0.4985/\sqrt{\delta}$ Hz (δ in m) ≈ $15.76/\sqrt{\delta}$ Hz (δ in mm); period $t_p = 2\pi\sqrt{(\delta/g)}$
- Effect of spring mass m_s: $\omega_n = \sqrt{(k/(m + m_s/3))}$
- Simple pendulum $\omega_n = \sqrt{(g/L)}$; compound pendulum $\omega_n = \sqrt{(gh/(k_G^2 + h^2))}$; equivalent length $L = (k_G^2 + h^2)/h$
- Torsional (single rotor): $\omega_n = \sqrt{(q/I)}$, $q = GJ/l$
- Springs in **series**: $1/k_{eq} = 1/k_1 + 1/k_2$; in **parallel**: $k_{eq} = k_1 + k_2$

Element	Stiffness k
Bar in axial load	**AE/L**
Cantilever, end load	**$3EI/L^3$**
Simply supported, central load	**$48EI/L^3$**
Fixed–fixed, central load	**$192EI/L^3$**
Propped cantilever, central load	$768EI/(7L^3)$
Shaft in torsion	**GJ/L** (N·m/rad)
Close-coiled helical spring	**$Gd^4/(8D^3n)$**

- Methods to find ω_n: **equilibrium (D'Alembert)**, **energy method** (KE + PE = constant → d/dt = 0), **Rayleigh's method** ($KE_{max} = PE_{max}$).
- **Rayleigh's method** gives an **upper bound** (slightly higher) of the fundamental frequency; **Dunkerley's method** gives a **lower bound**. True value lies between them.

Damped free vibration

- Viscous damping force = $c\cdot\dot{x}$. Equation: $m\ddot{x} + c\dot{x} + kx = 0$.
- **Critical damping coefficient $c_c = 2\sqrt{(km)} = 2m\omega_n$; damping ratio (factor) $\zeta = c/c_c$.**
- Damped natural frequency **$\omega_d = \omega_n\sqrt{(1 - \zeta^2)}$** (always < ω_n).
- **Coulomb (dry friction) damping**: amplitude decays **linearly** (by 4F/k per cycle; frequency unchanged). **Viscous**: decays **exponentially** (geometric progression).

ζ	Case	Response
ζ > 1	Over-damped	Aperiodic; returns slowly, no oscillation
ζ = 1	Critically damped	Aperiodic; returns to equilibrium in **shortest time** without oscillation – **gun recoil mechanism**, door closers, measuring instruments
ζ < 1	Under-damped	Oscillatory with exponentially decaying amplitude (most real systems)
ζ = 0	Undamped	Constant amplitude (ideal)

KEY FORMULAE – LOGARITHMIC DECREMENT

- **$\delta = \ln(x_n/x_{n+1}) = 2\pi\zeta/\sqrt{(1 - \zeta^2)} \approx 2\pi\zeta$** (small ζ)
- Over n cycles: $\delta = (1/n)\ln(x_0/x_n)$
- $\zeta = \delta/\sqrt{(4\pi^2 + \delta^2)}$

Nandyal Banking Coaching Center

Forced vibration, resonance and isolation

- Frequency ratio **$r = \omega/\omega_n$**. Equation: $m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$.
- **Magnification factor (MF)** = X/X_{st} = **$1/\sqrt{[(1 - r^2)^2 + (2\zeta r)^2]}$**, $X_{st} = F_0/k$.
- Phase angle **$\tan\varphi = 2\zeta r/(1 - r^2)$**: $\varphi \approx 0°$ for $r \ll 1$; **$\varphi = 90°$ at resonance ($r = 1$) for all ζ**; $\varphi \to 180°$ for $r \gg 1$.
- At resonance MF = **$1/(2\zeta)$**; peak MF actually occurs at $r = \sqrt{(1 - 2\zeta^2)}$ (slightly below 1); for $\zeta \geq 1/\sqrt{2}$ no peak.
- Rotating unbalance m_0e: $X/(m_0e/m) = r^2/\sqrt{[(1 - r^2)^2 + (2\zeta r)^2]} \to$ tends to 1 at high r.
- **Transmissibility** $TR = F_T/F_0$ = **$\sqrt{[1 + (2\zeta r)^2] / [(1 - r^2)^2 + (2\zeta r)^2]}$** (same expression for force and for base-motion displacement transmissibility).
- **TR = 1 at $r = \sqrt{2}$ for every ζ; isolation (TR < 1) only when $r > \sqrt{2}$**, i.e. $\omega_n < \omega/\sqrt{2} \to$ use **soft mountings**.
- For $r > \sqrt{2}$, damping **increases** TR (worsens isolation) – but some damping is needed to limit amplitude while **passing through resonance** during start-up. For $r < \sqrt{2}$, damping reduces TR.
- Isolation efficiency = 1 − TR. Practical isolators are designed for r ≈ 2.5–5.

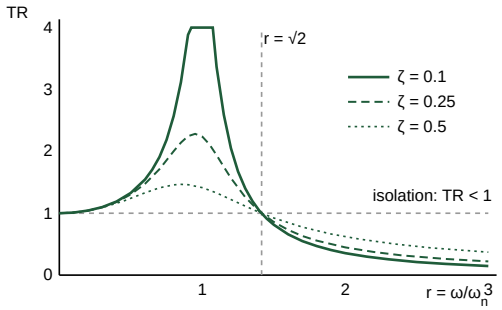


Fig 2.1 Transmissibility vs frequency ratio: all curves pass through TR = 1 at $r = \sqrt{2}$; beyond √2, lower damping gives better isolation.

Whirling of shafts and torsional vibration

- **Whirling (critical) speed** = speed at which a rotating shaft deflects violently; it equals the **natural frequency of transverse vibration**: $\omega_c = \sqrt{(k/m)} = \sqrt{(g/\delta)}$; **$N_c \approx 29.9/\sqrt{\delta}$ rpm** (δ in m).
- Causes: eccentricity of rotor, initial bend, unbalance. Below critical speed, heavy side of disc is **outward** (in phase); above critical speed, heavy side **inward** – at very high speed the rotor spins about its **centre of mass** (self-centring). Turbines often run **above** the first critical speed and pass through it quickly.
- Simply supported shaft with UDL: $\delta = 5wL^4/384EI$; f = **$0.5615/\sqrt{\delta}$** Hz (δ = max static deflection due to UDL).
- **Dunkerley's method** (several loads): **$1/f_n^2 = 1/f_1^2 + 1/f_2^2 + \ldots + 1/f_s^2$** ($f_s$ = shaft alone) → $f_n = 0.4985/\sqrt{(\delta_1 + \delta_2 + \ldots + \delta_s/1.27)}$.
- **Two-rotor system** (rotors I_A, I_B on shaft of length l): one **node** where shaft does not twist; **$I_AI_A = I_BI_B$**, $I_A + I_B = l$ → node is **nearer the heavier rotor**; $f = (1/2\pi)\sqrt{(GJ/(I_AI_A))}$. Rotors vibrate in **opposite** phase.
- **Three-rotor system**: **two nodes** possible (two natural frequencies other than zero). An N-rotor free system has up to N − 1 nodes.
- Stepped shaft – **equivalent length** (referred to diameter d_1): $l = l_1 + l_2(d_1/d_2)^4 + l_3(d_1/d_3)^4$.
- Geared system: inertia of rotor B referred to shaft A = **G^2I_B**, stiffness referred = G^2q ($G = N_B/N_A$).

Quantity	Formula / value
Natural frequency	$\omega_n = \sqrt{(k/m)}$; $f_n = 0.4985/\sqrt{\delta}$
Critical damping	$c_c = 2\sqrt{(km)} = 2m\omega_n$
Log decrement	$2\pi\zeta/\sqrt{(1 - \zeta^2)}$
MF at resonance	$1/2\zeta$
Phase at resonance	**90°**
Isolation	**$r > \sqrt{2}$**
Node in two-rotor	$I_AI_A = I_BI_B$
Bounds	Dunkerley ≤ true f_n ≤ Rayleigh

MEMORY TRICK

"**Root-2 is the Toll Gate**" – beyond √2 every isolator works, and there **less damping is better**. "**Dunkerley Down, Rayleigh Raised**" – lower and upper bounds.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Static deflection 1 mm – natural frequency? → **≈ 15.8 Hz**.
- Vibration isolation is possible when frequency ratio is? → **Greater than √2**.
- Phase angle between force and displacement at resonance? → **90°**.
- Critical speed of a shaft equals? → **Natural frequency of transverse vibration**.
- In a two-rotor system the node lies nearer to? → **The rotor with larger moment of inertia**.
- Gun barrels are designed for which damping? → **Critical damping** ($\zeta = 1$).

Nandyal Banking Coaching Center

3. Machine Design

NOTE

Machine Design (Design of Machine Elements) typically gives 5–8 questions in PSU/GATE-type papers. Most asked: factor of safety basis, Kf = 1 + q(Kt − 1), Goodman/Soderberg lines, rivet/weld strength, Pi = 2840d, screw efficiency & self-locking, ASME shaft code, key proportions, bearing life $(C/P)^k$, clutch torque (uniform wear), Wahl factor, Lewis equation and thin/thick cylinder formulae. Learn the standard proportions (d/4, 0.707s, 3.5d …) – they are direct one-mark questions.

3.1 Design Fundamentals ★★★

- **Design process (sequence)**: recognition of need → **problem definition / specifications** → synthesis (mechanism selection) → analysis of forces → **material selection** → design of elements (size) → modification → detailed drawing → production.
- **Factor of safety (FOS)** = failure stress ÷ allowable (working) stress. Always **> 1**.
- **Ductile materials** (static load): failure stress = **yield strength S_{yt}** → $\sigma_{allow} = S_{yt}/FOS$.
- **Brittle materials** (cast iron): no definite yield point → failure stress = **ultimate strength S_{ut}**.
- **Fatigue loading**: failure stress = **endurance limit S_e**.
- Typical FOS: steel under static load **1.5–2**; cast iron **3–4** (higher due to non-homogeneity); shock loads higher. Unwin's older table gave larger values (steel ≈ 4 dead load, ~12 shock) – based on ultimate strength.
- **Margin of safety** = FOS − 1.
- Theories of failure (cross-ref Strength of Materials): ductile → **maximum shear stress (Tresca/Guest)** or **distortion energy (von Mises–Hencky)**; brittle → **maximum principal stress (Rankine)** / Coulomb–Mohr. In pure shear, Tresca gives S_{sy} = **0.5 S_{yt}**; von Mises gives S_{sy} = **0.577 S_{yt}**.

Stress concentration

- **Stress concentration** = localised high stress near discontinuities (holes, fillets, keyways, grooves, threads, notches).
- **Theoretical stress concentration factor $K_t = \sigma_{max} \div \sigma_{nominal}$.** Depends only on **geometry and type of loading** – **not on material**.
- Small circular hole in an infinite plate under uniaxial tension: **K_t = 3**. Elliptical hole (axis 2a ⊥ load, 2b along load): **$K_t = 1 + 2a/b$** → crack (b → 0) gives $K_t \to \infty$.
- **Fatigue stress concentration factor K_f** = endurance limit of notch-free specimen ÷ endurance limit of notched specimen; **$1 \le K_f \le K_t$**.
- **Notch sensitivity q** = $(K_f - 1)/(K_t - 1)$; **$0 \le q \le 1$**. q = 0 → K_f = 1 (no notch effect); q = 1 → $K_f = K_t$ (fully sensitive).
- q increases with **notch radius** and with **material strength/hardness** (high-strength steels are more notch-sensitive). **Grey cast iron q ≈ 0** (graphite flakes already act as internal notches).
- Static load: **ductile** material → K_t usually **ignored** (local yielding redistributes stress); **brittle** material → K_t **considered**. Fatigue load → K_f applied for both.
- Reducing stress concentration: **generous fillet radius**, relief grooves, additional (auxiliary) holes/notches, **undercut** in shoulders, avoiding abrupt changes in section; rolled threads better than cut threads.

KEY FORMULAE

- $K_t = \sigma_{max}/\sigma_{nom}$; $K_f = 1 + q(K_t - 1)$; $q = (K_f - 1)/(K_t - 1)$
- Elliptical hole: $K_t = 1 + 2a/b$; circular hole (a = b): $K_t = 3$
- $\sigma_{allow} = S_{yt}/N$ (ductile) ; S_{ut}/N (brittle)

Preferred numbers & standardisation

- **Preferred numbers** are geometric series (proposed by **Charles Renard**, hence **R-series**); Indian standard **IS 1076** (ISO 3, ISO 17). Basic series R5, R10, R20, R40; derived series e.g. R10/3.
- Series ratio = $\mathbf{10^{1/n}}$ for Rn series (n terms per decade, from 1 to 10).
- Used for sizes of motors, shafts, bolts, pumps, machine-tool speeds, sheet thicknesses etc. → variety reduction.
- **Standardisation** advantages: interchangeability, mass production, lower cost & inventory, easy replacement. Standards: BIS (IS), ISO, DIN, ASTM, ANSI, JIS.

Series	Ratio ($10^{1/n}$)	Step increase	Values (1–10)
R5	**1.58**	≈ 60%	1, 1.6, 2.5, 4, 6.3, 10
R10	**1.25 (1.26)**	≈ 25%	1, 1.25, 1.6, 2, 2.5, 3.15, 4, 5, 6.3, 8, 10
R20	**1.12**	≈ 12%	1, 1.12, 1.25, 1.4, 1.6, 1.8, 2 …
R40	**1.06**	≈ 6%	1, 1.06, 1.12, 1.18, 1.25 …
R80	1.03	≈ 3%	supplementary series

Material designation (BIS)

Designation	Meaning
C40 (old IS 1570)	Plain carbon steel, **0.40% carbon** (avg.)
40C8 (new IS 1570)	Plain carbon steel, **0.40% C** and **0.8% Mn** (avg., 8 ÷ 10)
Fe 410 / E 250	Steel by **min. tensile strength 410 N/mm²** / **yield strength 250 N/mm²** (IS 2062)
FG 200	Grey cast iron, **min. tensile strength 200 N/mm²** (IS 210); grades FG 150–FG 400
SG 400/15	Spheroidal graphite (ductile) iron: **400 N/mm²** UTS, **15% elongation** (IS 1865)
BM / WM (e.g. BM 340)	Black-heart / white-heart malleable cast iron, UTS in N/mm²
40Cr4Mo2	Alloy steel: 0.40% C, Cr = 4/4 = 1%, Mo = 2/10 = 0.2%
X15Cr25Ni12	High-alloy steel (prefix **X**): 0.15% C, **25% Cr, 12% Ni** – actual percentages, no multiplying factor
XT75W18Cr4V1	High-speed tool steel (**T** = tool steel): 0.75% C, 18% W, 4% Cr, 1% V (18-4-1 HSS)
25C12S14	Free-cutting steel: 0.25% C, 1.2% Mn, 0.14% S

- Low-alloy steel multiplying factors (IS 1762): **Cr, Co, Ni, Mn, Si, W → 4**; **Al, Be, V, Pb, Cu, Nb, Ti, Ta, Zr, Mo → 10**; **P, S, N → 100**. Carbon always ÷ 100.

Nandyal Banking Coaching Center

- **Tolerances, limits & fits** (IS 919 / ISO 286, hole-basis system, IT grades, H7/g6 etc.) – see the Metrology chapter.

MEMORY TRICK

"**Duck Yields, Brick Ultimates**" – Ductile → yield, Brittle → ultimate, Fatigue → endurance. Preferred-number ratios: **R5-1.6, R10-1.25, R20-1.12, R40-1.06** (from R10 onwards each step roughly halves the % increase: 25 → 12 → 6).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- FOS for a ductile material under static load is based on? → **Yield strength**.
- K_t for a small circular hole in a wide plate in tension? → **3**.
- $K_t = 2$, $q = 0.8$; K_f = ? → $1 + 0.8(1)$ = **1.8**.
- Ratio of R10 series of preferred numbers? → $\mathbf{10^{1/10} \approx 1.25}$.
- In 40C8, "8" indicates? → **0.8% manganese**.
- Notch sensitivity of grey cast iron is nearly? → **Zero**.

3.2 Fatigue & Endurance ★★★

- **Fatigue** = failure under repeated/fluctuating stresses at stress levels **below S_{ut} (often below S_{yt})**. Fracture surface: smooth **beach marks** (crack-growth zone) + rough, crystalline final fracture zone. Cracks start at the **surface** / stress raisers.
- **S-N curve (Wöhler curve)**: stress amplitude vs number of cycles (log scale), obtained on **R.R. Moore rotating-beam** machine (completely reversed bending). Specimen: highly polished, ~7.5 mm dia.
- **Endurance limit (S_e')** = stress below which the material can withstand **infinite cycles**. Steels: knee at about **10^6–10^7 cycles**.
- **Non-ferrous metals (Al, Cu, Mg alloys)** have **no definite endurance limit** → specify **fatigue strength** at a given life (e.g. 5×10^8 cycles for Al).
- **Low-cycle fatigue**: $N < 10^3$ (designed on static strength); **high-cycle fatigue**: $N > 10^3$. At 10^3 cycles, fatigue strength ≈ **0.9 S_{ut}** (Shigley/Bhandari).
- **Steel (wrought)**: **$S_e' \approx 0.5\ S_{ut}$** (for $S_{ut} \le$ ~1400 MPa; above that S_e' ≈ 700 MPa). **Cast iron/cast steel**: $S_e' \approx$ **0.4 S_{ut}**.
- Endurance limit in reversed axial load ≈ **0.8 S_e'** (Bhandari; Shigley uses load factor 0.85); in reversed torsion ≈ **0.55–0.58 S_e'**.

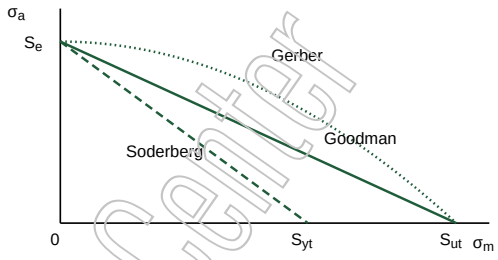
Modifying (Marin) factors

Factor	Symbol	Key facts
Surface finish	K_a	Mirror-polished = **1** > ground > machined/cold drawn > hot rolled > **as-forged (lowest)**; effect larger for high-strength steel
Size	K_b	$d \le 7.5$ mm → 1; $7.5 < d \le 50$ mm → **0.85** (Bhandari); larger size → lower S_e. Axial load: $K_b = 1$
Reliability	K_c	50% → 1.0; 90% → **0.897**; 99% → **0.814**; 99.9% → 0.753
Temperature	K_d	1 up to ~ 350 °C (Bhandari) ; reduces at higher temperature
Load / modifying for stress conc.	K_e	$K_e = 1/K_f$ (Bhandari); load factor: bending 1, axial ≈ 0.85, torsion ≈ 0.59 (Shigley)

Symbols for the factors differ between Shigley and Bhandari – learn the physical factor, not the letter.

Fluctuating stresses

Type	σ_m	$R = \sigma_{min}/\sigma_{max}$
Completely reversed	**0**	**−1**
Repeated (zero to max)	$\sigma_a = \sigma_m = \sigma_{max}/2$	**0**
Fluctuating (general)	≠ 0	between −1 and 1
Static	$\sigma_m = \sigma$, $\sigma_a = 0$	+1



Fatigue failure lines: Soderberg (S_e–S_{yt}, most conservative), Goodman (S_e–S_{ut}), Gerber parabola (S_e–S_{ut}, least conservative).

KEY FORMULAE

- $\sigma_m = (\sigma_{max} + \sigma_{min})/2$; $\sigma_a = (\sigma_{max} - \sigma_{min})/2$; stress ratio $R = \sigma_{min}/\sigma_{max}$; amplitude ratio $A = \sigma_a/\sigma_m$
- Corrected endurance limit: $S_e = K_a K_b K_c K_d K_e\ S_e'$
- **Goodman**: $\sigma_a/S_e + \sigma_m/S_{ut} = 1/N$
- **Soderberg**: $\sigma_a/S_e + \sigma_m/S_{yt} = 1/N$
- **Gerber**: $N\sigma_a/S_e + (N\sigma_m/S_{ut})^2 = 1$
- ASME elliptic: $(\sigma_a/S_e)^2 + (\sigma_m/S_{yt})^2 = 1/N^2$; Langer (first-cycle yield) line: $\sigma_a + \sigma_m = S_{yt}$
- **Miner's rule** (cumulative damage): $\Sigma\ n_i/N_i = 1$ (n_i = cycles applied, N_i = life at that stress)
- Basquin (finite life, log–log straight line): $S_f = a\ N^b$

- **Soderberg**: straight line S_e to **S_{yt}** → **most conservative**, safe against yielding too.
- **Goodman**: straight line S_e to **S_{ut}** → conservative, widely used (Bhandari recommends **modified Goodman** = Goodman + yield line).
- **Gerber**: **parabola** S_e to S_{ut} → fits test data of ductile materials best; **least conservative**.
- Effect of mean stress: **tensile σ_m reduces** allowable amplitude; compressive σ_m has little harmful effect.
- **Miner's rule** (Palmgren–Miner, linear damage) ignores order of loading.

Increases fatigue life	Decreases fatigue life
Shot peening (compressive residual stress), cold rolling of threads/fillets, surface rolling	Tensile residual stress (e.g. grinding burn, welding)
Polishing, fine surface finish	Rough surface, tool marks, as-forged skin
Nitriding, carburising, induction hardening	**Decarburisation** of surface
Generous fillets, removing stress raisers	**Corrosion** (corrosion fatigue – no endurance limit), fretting
Over-stressing below yield ("coaxing", understressing)	**Electroplating** (Cr, Ni plating) – tensile residual stress

Nandyal Banking Coaching Center

MEMORY TRICK

"**SO**derberg = **S**afest, goes to **yield** (shortest line); **G**oodman = **G**eneral straight line; **G**erber = **G**enerous curve." Line order from origin: Soderberg < Goodman < Gerber.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Most conservative fatigue criterion? → **Soderberg**.
- Gerber line is a? → **Parabola** (between S_e and S_{ut}).
- Endurance limit of a polished steel specimen ≈ ? → **0.5 S_{ut}**.
- For completely reversed stress, mean stress = ? → **Zero** (R = −1).
- Shot peening improves fatigue life by? → Inducing **compressive residual stresses** at the surface.
- Component runs 10^4 cycles at a stress with life 4×10^4; remaining damage capacity? → 1 − 0.25 = **0.75** (Miner).

3.3 Joints – Riveted, Welded & Threaded ★★★

Riveted joints

- Rivet parts: head, shank, tail (point). Heads: **snap (button)** – most common (boilers, structures); **pan**; **countersunk** – flush surface (ship hulls); conical. Rivet material: **low-carbon (mild) steel**, wrought iron, Cu, Al.
- **Lap joint** (plates overlap; single shear) and **butt joint** (plates butt, one or two cover straps; double strap → rivets in **double shear**).
- **Chain riveting** (rivets opposite) vs **zig-zag riveting** (staggered). **Diamond (lozenge) joint** used in structures – high efficiency (number of rivets increases towards middle row).
- **Unwin's formula**: rivet hole dia **d = 6√t** (d, t in mm), used when **t > 8 mm**; for thinner plates d is found from crushing = shearing equality.
- Margin (edge distance) **m = 1.5 d**. Pitch minimum **p ≥ 2d** (for proper head formation). Back pitch ≈ 0.33p + 0.67d (zig-zag, Khurmi).
- **Caulking**: making the joint **leak-proof (fluid-tight)** by hammering the bevelled edge (≈ 75–80°) of the plate with a **caulking tool** (blunt, chisel-like). **Fullering**: better method using a **fullering tool** whose edge thickness = plate thickness – presses the whole edge.
- **Boiler joints**: **longitudinal joint → butt joint** (resists hoop stress, which is **twice** longitudinal stress); **circumferential joint → lap joint**. Design per **Indian Boiler Regulations (IBR)**.
- IBR: a rivet in double shear is taken as **1.875 ×** single-shear strength (theoretically 2×).

Failure mode	Strength per pitch length
Tearing of plate between rivets	$P_t = (p - d)\, t\, \sigma_t$
Shearing of rivets	$P_s = n\,(\pi/4)\, d^2\, \tau$ (single); ×2 (or 1.875 IBR) for double shear
Crushing of rivets / plate	$P_c = n\, d\, t\, \sigma_c$
Tearing at margin	Avoided by m = 1.5d
Strength of solid plate	$P = p\, t\, \sigma_t$

KEY FORMULAE

- Efficiency η = (least of P_t, P_s, P_c) ÷ ($p\, t\, \sigma_t$)
- Tearing efficiency alone = (p − d)/p
- Unwin: d = 6√t (mm) ; boiler shell $t = pD/(2\sigma_t\eta_l) + 1$ mm (corrosion allowance)

Joint	Typical efficiency
Lap – single riveted	**45–60%**
Lap – double riveted	63–70%
Lap – triple riveted	72–80%
Butt (double strap) – single riveted	55–60%
Butt – double riveted	70–83%
Butt – triple riveted	80–90%
Butt – quadruple riveted	85–94%

Welded joints

- **Fillet weld**: leg (size) s; **throat t = s cos 45° = 0.707 s**; minimum area at throat; failure assumed along throat.
- **Transverse fillet** (load ⊥ weld line) – designed for tension (Khurmi); **parallel (side) fillet** (load ∥ weld line) – designed for **shear**. Bhandari designs both in shear (conservative). Transverse fillet is actually **stronger** than a parallel fillet of the same size.
- **Butt weld**: throat = plate thickness t (reinforcement ignored). Square butt for thin plates (< ~5 mm); single-V/U for medium; **double-V/U for thick plates**.
- Parallel fillet: add **≈ 12.5 mm** to calculated length for starting/stopping (Khurmi).
- **Eccentric loading** (load in plane of weld): **primary (direct) shear** $\tau_1 = P/A$, **secondary (torsional) shear** $\tau_2 = P\, e\, r/J$; resultant by vector addition; maximum at the **farthest point** from the centroid.
- Eccentric load ⊥ plane of weld: direct shear + bending stress $\sigma_b = M/Z$; combine as $\tau_{max} = \sqrt{(\tau^2 + (\sigma_b/2)^2)}$.
- Weld as a line: $J = A\, r^2$-type unit properties; e.g. circular fillet weld around a shaft of dia d under torque T: τ = **$2T/(\pi\, t\, d^2)$** = $2.83T/(\pi\, s\, d^2)$.

KEY FORMULAE

- Single transverse fillet: P = 0.707 s l σ_t ; double transverse: P = 1.414 s l σ_t
- Double parallel fillet: P = 1.414 s l τ ; single parallel: P = 0.707 s l τ
- Butt weld: P = t l σ_t
- Eccentric: $\tau_{res} = \sqrt{(\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos\theta)}$

Threaded fasteners (bolts)

- Designation **M16 × 1.5**: ISO metric, nominal (major) dia 16 mm, **fine pitch** 1.5 mm; **M16** alone → coarse pitch (2 mm). ISO metric thread angle **60°**; BSW **55°**.
- **Tensile stress area** $A_t = (\pi/4)[(d_p + d_c)/2]^2$ (mean of pitch and minor dia); core dia $d_c \approx$ **0.84 d** (coarse).
- **Initial tightening load** (Khurmi): **P_i = 2840 d N** (d in mm) for **fluid-tight** joints; **P_i = 1420 d N** for non-fluid-tight. Small bolts (below about M16) may be over-stressed during tightening (Khurmi).

Nandyal Banking Coaching Center

- Tightening torque T = **0.2 P_i d** (torque coefficient ≈ 0.2).
- **Bolt of uniform strength** (for **shock/impact**): shank reduced to **core diameter** (turning down) or an **axial hole drilled** → stress uniform along length, more strain energy absorbed, failure not concentrated at threads. Examples: cylinder-head and connecting-rod bolts.
- Stresses in bolt: initial tension, external load, torsional shear during tightening, bending (non-parallel faces), shear.
- **Eccentric load** in plane of bolts → primary shear + secondary shear ∝ distance from c.g. (like welds). Bracket with load ⊥ bolt axis (tilting about edge) → bolt load ∝ distance from tilting edge: $W_{max} = W e L_{max}/\Sigma L_i^2$.
- Resultant bolt load with gasket/elastic parts: $P = P_i + K P_e$, $K = k_b/(k_b + k_c)$ (stiff bolt/soft gasket → K → 1).
- Locking devices: jam (lock) nut, castle nut + split pin, Simmonds nut (fibre ring), spring washer, sawn nut, ring/grooved nut, set screws.

Power screws

Thread	Angle	Features / use
Square	0°	**Highest efficiency**, no radial (bursting) pressure; difficult to cut; **screw jacks**, presses, clamps
ACME	**29°**	Easy to cut, wear compensation with **split nut**; **lathe lead screw**
Trapezoidal (ISO metric)	**30°**	Indian/ISO equivalent of ACME, designated Tr 40 × 7
Buttress	one flank nearly ⊥ axis (≈ 45° total, ISO 3°/30°)	Load in **one direction only**; high efficiency + strength; vices, gun breech, aircraft hoists

KEY FORMULAE

- tan α = L/(π d_m) ; lead L = (no. of starts) × pitch ; μ = tan φ
- Raising torque T = W (d_m/2) tan(φ + α) ; lowering torque T = W (d_m/2) tan(φ − α)
- Efficiency (square, collar friction neglected) **η = tan α / tan(α + φ)**
- Max efficiency at α = 45° − φ/2 : **η_{max} = (1 − sin φ)/(1 + sin φ)**
- Trapezoidal/ACME: replace μ by μ sec β (β = half thread angle; 14.5° ACME, 15° trapezoidal)
- Collar friction torque $T_c = \mu_c W R_m$, $R_m = (R_1 + R_2)/2$ (uniform wear)
- Overall η = W L/(2π T_{total})

- **Self-locking**: load does not lower by itself → **φ > α** (μ > tan α). Self-locking screw has **efficiency < 50%**.
- **Overhauling**: φ < α → load descends on its own (lowering torque negative).
- Multi-start threads → larger α → higher efficiency but lose self-locking (used where speed matters).
- Bearing pressure check on nut threads: p = W/[π d_m h z]; screw-jack nut: bronze/phosphor bronze; screw: steel.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Unwin's formula for rivet diameter? → **d = 6√t** (mm).
- Throat of a fillet weld of size s? → **0.707 s**.
- Initial tension in a bolt for a fluid-tight joint? → **P_i = 2840 d N**.
- Condition of self-locking of a screw? → **φ > α** (friction angle > helix angle); η < 50%.
- Thread angle of ACME thread? → **29°**.
- Longitudinal joint of a boiler shell is generally? → **Butt joint** (circumferential = lap joint).

3.4 Shafts, Keys & Couplings ★★★

- **Shaft** – rotating, transmits power (torsion + bending); **axle** – supports rotating elements, no torque (bending only); **spindle** – short shaft (machine tools); **counter / jack shaft** – secondary shaft; **line shaft** – long shaft driving several machines.
- Materials: **40C8, 45C8, 50C4, 50C12** (plain carbon); alloy steels (Ni, Ni-Cr, Cr-V) for high strength. Small shafts – cold drawn; large – hot rolled/forged.
- **Hollow shaft**: for same weight, stronger and stiffer than solid (material near the axis carries little stress).
- Strength design: **Max shear stress (Guest)** theory for ductile → equivalent twisting moment; **Max normal stress (Rankine)** for brittle → equivalent bending moment.

KEY FORMULAE

- Power P = 2πNT/60 (W, N in rpm, T in N·m)
- Torsion: τ = 16T/(πd^3) ; hollow: τ = 16T d_o/[π(d_o^4 − d_i^4)]
- Bending: σ_b = 32M/(πd^3)
- Equivalent twisting moment $T_e = \sqrt{(M^2 + T^2)} = (\pi/16)\,\tau\, d^3$
- Equivalent bending moment $M_e = ½[M + \sqrt{(M^2 + T^2)}] = (\pi/32)\,\sigma_b\, d^3$
- **ASME code**: $d^3 = (16/\pi\tau)\sqrt{[(K_b M)^2 + (K_t T)^2]}$
- Torsional rigidity: θ = TL/(GJ) (θ in rad) ; J = πd^4/32

- **ASME code** allowable shear stress τ = **0.30 S_{yt}** or **0.18 S_{ut}**, whichever is **smaller**; reduce by **25%** if **keyways** are present.
- **K_b** = combined shock & fatigue factor for **bending**; **K_t** = for **torsion**.
- **Rigidity design**: permissible angle of twist ≈ **0.25° per metre** length for **machine-tool shafts**; line/transmission shafts ≈ **2.5–3° per metre**; common general limit **1° in a length of 20 d**. Lateral deflection limited so that gears mesh properly and bearings don't edge-load.
- **Critical (whirling) speed** of shaft = natural frequency of transverse vibration; $N_c \propto \sqrt{(g/\delta)}$ (cross-ref Theory of Machines).

Nature of load	K_b	K_t
Stationary shaft		
Gradually applied	1.0	1.0
Suddenly applied	1.5–2.0	1.5–2.0
Rotating shaft		
Gradually applied / steady	**1.5**	**1.0**
Suddenly applied, minor shock	1.5–2.0	1.0–1.5
Suddenly applied, heavy shock	2.0–3.0	1.5–3.0

Nandyal Banking Coaching Center

Keys

- **Key**: temporary (detachable) fastener between shaft and hub to transmit torque; usually of **mild steel / 40C8**, made **weaker** than shaft and hub so that it fails first (cheap to replace).
- **Sunk key** (half in shaft, half in hub): **rectangular** w = d/4, t = 2w/3 = **d/6**; **square** w = t = **d/4**. Taper on top **1 in 100**.
- **Gib-head key**: taper sunk key with head for easy removal.
- **Feather key**: parallel key fixed to shaft or hub → **permits axial sliding** of hub (gear-change, clutches).
- **Woodruff key**: **semicircular disc**; adjusts itself to taper of hub (**self-aligning**); used in **machine tools & automobiles** on tapered shafts; deep keyway **weakens the shaft** more.
- **Saddle key**: fits in hub keyway only. **Hollow saddle key** – concave bottom, power transmitted by **friction** only (light duty). **Flat saddle key** – sits on a flat on shaft; slightly stronger.
- **Tangent keys**: pair at **90°**, each transmits torque in one direction (heavy duty). **Kennedy keys**: pair of square taper keys fitted **diagonally** (at 90° to each other) – heavy duty.
- **Round key / pin**: circular section, light duty.
- **Splines**: keys integral with shaft (4, 6, 10, 16 splines); straight-sided or **involute**; higher torque, **axial sliding** – automobile gear boxes, propeller shafts, machine-tool spindles.
- **Effect of keyway** (H.F. Moore): shaft strength factor **e = 1 – 0.2(w/d) – 1.1(h/d)** (w = keyway width, h = keyway depth).

KEY FORMULAE

- Shearing of key: $T = l\, w\, \tau\, (d/2)$
- Crushing of key: $T = l\, (t/2)\, \sigma_c\, (d/2)$
- Equally strong in shear & crushing: $w/t = \sigma_c/(2\tau)$ → **square key (w = t) when $\sigma_c = 2\tau$**
- Key as strong as shaft in shear (w = d/4, same τ): **$l = \pi d/2 \approx 1.571\, d$**
- Usual hub/key length ≈ 1.5 d

Couplings

Coupling	Type	Key points / proportions
Muff / sleeve	Rigid	Sleeve D = **2d + 13 mm**, L = **3.5 d**; cast iron
Split-muff / clamp (compression)	Rigid	D = 2d + 13 mm, L = 3.5 d (same as muff, Khurmi; some texts D ≈ 2.5 d); easy assembly without shifting shafts; torque by friction
Flange coupling	Rigid	Hub dia 2d, hub length 1.5d, bolt-circle dia **3d**, outer dia **4d**, flange thickness **0.5d**; protected type has shrouding
Bush-pin (pin-bush) flexible	Flexible	**Rubber/leather bushes** over pins → absorbs **shock & slight misalignment**; motor–pump drives
Oldham	Flexible	**Parallel shafts with lateral offset**; inversion of double slider crank; equal speeds
Universal (Hooke's)	Flexible	**Intersecting shafts**; speed of driven shaft fluctuates; **two joints** give constant velocity ratio; automobile propeller shaft
Fluid coupling	Flexible	Torque through oil, smooth start, no torque multiplication

- Rigid couplings need **perfect alignment**; flexible couplings tolerate misalignment and damp shocks.
- Flange bolts are designed in **shear**: $T = n\, (\pi/4)\, d_b^2\, \tau\, (D_1/2)$, D_1 = pitch-circle dia.

MEMORY TRICK

Keys: "**Four-Six**": rectangular key w = d/**4**, t = d/**6**. Flange coupling "**2-3-4, half**": hub 2d, bolt circle 3d, outer 4d, flange ½d. "**Oldham Offset, Hooke Hits angle**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Allowable shear stress in ASME code? → **0.3 S_{yt} or 0.18 S_{ut}**, whichever is less (–25% with keyway).
- Width of a rectangular sunk key for shaft dia d? → **d/4** (thickness d/6).
- Key that permits axial movement of hub? → **Feather key** (or splines).
- Key used in automobiles / machine tools, semicircular? → **Woodruff key**.
- Square key equally strong in shear and crushing if? → **$\sigma_c = 2\tau$**.
- Coupling for parallel shafts with small lateral misalignment? → **Oldham coupling**.

3.5 Bearings ★★★

Sliding contact (journal) bearings

- Lubrication regimes: **thick-film (hydrodynamic)** – surfaces fully separated, pressure generated by **relative motion + wedge-shaped (converging) film** + viscosity; **thin-film (boundary)** – partial metal contact; **hydrostatic** – oil supplied at high pressure by external pump (works even at zero speed); **elastohydrodynamic (EHL)** – rolling bearings, gears, cams.
- Conditions for hydrodynamic film: **relative motion**, **converging wedge**, adequate **viscosity** and continuous oil supply. Journal runs **eccentric**; minimum film thickness h_0 on the loaded side.
- **Squeeze-film** lubrication: surfaces approaching each other (e.g. gudgeon pin).
- **Petroff's equation**: for lightly loaded, concentric journal (no eccentricity) – gives friction coefficient.
- **Sommerfeld number S** – dimensionless bearing characteristic number: **$S = (r/c)^2\, \mu n_s/p$**. Larger S → thicker film, lower eccentricity.
- **Bearing characteristic number / bearing modulus** = **ZN/p** (Z = absolute viscosity, N = speed, p = bearing pressure). Graph of μ vs ZN/p (**McKee / Stribeck curve**): minimum friction at **K**; left of K → boundary/thin-film; right → hydrodynamic.
- Operating value: **ZN/p ≈ 3K** (to cope with fluctuating loads); for heavy loads ≈ 15K (Khurmi).
- Ratio **l/d**: 1–2 typical; short bearing l/d < 1 (automobile engines ≈ 0.6–1.0). Diametral clearance ratio c/d ≈ **0.001**.
- Bearing types: full journal (360°), partial (< 360°, e.g. 120°), fitted (clearance zero); thrust bearings – footstep/pivot, collar; **Kingsbury/Michell tilting-pad** thrust bearing.
- **Bearing materials**: **Babbitt (white metal)** – tin-base (Sn-Sb-Cu) or lead-base, excellent **conformability & embeddability**, low fatigue strength; bronzes (phosphor bronze – high load); Cu-Pb, Al alloys; PTFE, nylon; sintered porous bronze (**oil-impregnated, self-lubricating**).

Nandyal Banking Coaching Center

KEY FORMULAE

- Unit bearing pressure p = W/(l d)
- **Petroff**: $\mu_f = 2\pi^2 (\mu n_s/p)(r/c)$ (n_s in rev/s)
- **Sommerfeld number** $S = (r/c)^2 (\mu n_s/p)$
- **McKee**: $\mu = (33/10^8)(ZN/p)(d/c) + k$; Z in kg/m·s (cP ÷ 1000), N in rpm, p in N/mm² ; k ≈ 0.002 for l/d = 0.75–2.8
- Heat generated $H_g = \mu W V$ (V = rubbing velocity = πdN/60)
- Heat dissipated $H_d = C A (t_b - t_a)$, A = l d (projected area)

Rolling contact (antifriction) bearings

Type	Load	Typical use
Deep groove ball (6xxx)	Radial + moderate axial (both directions)	Most widely used – motors, gearboxes
Self-aligning ball (1xxx)	Radial; tolerates **misalignment** (spherical outer race)	Long shafts, agricultural machinery
Angular contact ball (7xxx)	Radial + **heavy axial in one direction**	Machine-tool spindles (used in pairs)
Cylindrical roller (N, NU)	**Heavy radial**, little/no axial	Large motors, rolling mills
Taper roller (3xxxx)	Heavy **radial + axial** combined	**Automobile wheel hubs**, differential, machine-tool spindles
Spherical roller (2xxxx)	Heavy radial + axial; self-aligning	Heavy shafts, vibrating screens
Needle roller (NA)	Radial; **limited radial space** (L/d > 4)	Gudgeon pins, universal joints, rocker arms
Thrust ball (5xxxx)	**Pure axial** only; not for high speed	Crane hooks, vertical shafts

- **Designation** (e.g. **6208**): 6 = deep groove ball; 2 = diameter series (**light**); last two digits × 5 = **bore = 40 mm**. Exceptions: 00 → 10 mm, 01 → 12 mm, 02 → 15 mm, 03 → 17 mm. Series: 60xx extra-light, 62xx light (200), 63xx medium (300), 64xx heavy (400).
- **Rating life L_{10}** = life (million revolutions) that **90%** of a group of identical bearings will reach or exceed (10% failure). Median life $L_{50} \approx 5 \times L_{10}$.
- **Dynamic load capacity C** = constant radial load which a bearing can carry for an L_{10} life of **1 million revolutions**.
- **Static load capacity C_0** = load producing total permanent deformation of **0.0001 × rolling-element diameter** (Stribeck's equation for balls: $C_0 = k n D^2/5$).
- **V = rotation factor**: 1 when **inner race rotates**, **1.2** when outer race rotates.
- Rolling bearing friction ≈ 0.001–0.005; low starting torque; but noisy, finite fatigue life, costly, poor shock tolerance compared with journal bearings.
- Failure mode: **surface fatigue – pitting/spalling**; Brinelling under static overload.

KEY FORMULAE

- $L_{10} = (C/P)^k$ million rev ; **k = 3 (ball), k = 10/3 (roller)**
- Life in hours $L_{10h} = L_{10} \times 10^6/(60\ n)$
- Equivalent dynamic load $P = X V F_r + Y F_a$ (× service/load factor K_s)
- Reliability (Weibull): $L/L_{10} = [\ln(1/R)/\ln(1/0.9)]^{1/b}$, b ≈ 1.17 (Bhandari)
- Doubling load (ball) → life becomes **1/8**; life doubled → C increases by $2^{1/3} = 1.26$
- Variable load (cubic mean): $P_e = [\Sigma(P_i^3 N_i)/\Sigma N_i]^{1/3}$

MEMORY TRICK

"**Ball = 3, Roller = 3⅓**" (rollers have line contact, slightly larger exponent). "**Inner turns → V = 1**". Bore code: "**10-12-15-17, then ×5**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Bore of bearing 6310? → 10 × 5 = **50 mm**.
- Life exponent for roller bearings? → **10/3**.
- Load on a ball bearing is doubled; life becomes? → **1/8**.
- Rating life is the life reached by what % of bearings? → **90%**.
- Bearing modulus is? → **ZN/p**; operate at about **3 times** its minimum friction value.
- Bearing for combined heavy radial and thrust loads (car wheel)? → **Taper roller bearing**.

3.6 Brakes & Clutches ★★★

Friction clutches

- **Uniform pressure theory** (p = constant): valid for **new** clutches (well-fitted surfaces).
- **Uniform wear theory** (p·r = constant): valid after initial wear; **maximum pressure at inner radius**. Gives **lower torque** → **used for design (conservative/safer)**.
- Number of pairs of contact surfaces n: single-plate clutch with both sides effective → **n = 2**; multi-plate: **$n = n_1 + n_2 - 1$** (n_1 driving, n_2 driven plates).
- For uniform wear, maximum torque for given outer radius when **$R_2 = R_1/\sqrt{3} = 0.577\ R_1$**.
- **Multi-plate clutch**: compact, larger torque for same diameter – **motorcycles**, scooters, automatic transmissions (wet type in oil).
- **Cone clutch**: semi-cone angle α ≈ **12.5°** (12–15°); smaller α → less axial force but risk of **jamming** (self-engaging if tan α < μ). Replaced by plate clutches in automobiles.
- **Centrifugal clutch**: shoes thrown out by centrifugal force; engagement starts at about **¾ of running speed** (spring force set accordingly); mopeds, small engines – automatic engagement.
- Friction lining materials: woven asbestos (now **asbestos-free** organic/ceramic/sintered), cork, leather; μ ≈ 0.25–0.4 (dry).
- Positive (jaw/claw, dog) clutches: no slip, cannot engage at high relative speed.

Nandyal Banking Coaching Center

KEY FORMULAE

- Uniform pressure: $T = n \mu W R_f$, $R_f = (2/3)(R_1^3 - R_2^3)/(R_1^2 - R_2^2)$; $W = \pi p (R_1^2 - R_2^2)$
- **Uniform wear**: $T = n \mu W R_m$, $\mathbf{R_m = (R_1 + R_2)/2}$; $W = 2\pi C (R_1 - R_2)$, $C = p_{max} R_2$
- Flat pivot (full disc), uniform pressure: $T = (2/3) \mu W R$; uniform wear: $T = ½ \mu W R$
- **Cone clutch** (uniform wear): $T = \mu W R_m / \sin \alpha = \mu W R_m \operatorname{cosec} \alpha$; engaging force $W_e = W_n(\sin \alpha + \mu \cos \alpha)$
- **Centrifugal clutch**: $P_c = m \omega^2 r_g$; $T = n \mu (P_c - P_s) R$ (n = no. of shoes)

Brakes

- Brake absorbs **kinetic/potential energy** and dissipates it as **heat**; capacity depends on unit pressure, μ, rubbing speed and heat-dissipation ability (**pV** value).
- **Single block (shoe) brake**: for short shoe (contact angle < 60°), $T_b = \mu R_n r$. For **long shoe** ($2\theta > 60°$) use equivalent $\mu' = \mathbf{4\mu \sin \theta/(2\theta + \sin 2\theta)}$.
- **Self-energising** brake: moment of friction force **assists** the applied force (reduces effort). **Self-locking**: applied force required ≤ 0, i.e. friction moment ≥ normal-force moment (for pivot offset a and lever arm x: **μ a ≥ x**) – brake applies itself; undesirable for vehicles, used in back-stops.
- **Double block brake**: two shoes diametrically opposite → **no bending load on shaft** (railway wheels, cranes, electric hoists).
- **Simple band brake**: one end of band fixed to fulcrum. **Differential band brake**: both ends attached to lever on either side of fulcrum – can be made **self-locking** (used for back-stop of hoists/conveyors).
- **Band-and-block brake**: wooden/friction blocks fixed inside band; each block subtends 2θ.
- **Internal expanding shoe brake**: automobiles (drum brakes); leading shoe (self-energising) and trailing shoe. Disc brakes – better heat dissipation, less fade.
- Temperature rise: **$\Delta T = E/(m c)$** (E = energy absorbed, m = mass of brake drum, c = specific heat).

KEY FORMULAE

- Band brake: $\mathbf{T_1/T_2 = e^{\mu\theta}}$; braking torque $T_b = (T_1 - T_2) r$ (r = drum radius + ½ band thickness)
- Band-and-block (n blocks): $T_1/T_2 = [(1 + \mu \tan \theta)/(1 - \mu \tan \theta)]^n$
- Kinetic energy absorbed: $E = ½ m (v_1^2 - v_2^2) + ½ I(\omega_1^2 - \omega_2^2)$ + potential energy
- Heat generated rate = $\mu R_n v$

Dynamometers

Absorption type (power wasted as heat)	Transmission type (power measured, transmitted)
Prony brake – wooden blocks on pulley, lever + weight	**Epicyclic-train** dynamometer
Rope brake – $P = (W - S) \pi(D + d) N/60$	**Belt transmission** (Froude / Tatham)
Hydraulic (Froude water brake) – high powers	**Torsion dynamometer** (Bevis–Gibson flash-light)
Eddy-current / electrical (DC swinging-field)	Strain-gauge torque meters

MEMORY TRICK

"**Wear = Worst-case = Wise for design**" (uniform wear gives lower torque). Clutch plates: "$\mathbf{n_1 + n_2 - 1}$". Dynamometers: "**Prony & Rope Absorb; Belt, Epicyclic & Torsion Transmit**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Which theory is used for design of clutches and why? → **Uniform wear**; gives smaller (safer) torque.
- Mean radius of friction surface under uniform wear? → $\mathbf{(R_1 + R_2)/2}$.
- Multi-plate clutch with 4 driving and 3 driven plates; pairs of contact surfaces? → 4 + 3 − 1 = **6**.
- Ratio of tensions in a band brake? → $\mathbf{e^{\mu\theta}}$.
- Which band brake can be self-locking? → **Differential band brake**.
- Prony brake is which type of dynamometer? → **Absorption**.

3.7 Springs, Belts & Gears Design ★★☆

Helical springs

- **Spring index C = D/d** (D = mean coil dia, d = wire dia); recommended **4 ≤ C ≤ 12** (C < 4 → difficult to coil, high stress; C > 12 → buckling/tangling). Preferred 6–9 (Bhandari).
- Close-coiled helical compression/tension spring: wire under **torsional shear** (+ direct shear + curvature effect).
- **Wahl's factor K** accounts for **direct shear + curvature**; shear stress factor K_s accounts for direct shear only.
- End styles & inactive coils: plain (0), plain-ground (½–1), squared (2), **squared & ground (2)** – total coils n′ = n + 2; best seating.
- **Solid length** = n′ d (squared & ground). Free length = solid length + max deflection + clearance (≈ 15% of δ_{max}).
- **Buckling**: compression spring likely to buckle when **free length / mean dia > 4** (Bhandari) → use guide rod or sleeve.
- **Surge**: resonance of spring when forcing frequency (e.g. cam/valve) coincides with its natural frequency → coils clash, failure. Remedy: natural frequency ≥ **15–20 ×** operating frequency; friction dampers; variable-pitch springs.
- **Springs in series**: $1/k = 1/k_1 + 1/k_2$; **parallel**: $k = k_1 + k_2$. Concentric (nested) springs = parallel; opposite hand of coiling to avoid tangling.
- Materials: patented & cold-drawn steel, **oil-hardened & tempered** wire, **music (piano) wire** (highest strength, small springs), stainless steel, phosphor bronze (corrosion/electrical), 50Cr1V23 (Cr-V) for valve springs. Shot peening improves spring fatigue life.

KEY FORMULAE

- $\tau = K (8WD)/(\pi d^3) = K (8WC)/(\pi d^2)$
- **Wahl's factor $\mathbf{K = (4C - 1)/(4C - 4) + 0.615/C}$** ; $K_s = 1 + 1/(2C)$
- Deflection $\delta = 8WD^3n/(Gd^4) = 8WC^3n/(Gd)$
- Stiffness $k = W/\delta = Gd^4/(8D^3n)$; energy $U = ½ W \delta$

Leaf (laminated) springs

- Used in automobile suspensions (semi-elliptic); **graduated leaves** + **extra full-length leaves** (master leaf with eyes).
- Without pre-stress, full-length leaves are stressed **50% more** than graduated leaves.

Nandyal Banking Coaching Center

- **Nipping**: giving full-length leaves a **greater radius of curvature** than graduated leaves (initial gap C) so that after assembly (with central/U-bolt clamp) all leaves are **equally stressed** under full load. Initial gap $C = 2WL^3/(n\,E\,b\,t^3)$ (Khurmi).
- Leaf spring of uniform strength = triangular/trapezoidal plate cut into strips.

KEY FORMULAE

- Equalised (with nipping) stress $\sigma = 6WL/(n\,b\,t^2)$ (cantilever, W at end, L = length, n = total leaves)
- Without nipping: $\sigma_F = 18WL/[bt^2(2n_G + 3n_F)]$, $\sigma_G = 12WL/[bt^2(2n_G + 3n_F)] \rightarrow \sigma_F/\sigma_G = 1.5$
- Deflection (cantilever) $\delta = 12WL^3/[E\,b\,t^3(2n_G + 3n_F)]$

Belt selection (cross-ref Theory of Machines)

- Flat belts selected from manufacturer's **power rating per mm width** (at 180° and v = 10 m/s), corrected by **load correction factor** and **arc-of-contact factor**.
- **V-belts (IS 2494)**: sections **A, B, C, D, E** (top width ≈ 13, 17, 22, 32, 38 mm); belt included angle **40°**; pulley groove 34–38°. V-belt $T_1/T_2 = e^{\mu\theta\,\mathrm{cosec}\,\beta}$ (β = half groove angle) → grips better than flat.
- Max power when centrifugal tension $T_c = T_{max}/3$, at $v = \sqrt{(T_{max}/3m)}$. Belt speed ≈ 20–25 m/s economical.
- Minimum arc of contact on small pulley; centre distance; number of belts = design power ÷ rating per belt.

Gear tooth strength

- **Lewis equation** (beam strength): tooth treated as **cantilever**; load at **tip**, carried by **one pair**; radial component and stress concentration ignored; static.
- **Lewis form factor Y** depends on **number of teeth** and **tooth profile** ($Y = \pi y$). 20° full depth: **$Y = \pi(0.154 - 0.912/T)$**; 14.5°: $Y = \pi(0.124 - 0.684/T)$. More teeth → larger Y.
- When pinion and gear are of different materials, weaker is the one with smaller **$\sigma_b \cdot Y$** product → design that one.
- **Barth velocity factor C_v** (v in m/s): **$3/(3 + v)$** ordinary cut gears ($v < 10$); **$6/(6 + v)$** carefully cut ($v < 20$); **$5.6/(5.6 + \sqrt{v})$** precision ($v > 20$).
- **Buckingham dynamic load**: $F_d = F_t + F_i$; accounts for tooth errors (C = deformation factor).
- **Wear (surface) strength** (Buckingham): $F_w = d_p\,b\,Q\,K$ – based on Hertz contact stress; failure = **pitting**.
- Design check: F_s (beam strength) ≥ F_d and $F_w \geq F_d$.
- Face width **b = 9.5 m to 12.5 m** (usually **10 m**); m = module.
- Minimum teeth on pinion to avoid interference: **20° full depth – 18** (theory 17 with rack); **14.5° – 32**; 20° stub – 14.
- **Helical gears**: helix angle ψ = **15°–25°** (single), 25°–45° for **herringbone/double helical** (axial thrust cancels). Virtual (formative) number of teeth **$T_v = T/\cos^3\psi$** → used for Lewis form factor. Normal module $m_n = m\cos\psi$.
- **Bevel gears**: $T_v = T/\cos\delta$ (δ = pitch angle) – Tredgold's approximation.
- **Worm gears**: large reduction (up to ≈ 100:1) in one stage, non-intersecting shafts at 90°; worm usually case-hardened steel, wheel **phosphor bronze**; **self-locking (irreversible)** when $\varphi > \lambda$; efficiency low (≈ 50–90%), heat dissipation is the limiting factor.
- Gear failures: **tooth breakage** (bending fatigue), **pitting** (surface fatigue), **scoring** (lubrication failure), abrasive wear, corrosive wear.

KEY FORMULAE

- **Lewis**: $\sigma_b = F_t/(b\,m\,Y) \Leftrightarrow F_s = \sigma_b\,b\,m\,Y$ (or $\sigma_b\,b\,p_c\,y$, $p_c = \pi m$)
- Tangential load $F_t = 2T/d = P/v$; with C_v: $F_t = \sigma_o\,C_v\,b\,m\,Y$
- **Buckingham**: $F_d = F_t + 21v(bC + F_t)/[21v + \sqrt{(bC + F_t)}]$ (v m/s, b mm, C N/mm)
- **Wear load**: $F_w = d_p\,b\,Q\,K$; $Q = 2T_G/(T_G + T_P)$ (external) ; $K = \sigma_{es}^2 \sin\varphi\,(1/E_P + 1/E_G)/1.4$
- Helical: $T_v = T/\cos^3\psi$; axial thrust $F_a = F_t \tan\psi$
- Worm: $\tan\lambda = L/(\pi\,d_w)$; $\eta = \tan\lambda/\tan(\lambda + \varphi)$ (pressure angle neglected) ; with normal pressure angle φ_n: $\eta = (\cos\varphi_n - \mu\tan\lambda)/(\cos\varphi_n + \mu\cot\lambda)$

Parameter	Typical value
Spring index C	**4–12**
Inactive coils, squared & ground	**2**
Gear face width	**≈ 10 m** (9.5–12.5 m)
Helix angle (helical / herringbone)	15–25° / 25–45°
V-belt included angle	**40°**
Min pinion teeth, 20° full depth	**18**

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Wahl's factor accounts for? → **Direct shear and curvature** of coil.
- Equalising stresses in leaves of a leaf spring is done by? → **Nipping**.
- In Lewis equation, the tooth is treated as? → **Cantilever beam** with load at the tip.
- Formative number of teeth for a helical gear? → **$T/\cos^3\psi$**.
- Two springs of stiffness k in series; equivalent stiffness? → **k/2**.
- Usual material of worm wheel? → **Phosphor bronze**.

3.8 Pressure Vessels & Flywheel Design ★☆☆

- **Thin cylinder**: $t < D/20$ ($D/t > 20$, Khurmi/Bhandari) – stress assumed **uniform** across thickness; radial stress neglected.
- Hoop (circumferential) stress = **2 ×** longitudinal stress → longitudinal seam is critical; cylinder bursts along a **longitudinal** line.
- Thin sphere: $\sigma = pD/(4t)$ (same as longitudinal stress of cylinder) → sphere needs **half** the thickness of a cylinder.
- **Thick cylinder**: stresses vary across wall – **Lamé's equations**; hoop stress **maximum at inner surface**; radial stress = $-p_i$ at inner and 0 at outer surface (external p = 0).
- Thick-cylinder thickness equations:

Equation	Basis / application
Lamé's	**Max principal stress** theory → **brittle** materials (cast iron, cast steel)
Clavarino's	**Max strain** theory, **closed ends** – ductile (steel)
Birnie's	**Max strain** theory, **open ends** – ductile (pump cylinders, gun barrels)
Barlow's	High-pressure oil/gas pipes, thin-ish: $t = pD_o/(2\sigma_t)$

Nandyal Banking Coaching Center

KEY FORMULAE

- Thin cylinder: $\sigma_t = pD/(2t)$; $\sigma_l = pD/(4t)$; design **$t = pD/(2\sigma_t\eta) + CA$** ($\eta$ = joint efficiency, CA = corrosion allowance, ≈ 1–3 mm)
- Thin sphere: $\sigma = pD/(4t)$
- Lamé: $\sigma_r = a - b/r^2$; $\sigma_t = a + b/r^2$
- Lamé thickness: $t = r_i[\sqrt{((\sigma_t + p)/(\sigma_t - p))} - 1]$
- Clavarino: $t = r_i[\sqrt{((\sigma_t + (1 - 2\nu)p)/(\sigma_t - (1 + \nu)p))} - 1]$
- Birnie: $t = r_i[\sqrt{((\sigma_t + (1 - \nu)p)/(\sigma_t - (1 + \nu)p))} - 1]$
- Lamé: if $p \geq \sigma_t$, no thickness can contain the pressure → use **compound cylinders** (shrink fit) or **autofrettage**

- **Compound cylinder**: outer shrunk on inner → initial compressive hoop stress in inner tube → more uniform stress distribution.
- **Autofrettage**: pre-pressurising beyond yield so inner layers get compressive residual stress (gun barrels, high-pressure tubing).
- Codes: unfired pressure vessels – **IS 2825** (ASME Sec VIII equivalent); boilers – **IBR**.

Flywheel design

- Flywheel stores energy and limits **speed fluctuation within a cycle** (governor controls mean speed over cycles) – cross-ref Theory of Machines (turning-moment diagram).
- **Rim** provides ≈ **90%** of moment of inertia (arms and hub ≈ 10%) – Khurmi assumes rim effect only, or rim = 0.9 I.
- Rim stress (rotating thin ring, arms neglected): **$\sigma_t = \rho v^2$** → limits rim speed: **cast iron ≈ 25–30 m/s**, steel up to ~ 50 m/s.
- **Arms**: usually **6** (4–8 depending on size); **elliptical cross-section**, major axis ≈ **2 ×** minor axis (in plane of rotation); designed for bending as cantilevers fixed at hub. Small flywheels – solid web (disc) type.
- Cast iron flywheels most common; rim split in halves for large wheels.

KEY FORMULAE

- Maximum fluctuation of energy $\Delta E = I\omega^2 C_s = m k^2 \omega^2 C_s = m v^2 C_s$ (rim)
- Coefficient of fluctuation of speed $C_s = (\omega_{max} - \omega_{min})/\omega$
- Rim hoop stress $\sigma = \rho v^2$

MEMORY TRICK

"**Lamé – Leaky (brittle); Clavarino – Closed; Birnie – Bare (open) ends**". Thin vessel: "**Hoop is Half-D-over-t**": $\sigma_h = pD/2t$.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Lamé's equation is used for thick cylinders of? → **Brittle materials** (max principal stress theory).
- Clavarino's equation applies to? → Ductile material, **closed-end** cylinders.
- Ratio of hoop to longitudinal stress in a thin cylinder? → **2**.
- Maximum hoop stress in a thick cylinder under internal pressure occurs at? → **Inner surface**.
- Hoop stress in a rotating flywheel rim? → **ρv^2**.
- Cross-section of flywheel arms is usually? → **Elliptical**.

Nandyal Banking Coaching Center

4. Fluid Mechanics & Fluid Machinery

> **NOTE**
> Usually **6–9 of 60** questions. Favourite areas: viscosity & its temperature effect, capillarity, centre of pressure, metacentric height, Bernoulli assumptions, Cd of flow meters, laminar flow results (f = 64/Re, V_{max} = 2V), max power transmission (h_f = H/3), jet efficiency (29.6% / 50% / 100%), specific speed ranges, Pelton speed ratio, draft tube, NPSH, affinity laws and air-vessel savings. Learn the "standard values" — they are the answers.

4.1 Fluid Properties & Statics ★★★

- **Fluid** = substance that deforms **continuously** under any shear stress, however small. A fluid at rest carries **no shear stress**.
- **Ideal fluid** = **inviscid + incompressible** (imaginary). **Real fluid** has viscosity.
- **Density** of water = **1000 kg/m³ at 4 °C** (maximum). Specific weight w = ρg = **9.81 kN/m³**. Specific gravity of mercury = **13.6**; sea water ≈ 1.025.
- **Specific volume** = 1/ρ (m^3/kg) – used mainly for gases.
- **Newton's law of viscosity**: **τ = μ (du/dy)**; μ = dynamic (absolute) viscosity.
- **Kinematic viscosity** ν = μ/ρ, unit **m^2/s**; CGS unit **stoke** = 1 cm^2/s = **10^{-4} m^2/s**.
- **Poise** = 1 dyne·s/cm^2 = **0.1 N·s/m^2 (Pa·s)**; 1 centipoise = 10^{-3} Pa·s.
- Water at 20 °C: μ ≈ **1.0×10^{-3} Pa·s (≈ 1 cP)**, ν ≈ **1.0×10^{-6} m^2/s**. Air at 20 °C: μ ≈ **1.8×10^{-5} Pa·s**, ν ≈ **1.5×10^{-5} m^2/s** (air has ~15 times the ν of water).
- **Temperature effect**: viscosity of **liquids decreases** with temperature (cause: **cohesion** falls); viscosity of **gases increases** with temperature (cause: **molecular momentum exchange** rises). Effect of pressure on viscosity is negligible (except at very high pressures).
- Dimensions: μ = **$ML^{-1}T^{-1}$**; ν = **L^2T^{-1}**.
- **Viscometers**: capillary tube (Hagen–Poiseuille), falling sphere (**Stokes' law**), rotating cylinder, Saybolt/Redwood/Engler (efflux time – empirical, "seconds").

Fluid type	τ – du/dy relation	Examples
Newtonian	Linear through origin, μ constant	Water, air, kerosene, petrol, thin oils
Pseudoplastic (shear-thinning)	$\tau = K(du/dy)^n$, **n < 1**	Blood, polymer solutions, paper pulp, paints
Dilatant (shear-thickening)	**n > 1**	Quicksand, starch (corn-flour) in water
Bingham plastic	$\tau = \tau_0 + \mu_p(du/dy)$; needs **yield stress**	Toothpaste, sewage sludge, drilling mud
Thixotropic	μ_{app} **decreases with time** of shearing	Printer's ink, ketchup, some paints
Rheopectic	μ_{app} **increases with time**	Gypsum suspension, bentonite sol
Ideal fluid	τ = 0 (lies on du/dy axis)	—
Ideal solid (elastic)	du/dy = 0 (lies on τ axis)	—

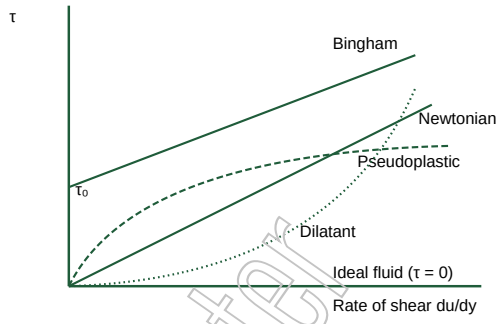


Fig 4.1 Rheological diagram (τ vs du/dy). Ideal solid lies on the τ-axis.

- **Surface tension** σ (N/m, dimension **MT^{-2}**) – due to **cohesion**; decreases with rise in temperature. Water–air at 20 °C ≈ **0.073 N/m**; mercury–air ≈ **0.48–0.51 N/m**.
- Pressure excess: **liquid droplet Δp = 4σ/d**; **soap bubble Δp = 8σ/d** (two surfaces); **liquid jet Δp = 2σ/d**.
- **Capillary rise h = 4σ cosθ/(ρgd)** ∝ 1/d. Water–glass θ ≈ **0°** → rise (adhesion > cohesion); mercury–glass θ ≈ **130–140°** → **capillary depression** (cohesion > adhesion).
- Between two parallel plates with gap t: h = 2σ cosθ/(ρgt) – half that in a tube of diameter t.
- To keep capillary error small, manometer tubes should have diameter ≥ **6 mm** (common textbook value).
- **Compressibility** β = 1/K; **bulk modulus K = −dp/(dV/V)**; K of water ≈ **2.1–2.2 GPa** (about 20,000 times that of air at 1 atm, whose isothermal K ≈ 0.1 MPa). For gases: isothermal K = **p**, adiabatic K = **γp**.
- **Sonic velocity** C = √(K/ρ); in water ≈ **1450–1500 m/s**; in gas C = √(γRT) ≈ **340 m/s** in air at 15–20 °C.
- **Vapour pressure** = partial pressure of vapour in equilibrium with liquid; rises with temperature. Water: ≈ **2.34 kPa at 20 °C**, **101.3 kPa at 100 °C**. Mercury has very low p_v → used in **barometers**.
- **Cavitation** = formation of vapour bubbles where local pressure falls **below vapour pressure**, followed by their **collapse** in high-pressure zone → pitting, noise, vibration, fall in efficiency. Sites: **turbine runner outlet / draft tube inlet**, **pump impeller eye (inlet)**, ship propellers, valves.
- **Pascal's law**: pressure at a point in a static fluid is equal in all directions. **Hydrostatic law**: dp/dz = −ρg → p = ρgh.
- **Absolute = gauge + atmospheric**; vacuum = atmospheric – absolute. Standard atmosphere = **101.325 kPa = 760 mm Hg = 10.33 m of water**.

Pressure device	Key point
Piezometer	Simplest; **cannot measure gas or negative (vacuum) pressure**, nor high pressure
Simple U-tube manometer	Measures +ve and −ve pressure, uses heavier liquid (Hg)
Differential U-tube	Pressure difference between two points (e.g. venturimeter)
Inverted U-tube	Uses **lighter liquid** (air/oil) – **small** differences
Inclined manometer	More **sensitive**; sensitivity ∝ **1/sinθ**
Micromanometer	Very small pressure differences

Nandyal Banking Coaching Center

Bourdon tube gauge	Mechanical; elliptical tube straightens under pressure; reads **gauge** pressure
Barometer	Atmospheric (absolute) pressure; Torricelli (mercury)

- **Total pressure** on a plane surface: **F = ρgA $\bar{x}$** ($\bar{x}$ = depth of centroid) – same for any inclination.
- **Centre of pressure** (vertical surface): **$h^* = \bar{x} + I_G/(A\,\bar{x})$**; inclined: $h^* = \bar{x} + I_G \sin^2\theta/(A\,\bar{x})$. Centre of pressure always lies **below the centroid**; approaches centroid as depth increases. For a **horizontal** surface, they coincide.
- Vertical rectangle with top edge at free surface: h^* = **2d/3**. Triangle, base at surface (apex down): h^* = **d/2**; apex at surface (base down): h^* = **3d/4**.
- Dam/wall of height H: force = **$\rho g H^2/2$** per unit width, acting at **H/3 above base**.
- **Curved surface**: horizontal component = force on **vertical projection**; vertical component = **weight of liquid** (real or imaginary) above the surface.
- **Archimedes' principle**: buoyant force = weight of fluid displaced; acts through **centre of buoyancy** (centroid of displaced volume).
- **Metacentre (M)** = point about which a floating body oscillates when given a small tilt. **GM = I/V – BG** (I = M.I. of water-plane area about the longitudinal axis, V = displaced volume).
- Stability: **submerged** body – stable if **B above G**; **floating** body – stable if **M above G** (GM > 0); neutral if M coincides with G; unstable if M below G.
- Experimental GM = **$w\,x/(W \tan\theta)$**. Period of rolling **$T = 2\pi\sqrt{(k^2/(g\cdot GM))}$** → large GM = "stiff" ship, quick uncomfortable rolling. Typical GM: ocean liners ≈ 0.3–0.7 m, warships ≈ 1–1.5 m.
- **Forced vortex** (rotating container): free surface is a **paraboloid**, height $Z = \omega^2 r^2/2g$; rise at wall = fall at centre = **$\omega^2R^2/4g$**. **Free vortex**: V·r = constant (whirlpool, flow outside impeller).

KEY FORMULAE

- $\tau = \mu\, du/dy$; $v = \mu/\rho$; $K = -dp/(dV/V)$
- $h = 4\sigma\cos\theta/(\rho g d)$; $\Delta p_{drop} = 4\sigma/d$; $\Delta p_{bubble} = 8\sigma/d$; $\Delta p_{jet} = 2\sigma/d$
- $F = \rho g A\,\bar{x}$; $h^* = \bar{x} + I_G \sin^2\theta/(A\,\bar{x})$
- $GM = I/V - BG$; $T = 2\pi\sqrt{(k^2/(g\,GM))}$

MEMORY TRICK

"**Liquids Lose, Gases Gain**" viscosity on heating. Drop–Bubble–Jet pressure = **4–8–2** σ/d.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- 1 stoke = ? → **10^{-4} m^2/s**.
- Viscosity of a gas with rise in temperature? → **Increases**.
- Toothpaste behaves as? → **Bingham plastic**.
- Pressure inside a soap bubble exceeds outside by? → **8σ/d**.
- Centre of pressure of a vertical rectangle with top at free surface? → **2d/3 below surface**.
- Floating body is stable when? → **Metacentre is above centre of gravity**.

4.2 Fluid Kinematics, Bernoulli & Momentum ★★★

Flow type	Condition	Example
Steady / unsteady	$\partial/\partial t = 0 / \neq 0$ at a point	Constant-head pipe / valve being closed
Uniform / non-uniform	$\partial V/\partial s = 0 / \neq 0$ along flow at an instant	Constant-dia pipe / tapering pipe
Laminar / turbulent	Layers / random eddies (Re)	Oil in thin tube / river
Compressible / incompressible	ρ varies / constant	Gas at Ma > 0.3 / liquids
Rotational / irrotational	$\omega \neq 0$ / $\omega = 0$	Forced vortex / free vortex
1-D, 2-D, 3-D	No. of space coordinates	—

- **Streamline** – tangent gives velocity direction at an instant; no flow across it. **Pathline** – path of one particle. **Streakline** – locus of particles that passed through one point (dye injection). In **steady flow all three coincide**.
- **Acceleration** = **local (temporal)** $\partial V/\partial t$ + **convective** $(V\cdot\nabla)V$. Steady flow → local = 0; uniform flow → convective = 0.
- **Continuity** (conservation of mass): $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$; incompressible: **$A_1V_1 = A_2V_2$**; differential form (3-D, incompressible): **$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0$**.
- **Rotation** $\omega_z = ½(\partial v/\partial x - \partial u/\partial y)$; **vorticity = 2ω**; **circulation** $\Gamma = \oint V\cdot ds$ = vorticity × area.
- **Stream function ψ** – exists for **2-D incompressible** flow (rotational or irrotational); automatically satisfies continuity. ψ satisfies Laplace equation only if flow is irrotational. Difference $\psi_2 - \psi_1$ = **discharge per unit width** between the streamlines.
- **Velocity potential φ** – exists **only for irrotational flow**; satisfies Laplace equation if flow is also incompressible (continuity).
- Sign conventions differ: Cengel uses $u = \partial\varphi/\partial x$, $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$; many Indian texts (Bansal, Rajput) use $u = -\partial\varphi/\partial x$ and $u = -\partial\psi/\partial y$, $v = \partial\psi/\partial x$. Check the convention given in the question.
- Lines of constant φ (equipotential) and constant ψ (streamlines) are **orthogonal → flow net**. Flow net is not valid near solid boundaries (viscous) or in rotational flow.
- **Euler's equation** (along a streamline): **dp/ρ + g dz + V dV = 0** – integrated for constant ρ gives Bernoulli.
- **Bernoulli's equation**: $p/\rho g + V^2/2g + z$ = constant (**energy per unit weight**, i.e. head in m). Assumptions: **steady, ideal (inviscid), incompressible, along a streamline**, no shaft work/heat transfer; valid across streamlines only if **irrotational**.
- Real fluid: $p_1/\rho g + V_1^2/2g + z_1 = p_2/\rho g + V_2^2/2g + z_2 + \mathbf{h_L}$.
- **Kinetic energy correction factor α**: laminar pipe = **2**; turbulent ≈ 1.03–1.1. **Momentum correction factor β**: laminar pipe = **4/3**; turbulent ≈ 1.01–1.04.
- **HGL** = $p/\rho g + z$ (piezometric head); **TEL** = HGL + $V^2/2g$. TEL always **falls** in direction of flow (unless a pump adds energy); HGL may rise (e.g. at an expansion). Vertical gap TEL–HGL = **velocity head**. Pipe above HGL → **negative (sub-atmospheric) pressure**.

Device	C_d (typical)	Notes

Nandyal Banking Coaching Center

Venturimeter	**0.95–0.99 (≈ 0.98)**	Convergent cone ≈ **20–22°**, divergent **5–7°** (avoids separation); throat d = **1/3 to 3/4 D** (often 0.5D); lowest head loss; costly; Cd independent of inclination
Orifice meter	**0.6–0.65 (≈ 0.62)**	Orifice d ≈ 0.5D; cheap, compact; high permanent loss; vena contracta downstream
Flow nozzle	**0.93–0.98**	Intermediate between the two
Pitot tube	C_v ≈ 0.98–0.99	Measures **point velocity** $V = C_v\sqrt{(2gh)}$; stagnation pressure; Pitot-static (Prandtl) tube
Rotameter	—	**Variable area** meter; float in tapered vertical tube
Sharp-edged orifice (tank)	C_c ≈ 0.64, C_v ≈ 0.97–0.98, **C_d ≈ 0.62**	Cd = Cc × Cv; vena contracta ≈ 0.5d from orifice
External cylindrical mouthpiece	**0.855**	Length 2–3 d
Borda's (re-entrant) mouthpiece	Running free **0.5**; running full **0.707**	Internal
Convergent–divergent mouthpiece	**1.0**	No loss at vena contracta

- Venturimeter: $\mathbf{Q = C_d\, a_1a_2\sqrt{(2gh)}/\sqrt{(a_1^2 - a_2^2)}}$; h = x($S_m$/S − 1) for a heavier manometric liquid.
- Discharge over notches: rectangular $Q = (2/3)C_dL\sqrt{(2g)}\, H^{3/2}$; triangular (V) $Q = (8/15)C_d \tan(\theta/2)\sqrt{(2g)}\, H^{5/2}$. Right-angled V-notch is most accurate for **small discharges**.
- **Impulse–momentum equation**: $\Sigma F = \rho Q(V_2 - V_1)$ – used for force on bends, nozzles, jets, vanes.

Jet (area a, velocity V) striking	Force in jet direction	Max efficiency
Fixed flat plate normal to jet	**ρaV^2**	—
Fixed inclined plate (angle θ with jet)	Normal: $\rho aV^2 \sin\theta$	—
Fixed symmetric curved vane, jet at centre (deflection 180° − θ)	$\rho aV^2(1 + \cos\theta)$; semicircular: **$2\rho aV^2$**	—
Single moving flat plate (speed u)	$\rho a(V - u)^2$	**8/27 = 29.6% at u = V/3**
Series of flat plates on a wheel	$\rho aV(V - u)$	**50% at u = V/2**
Series of semicircular (curved) vanes	$2\rho aV(V - u)$	**100% at u = V/2** (theoretical)

KEY FORMULAE

- Continuity: $A_1V_1 = A_2V_2$; Bernoulli: $p/\rho g + V^2/2g + z = H$
- $Q_{venturi} = C_d a_1 a_2\sqrt{(2gh)}/\sqrt{(a_1^2 - a_2^2)}$; Pitot $V = C_v\sqrt{(2gh)}$
- Force exerted by jet on vane $F = \rho Q(V_1 - V_2)$; $\eta_{series\ flat} = 2u(V - u)/V^2$

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Velocity potential function exists only for? → **Irrotational flow**.
- Cd of a venturimeter is about? → **0.98**; of an orifice meter? → **0.62**.
- Divergent cone angle of venturimeter? → **5–7°** (to avoid flow separation).
- Max efficiency of jet on a single moving flat plate? → **29.6% (u = V/3)**.
- Kinetic energy correction factor for laminar pipe flow? → **2**.
- Pitot tube measures? → **Velocity at a point (stagnation pressure)**.

4.3 Flow Through Pipes, Boundary Layer, Drag & Lift ★★★

- **Reynolds number** Re = ρVD/μ = VD/ν = inertia/viscous force (**Osborne Reynolds, 1883**, dye experiment).

Situation	Laminar	Transition	Turbulent
Pipe flow (Re on D)	**< 2000** (some texts 2300)	2000–4000	**> 4000**
Open channel (Re on R)	< 500	500–2000	> 2000
Flat plate boundary layer (Re_x)	**< 5×10^5**	Transition ≈ 5×10^5; turbulent beyond	
Flow past sphere (Stokes' law)	Valid for **Re < 0.2** (≈ 1 in some texts); $C_D = 24/Re$		

Laminar flow result	Circular pipe	Between fixed parallel plates (gap B)
Velocity profile	**Parabolic**	Parabolic
V_{max}/V_{mean}	**2**	**1.5**
Pressure drop	**$\Delta p = 32\mu VL/D^2$** (Hagen–Poiseuille)	**$\Delta p = 12\mu VL/B^2$**
Shear stress	Linear: **zero at centre, max at wall**	Zero at mid-plane, max at plates
α / β	2 / 4/3	1.543 / 1.2

- Hagen–Poiseuille discharge: **$Q = \pi D^4\Delta p/(128\mu L)$** → $Q \propto D^4$ (halving D cuts Q to 1/16 for same Δp).
- Laminar friction factor (Darcy) **f = 64/Re**; Fanning f′ = **16/Re** (f = 4f′). Head loss ∝ V (laminar) vs ∝ $V^{1.75–2}$ (turbulent).
- Radius where local velocity = mean velocity (laminar pipe) = **R/√2 = 0.707R**.
- **Darcy–Weisbach**: **$h_f = fLV^2/(2gD)$** (f = Darcy) = $4f'LV^2/(2gD)$ (f′ = Fanning, used in Rajput/Bansal as "f"). In terms of Q: $h_f = fLQ^2/(12.1D^5)$ → **$h_f \propto 1/D^5$** at constant Q.
- **Blasius** (smooth pipe, Re 4000–10^5): **$f = 0.316/Re^{0.25}$**. **Colebrook–White** covers the transition zone; the **Moody diagram** plots f vs Re for various ε/D.
- Moody zones: laminar – f depends on **Re only**; transition; **fully rough** – f depends on **ε/D only** (independent of Re).
- Hydraulically **smooth** if roughness k < laminar sub-layer thickness δ′ ($\delta' = 11.6\nu/V_*$, $V_* = \sqrt{(\tau_0/\rho)}$ = shear/friction velocity). Same pipe can be smooth at low Re and rough at high Re.
- Turbulent profile flatter: **1/7th power law**; $V_{max}/V \approx$ **1.2**. Turbulent shear: Boussinesq eddy viscosity; **Prandtl mixing**

Nandyal Banking Coaching Center

length $\tau = \rho l^2(du/dy)^2$.

- Chezy: $V = C\sqrt{(mi)}$; m = hydraulic mean depth = A/P = **D/4** for full pipe.

Minor loss	Head loss
Sudden enlargement	$\mathbf{(V_1 - V_2)^2/2g}$ (Borda–Carnot)
Sudden contraction	$(1/C_c - 1)^2V_2^2/2g$; = $\mathbf{0.375\ V_2^2/2g}$ for $C_c = 0.62$; taken as $\mathbf{0.5\ V_2^2/2g}$ when C_c is not given
Entrance (sharp)	$\mathbf{0.5\ V^2/2g}$
Exit (into reservoir)	$\mathbf{V^2/2g}$
Bend / fitting	$kV^2/2g$; equivalent length $L_e = kD/f$

- Minor losses are "minor" only in **long pipes**; neglected if L > ~1000D.
- **Series**: same Q, $H = \Sigma h_f$ + minor losses. **Parallel**: same h_f in each branch, $Q = \Sigma Q_i$.
- **Equivalent pipe (Dupuit's equation)**: $\mathbf{L/D^5 = L_1/D_1^5 + L_2/D_2^5 + ...}$ (same f, same Q and h_f).
- **Siphon**: highest point (summit) is at **sub-atmospheric pressure**; summit should not be more than about **7–8 m** above the upstream liquid level (theoretical 10.3 m) to avoid air release/vapour lock.
- **Power transmitted** through a pipe: $P = \rho gQ(H - h_f)$; **maximum when $h_f = H/3$**; $\mathbf{\eta_{max} = 2/3 = 66.7\%}$.
- **Water hammer** = pressure rise due to sudden stoppage of flow (valve closure). Pressure wave speed $C = \sqrt{(K/\rho)}$ (rigid pipe) ≈ 1400–1500 m/s for water.
- Elastic pipe: $1/K_{eff} = 1/K + D/(Et)$. **Rapid closure** if closure time **t < 2L/C**; **gradual** if t > 2L/C.
- Protection: **surge tank** (in hydro plants between reservoir/tunnel and penstock), slow closing valves, relief valves, air chambers.

KEY FORMULAE

- Sudden closure (rigid pipe): **p = ρVC**; gradual closure (time t): p = ρLV/t
- $h_f = fLV^2/2gD$; laminar f = 64/Re; $\Delta p = 32\mu VL/D^2$
- Nozzle at end of pipe for max power (Rajput form, $h_f = 4fLV^2/2gD$): $d = (D^5/8fL)^{1/4}$

- **Boundary layer** (Prandtl, 1904) = thin region near a wall where velocity rises from 0 (no-slip) to 99% of free stream U. Thickness grows downstream.
- **Laminar**: $\mathbf{\delta/x = 5/\sqrt{Re_x}}$ (Blasius exact ≈ 4.91; Pohlhausen cubic 4.64) → $\mathbf{\delta \propto \sqrt{x}}$. **Turbulent**: $\delta/x = 0.37/Re_x^{1/5}$ → $\mathbf{\delta \propto x^{4/5}}$.
- **Displacement thickness** $\delta^* = \int(1 - u/U)dy$; **momentum thickness** $\theta = \int(u/U)(1 - u/U)dy$; energy thickness $\delta_e = \int(u/U)(1 - u^2/U^2)dy$. Order: $\mathbf{\delta > \delta^* > \delta_e > \theta}$.
- Linear profile: $\delta^* = \delta/2$, $\theta = \delta/6$. Blasius: $\delta^* = 1.72x/\sqrt{Re_x}$, $\theta = 0.664x/\sqrt{Re_x}$; **shape factor $H = \delta^*/\theta \approx 2.59$** (laminar), ≈ 1.3–1.4 (turbulent).
- Laminar skin-friction (Blasius) local $C_f = 0.664/\sqrt{Re_x}$; average $C_D = \mathbf{1.328/\sqrt{Re_L}}$.
- **Separation** occurs in an **adverse pressure gradient ($\partial p/\partial x > 0$)**; at separation point $\mathbf{(\partial u/\partial y)_{y=0} = 0}$. Control: streamlining, suction, blowing, rotating cylinder, vortex generators.
- **Drag $F_D = C_D A\ \rho U^2/2$**; **Lift $F_L = C_L A\ \rho U^2/2$**. Drag = skin-friction (viscous) + pressure (form) drag. **Streamlined** body: friction drag dominates; **bluff** body: pressure drag dominates.
- **Stokes' law**: $F_D = 3\pi\mu DV$; terminal velocity $V = D^2(\rho_s - \rho)g/(18\mu)$.
- **Drag crisis**: sudden fall in C_D of sphere/cylinder at Re ≈ $\mathbf{3 \times 10^5}$ when BL becomes turbulent and separation moves rearward (reason for dimples on golf balls).
- **Magnus effect** = lift on a rotating cylinder/ball in a stream; **Kutta–Joukowski**: L = ρUΓ per unit span. Aerofoil **stalls** at high angle of attack (≈ 15°) due to separation.

MEMORY TRICK

"**64 – 2 – 3**": laminar f = 64/Re, $V_{max} = 2V$, max power when loss = H/3. Plates are "**12 and 1.5**", pipe is "**32 and 2**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Friction factor for laminar pipe flow? → **64/Re**.
- Max efficiency of power transmission through a pipe? → **66.67% ($h_f = H/3$)**.
- In fully rough turbulent flow, f depends on? → **Relative roughness only**.
- Laminar boundary layer thickness varies as? → $\mathbf{x^{1/2}}$ (turbulent $x^{4/5}$).
- Pressure rise for sudden valve closure in rigid pipe? → **ρVC**.
- Flow separation occurs when? → **Adverse pressure gradient, wall shear = 0**.

4.4 Dimensional Analysis & Similitude ★★☆

- **Dimensional homogeneity** (Fourier principle): every term of a physical equation has the same dimensions.
- **Rayleigh's method** – suitable for ≤ 3–4 variables. **Buckingham π-theorem**: n variables, m fundamental dimensions → **(n – m) dimensionless π-groups**.
- Repeating variables (= m): together must contain **all fundamental dimensions**, must **not form a dimensionless group** themselves, and the **dependent variable must not be chosen**. Usual choice: one geometric (D), one kinematic (V), one fluid property (ρ).
- Dimensions: force MLT^{-2}; pressure/stress/K $ML^{-1}T^{-2}$; power ML^2T^{-3}; σ MT^{-2}; μ $ML^{-1}T^{-1}$; ν L^2T^{-1}; Q L^3T^{-1}.
- **Similitude**: geometric (length ratio), kinematic (velocity ratio), dynamic (**force ratio**) similarity. Dynamic similarity requires equality of the governing dimensionless number.

Number	Definition	Force ratio	Model law used for
Reynolds Re	ρVL/μ	Inertia/viscous	**Pipe flow**, submarines, low-speed aircraft, flow meters, fully submerged bodies
Froude Fr	V/√(gL)	√(Inertia/gravity)	**Free-surface flows**: open channels, spillways, weirs, ships (wave resistance), hydraulic jump
Euler Eu	V/√(p/ρ)	√(Inertia/pressure)	Pressure-dominated flows, **cavitation** studies, pipe flow with pressure surges

Nandyal Banking Coaching Center

Weber We	$V/\sqrt{(\sigma/\rho L)}$	√(Inertia/surface tension)	**Capillary** flows, droplets, thin sheets over weirs at very low head
Mach Ma	V/C	√(Inertia/elastic)	**Compressible** flow: high-speed aircraft, projectiles, rockets, water hammer
Cauchy Ca	$\rho V^2/K$	Inertia/elastic	= Ma^2
Strouhal St	fL/V	—	**Vortex shedding** (Kármán street) frequency

- Mach regimes: incompressible treatment if **Ma < 0.3**; subsonic < 1; **transonic ≈ 0.8–1.2**; supersonic > 1; **hypersonic > 5**. Mach angle sin α = 1/Ma.
- **Froude model** (scale ratio L_r): $V_r = \sqrt{L_r}$, $T_r = \sqrt{L_r}$, $a_r = 1$, $Q_r = L_r^{2.5}$, $F_r = L_r^3$, $P_r = L_r^{3.5}$.
- **Reynolds model** with same fluid: $V_r = 1/L_r$ → model velocity is higher than prototype; $Q_r = L_r$; $T_r = L_r^2$.
- Ship models: wave resistance by Froude law, skin friction by Reynolds law – both cannot be satisfied with water → **Froude's method** computes friction separately.
- **Distorted models** (different horizontal and vertical scales) used for **rivers, harbours** to keep depths measurable and flow turbulent.
- **Scale effect** = discrepancy between model and prototype behaviour due to incomplete similarity.

KEY FORMULAE

- π-groups = n − m; Froude law: $Q_r = L_r^{5/2}$, $P_r = L_r^{7/2}$
- Reynolds law: $(VL/\nu)_m = (VL/\nu)_p$

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Model law for spillway / ship testing? → **Froude**.
- Model law for flow through pipes / submarine? → **Reynolds**.
- Discharge scale ratio in Froude model? → $L_r^{2.5}$.
- Number of π-terms with 7 variables and 3 fundamental dimensions? → **4**.
- Ratio of inertia to surface-tension force? → **Weber number** (squared).

4.5 Hydraulic Turbines ★★★

- Hydraulic turbine converts **hydraulic energy → mechanical energy**. Gross head H_g = headrace level − tailrace level; **net (effective) head** H = H_g − penstock losses.
- **Impulse turbine**: entire head converted to KE in **nozzle**; runner at **atmospheric pressure**; no pressure change across runner; casing has no hydraulic function; can be partially admitted; **no draft tube**. E.g. **Pelton**, Turgo, cross-flow (Banki–Michell).
- **Reaction turbine**: part pressure + part kinetic energy at runner inlet; pressure falls across runner; runner runs **full** and enclosed; **draft tube** essential. E.g. **Francis, Kaplan**, propeller, Deriaz, bulb/tubular.
- **Degree of reaction R** = change of pressure energy in runner/total energy change; impulse R = 0.
- **Euler (work) equation**: work per unit weight = $(V_{w1}u_1 \pm V_{w2}u_2)/g$. Max efficiency when $\mathbf{V_{w2} = 0}$ **(radial/axial discharge)** → $\eta_h = V_{w1}u_1/(gH)$.

Feature	Pelton	Francis	Kaplan
Type	Impulse	Reaction	Reaction
Flow direction	**Tangential**	Radial inward → **mixed** (modern)	**Axial**
Head	**High (> 250 m)**	**Medium (60–250 m)**	**Low (< 60 m)**
Discharge	Low	Medium	High
Specific speed (SI, kW)	**10–35 (single jet)**; 35–60 multi-jet	**60–300**	**300–1000**
Speed ratio $K_u = u/\sqrt{(2gH)}$	**0.43–0.47 (≈ 0.46)**	0.6–0.9	1.4–2.0
Flow ratio $K_f = V_f/\sqrt{(2gH)}$	—	**0.15–0.30**	**0.35–0.75**
Governing	**Spear (needle)** in nozzle + **deflector**	Guide (wicket) vanes	Guide vanes + runner blades (**double regulated**)
Part-load efficiency	Good (flat curve)	Poor	**Best**
Draft tube	Not used	Used	Used (elbow type)

- Specific-speed ranges differ between books — Bansal: Pelton **8.5–30** (single jet), 30–51 (two jets); Francis **51–255**; Kaplan **255–860**. Rajput ranges are given in the table. Head limits also vary (some books: Pelton > 300 m).
- **Pelton wheel** (Lester Pelton): jet velocity $V_1 = C_v\sqrt{(2gH)}$, $\mathbf{C_v \approx 0.97–0.99}$. Double-hemispherical buckets with a central **splitter**; jet deflected through ≈ **165°** (160–165°, not 180°, so that water leaving does not hit the next bucket).
- Pelton theoretical max hydraulic efficiency when $\mathbf{u = V_1/2}$: $\eta_{max} = (1 + k\cos\varphi)/2$, which = **100%** for φ = 0 (180° deflection) and k = 1. In practice best efficiency at $u/V_1 \approx$ **0.46** (friction, windage).
- **Jet ratio m = D/d** (pitch circle dia/jet dia) ≈ **12** (usually 11–14; minimum ≈ 10). Number of buckets (Tygun): **Z = 15 + m/2 = 15 + D/(2d)**. Max **6 jets** (vertical shaft).
- **Deflector** diverts jet on sudden load reduction while spear closes slowly → avoids **water hammer**. **Braking jet** (directed on back of buckets) brings the wheel to rest.
- **Francis turbine** (James B. Francis): components – **spiral (scroll) casing** (decreasing area for uniform velocity) → **stay vanes** → **guide (wicket) vanes** → runner → **draft tube**. Number of guide vanes ≈ 16–24. **Relief valve** (pressure regulator) bypasses water on load rejection.
- **Kaplan turbine** (Viktor Kaplan): axial-flow propeller with **adjustable** runner blades (typically 3–8, commonly 4–6); **propeller turbine** has fixed blades. Flow area $\mathbf{Q = (\pi/4)(D_o^2 - D_b^2)V_f}$; hub dia $D_b \approx$ **0.35–0.6** $\mathbf{D_o}$.
- **Deriaz** = diagonal (mixed) flow with adjustable blades – reversible pump-turbine for **pumped storage**. **Bulb/tubular** turbines for very low (tidal, run-of-river) heads.
- Efficiencies: **hydraulic** η_h = runner power/water power; **mechanical** η_m = shaft power/runner power; **overall** $\eta_o = \eta_h \times \eta_m$ (volumetric also for reaction turbines). Large turbines $\eta_o \approx$ 85–94%.

Nandyal Banking Coaching Center

KEY FORMULAE

- Water power = ρgQH; shaft power P = η_o ρgQH
- **Specific speed $N_s = N\sqrt{P}/H^{5/4}$** (N rpm, P kW, H m) – speed of a geometrically similar turbine producing **1 kW under 1 m head**. N_s is **dimensional** – its value depends on the units used.
- Unit speed $N_u = N/\sqrt{H}$; unit discharge $Q_u = Q/\sqrt{H}$; unit power $P_u = P/H^{3/2}$ (values under 1 m head)
- Same turbine, different head: $N \propto \sqrt{H}$, $Q \propto \sqrt{H}$, $P \propto H^{3/2}$
- Pelton: $\eta_h = 2u(V_1 - u)(1 + k\cos\varphi)/V_1^2$
- Thoma's cavitation factor **$\sigma = (H_{atm} - H_v - H_s)/H$**; no cavitation if $\sigma > \sigma_c$
- Draft tube efficiency $\eta_d = [(V_2^2 - V_3^2)/2g - h_f]/(V_2^2/2g)$

- Correction: P in metric hp gives N_s about $\sqrt{1.36} \approx$ **1.17 times** the kW value (since 1 kW = 1.36 PS). Low N_s → Pelton; high N_s → Kaplan. N_s increases as head decreases.
- **Draft tube** functions: (i) allows turbine to be set **above tailrace** without loss of head (creates **negative pressure** at runner exit); (ii) **recovers KE** of exit water as pressure energy (diverging tube). Outlet is always **submerged** below tailrace level.
- Draft tube types: **conical** (straight divergent) – η ≈ **90%**, cone half-angle ≈ 4° (total ≤ 8°), Francis; **simple elbow** – η ≈ **60%**; **Moody spreading (hydracone, bell-mouthed)** – η ≈ **85–88%**, reduces whirl; **elbow with circular inlet & rectangular outlet** – used with **Kaplan** (low setting).
- Draft tube height limited by **cavitation**: exit-of-runner pressure must stay above vapour pressure (practical limit ≈ 2.5 m water absolute).
- **Cavitation** in reaction turbines occurs at **runner blade outlet / draft tube inlet** (lowest pressure). Prevented by: lower setting (smaller H_s), stainless/chromium steel runners, keeping $\sigma > \sigma_c$. σ_c increases with N_s → Kaplan more prone than Francis.
- **Governing** = keeping speed constant (synchronous) under varying load by regulating Q. Oil-pressure governor: **centrifugal (fly-ball) governor → relay/distribution valve → servomotor** → spear/guide vanes. Surge tank protects penstock.
- **Characteristic curves**: main (constant head; vs N_u), operating (constant speed; η vs load), **Muschel / iso-efficiency curves** (constant efficiency contours).
- **Runaway speed** = max speed at full gate when load is suddenly lost: Pelton ≈ 1.8–1.9 N, Francis ≈ 2–2.2 N, Kaplan ≈ 2.5–3 N (approximate).

MEMORY TRICK

Head goes **High–Medium–Low** as turbines go **P–F–K** (Pelton–Francis–Kaplan), while N_s and discharge go up. Pelton's magic numbers: "**0.46, 12, 165°, 15 + m/2**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Specific speed of a turbine is defined as? → **$N\sqrt{P}/H^{5/4}$**.
- Kaplan turbine is a? → **Low head, axial-flow reaction turbine with adjustable blades**.
- Pelton hydraulic efficiency is maximum when bucket speed equals? → **Half the jet velocity**.
- Draft tube is not used in? → **Pelton wheel**.
- Most efficient draft tube type? → **Conical (≈ 90%)**.
- Function of a deflector in Pelton wheel? → **Divert the jet to prevent water hammer**.

4.6 Pumps ★★★

- Pump converts **mechanical energy → hydraulic (pressure) energy**. **Rotodynamic** (centrifugal, axial, mixed) vs **positive displacement** (reciprocating, gear, vane, screw, lobe).
- **Centrifugal pump** works on **forced vortex** principle: pressure rise $\propto \omega^2 r^2$. Components: **impeller**, **casing**, suction pipe with **foot valve & strainer**, delivery pipe with **delivery valve**.
- Casing types: **volute** (spiral, area increasing – some loss), **vortex (whirlpool) chamber** (circular chamber between impeller and volute), **casing with guide blades (diffuser)** – "turbine pump", highest efficiency.
- Impellers: **shrouded (closed)** – clean water; **semi-open**; **open** – sewage, slurries. Vanes usually **backward-curved** ($\beta_2 \approx$ 20–25°): non-overloading power, stable head.
- Water enters impeller **radially** ($V_{w1} = 0$) → work per unit weight = **$V_{w2}u_2/g$**.
- **Manometric head H_m** = head against which the pump works = $h_s + h_d + h_{fs} + h_{fd} + V_d^2/2g$ (= reading of gauges at outlet and inlet + datum difference).
- **Priming** = filling suction pipe, casing and impeller with liquid to expel air. Needed because head developed is in metres of the fluid present; air (density ≈ 1/800 of water) gives a negligible pressure rise. Reciprocating pumps do not need priming.
- Centrifugal (radial) pump is started with **delivery valve closed** (shut-off power minimum). **Axial-flow** pumps are started with valve **open** (power highest at shut-off).
- **Minimum starting speed**: centrifugal head $(u_2^2 - u_1^2)/2g \geq H_m$ → **$N_{min} = 120\,\eta_{man}V_{w2}D_2/[\pi(D_2^2 - D_1^2)]$**.

KEY FORMULAE

- Manometric efficiency **$\eta_{man} = gH_m/(V_{w2}u_2)$**
- Mechanical η_m = power at impeller/shaft power; **overall $\eta_o = \rho gQH_m/P_{shaft} = \eta_{man} \times \eta_m$** (× η_v)
- **Specific speed $N_s = N\sqrt{Q}/H_m^{3/4}$** – speed of a similar pump delivering **1 m³/s against 1 m head** (Q per eye, H per stage)
- Affinity laws: **$Q \propto ND^3$, $H \propto N^2D^2$, $P \propto N^3D^5$**; same pump (D fixed): **$Q \propto N$, $H \propto N^2$, $P \propto N^3$**
- **$NPSH_A = p_{atm}/\rho g - p_v/\rho g - h_s - h_{fs}$** (suction lift h_s); must be ≥ **$NPSH_R$**
- Thoma's σ (pump) = $NPSH/H_m$

Pump type (by N_s, SI, Q in m³/s)	N_s range	Character
Radial – slow speed	10–30	High head, low Q
Radial – medium speed	30–50	—
Radial – high speed	50–80	—

Nandyal Banking Coaching Center

Mixed flow	**80–160**	Medium head
Axial flow (propeller)	**160–500**	**Low head, high Q**

- **Multistage – series** (impellers on one shaft): **head adds** (H_{total} = nH), same Q – high-head duty (boiler feed, deep well). **Parallel**: **discharge adds** (Q_{total} = nQ), same H – large-flow duty.
- **NPSH** = net positive suction head = total suction head above vapour pressure at pump inlet. Cavitation if $NPSH_A < NPSH_R$. Cure: lower the pump (reduce suction lift), larger/shorter suction pipe, lower liquid temperature, lower speed, booster/inducer.
- Max practical **suction lift** for water ≈ **6–7 m** (theoretical 10.3 m).
- **Characteristic curves**: main (H, P, η vs Q at various N), **operating** (at constant design N), **iso-efficiency** (Muschel). Best efficiency point (BEP) = design point. System curve $H = H_{static} + kQ^2$; operating point = intersection with pump H–Q curve.
- Vane shape effect on H–Q (theoretical): **backward** – falling H, power has a maximum (**non-overloading**); **radial** – constant theoretical H; **forward** – rising H and power rises continuously (**overloading**, unstable).
- Other troubles: **surging** – unstable operation on rising part of H–Q curve; **pump not delivering** – not primed, air leak, speed too low, suction lift too high.

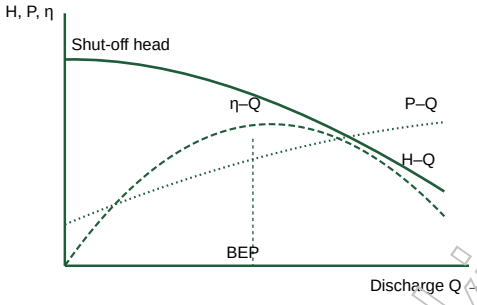


Fig 4.2 Operating characteristics of a centrifugal pump (backward vanes, constant N): H falls with Q, η peaks at BEP, power rises and flattens (non-overloading).

- **Reciprocating pump** – positive displacement; suited to **small discharge, high head**; pulsating flow; self-priming. Single-acting: one delivery per revolution; double-acting: two.
- Theoretical discharge: single-acting Q_{th} = **ALN/60**; double-acting Q_{th} = (2A – a)LN/60 ≈ **2ALN/60** (a = piston-rod area).
- **Slip** = $(Q_{th} - Q_{act})/Q_{th}$; $C_d = Q_{act}/Q_{th}$ ≈ **0.96–0.98** (slip 2–4%) due to valve leakage/delay.
- **Negative slip** ($Q_{act} > Q_{th}$, $C_d > 1$): occurs with **long suction pipe, short delivery pipe and high speed** – inertia of suction water keeps the delivery valve open before the suction stroke ends.
- Work done (single-acting, ideal) = $\rho gALN(h_s + h_d)/60$; power $P = \rho g Q_{th}(h_s + h_d)$.
- **Acceleration head** $h_a = (L/g)(A/a)\omega^2 r \cos\theta$ – **max at start and end** of stroke, zero at mid-stroke. **Friction head** $h_f = (4f'L/2gd)[(A/a)\omega r \sin\theta]^2$ – **max at mid-stroke**, zero at ends.
- **Separation** (cavitation) occurs at the **beginning of the suction stroke** when pressure falls below ≈ **2.5 m of water (absolute)** → limits maximum speed.
- **Indicator diagram** (pressure head in cylinder vs stroke): area ∝ **work done per cycle**. Ideal = rectangle (height $h_s + h_d$). Acceleration head tilts suction/delivery lines → parallelogram-like figure, **area unchanged** (no net work by acceleration).

Friction adds **parabolic** curves → area increases by $(2/3)h_{fs}$ and $(2/3)h_{fd}$ terms.

- **Air vessels** (closed chambers on suction and delivery, close to the cylinder): give **nearly uniform flow** in pipes, reduce acceleration head, permit higher speed, allow longer suction pipe, and **save friction work**.
- Work saved against friction by fitting air vessels: **single-acting = 84.8%**; **double-acting = 39.2%**. (With vessel mean $h_f = h_{f,max}/\pi^2$ vs $(2/3)h_{f,max}$ without for single-acting.)

Aspect	Centrifugal pump	Reciprocating pump
Principle	Rotodynamic (forced vortex)	Positive displacement
Discharge / head	**Large Q, moderate H**	**Small Q, high H**
Flow	Continuous, uniform	Pulsating (air vessels used)
Priming	**Required**	Not required
Viscous / dirty liquids	Handles sewage, slurries	Not suitable for dirty liquids (valves)
Speed, size, cost	High speed, compact, cheap	Low speed, bulky, costly; more wear parts
Efficiency	Lower at small Q	Higher at high head
Closing delivery valve	Safe (shut-off head)	**Dangerous** – needs **relief valve**

Other pump	Key fact
Gear pump (external/internal)	Rotary positive displacement; **fixed displacement**; lubricating oil, fuel, hydraulic circuits
Vane pump	Rotor eccentric in casing; **unbalanced type can be variable displacement**; balanced type fixed
Screw / lobe pump	Viscous liquids, smooth flow (screw); food industry (lobe)
Jet pump (ejector)	High-velocity jet entrains fluid; no moving parts; with centrifugal pump for **deep wells**; low efficiency
Hydraulic ram	Uses **water hammer** of a large quantity at low head to lift a small quantity to a greater height; no external power
Submersible pump	Multistage centrifugal + sealed motor immersed in **borewell**; no priming, no suction lift problem
Air-lift pump	Compressed air lowers mixture density; for oil/water wells, sandy water
Diaphragm pump	Flexible diaphragm; slurries, chemicals; no leakage
Deep-well turbine pump	Vertical multistage with diffusers; motor on surface

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Specific speed of a centrifugal pump? → **$N\sqrt{Q}/H^{3/4}$**.
- If speed of a pump is doubled, power becomes? → **8 times** (head 4 times).
- Pumps in series are used to obtain? → **Higher head**.
- Negative slip occurs when? → **Long suction, short delivery, high speed**.
- Work saved by air vessel in single-acting reciprocating pump? → **84.8%**.
- Centrifugal pump is started with delivery valve? → **Closed**.

4.7 Hydraulic Systems & Fluid Power ★★☆

- **Hydraulic press (Bramah press)** – based on **Pascal's law**: W/A = F/a → **W = F(A/a)**; mechanical advantage = A/a (× lever ratio). Used for forging, baling, punching.
- **Hydraulic accumulator** – **stores pressure energy** during light load and supplies it at peak demand (lifts, cranes, presses). **Capacity = pAL** (p = pressure, A = ram area, L = stroke). Types: **weight-loaded** (constant pressure), **differential** (higher pressure with small load), **spring-loaded**, **gas-charged** (bladder/piston/diaphragm with **nitrogen**) – most common in modern circuits; also absorbs shocks and leakage.
- **Hydraulic intensifier** – **raises pressure** of liquid (for presses, riveting): $p_2 = p_1A_1/A_2$ (fixed ram inside a sliding cylinder). Ratio of intensification = A_1/A_2.
- **Hydraulic jack, hydraulic lift** (direct-acting or suspended), **hydraulic crane**: all use Pascal's law.
- **Fluid (hydraulic) coupling**: pump impeller + turbine runner in one oil-filled casing; **no reactor → output torque = input torque**; slip ≈ **2–4%** → **η = 1 – slip ≈ 96–98%**. Smooth start, shock damping.
- **Torque converter**: impeller + turbine + **fixed stator (reactor)** → **torque multiplication** (output torque > input, max at stall, ≈ 2–3 times in automotive units); output speed < input. Max efficiency ≈ **85–90%** (falls at both ends of speed range). Becomes a coupling at "coupling point" (stator freewheels – one-way clutch).

Aspect	Hydraulic system	Pneumatic system
Medium	Oil (mineral/synthetic) – **incompressible**	Air – **compressible**
Working pressure	High: **~70–350 bar** (up to ~700)	Low: **~6–7 bar** (≈ 0.6 MPa)
Force / power	**Very high**	Low–medium
Speed	Slower	**Faster** actuation
Accuracy / stiffness	**Precise positioning**, rigid	Spongy, less precise
Return line	Required (to tank)	Exhausts to atmosphere
Fire/leak hazard	Oil leakage, fire risk	Clean, **fire/explosion safe**
Typical use	Presses, excavators, machine tools, aircraft controls	Clamping, packaging, pick-and-place, tools, brakes of trucks

- Pneumatic air preparation: **FRL unit = Filter–Regulator–Lubricator** (in that order). Pneumatic logic: **shuttle valve = OR**, **twin-pressure (dual-pressure) valve = AND**.
- Hydraulic fluid properties: adequate viscosity, high **viscosity index** (small change with temperature), lubricity, low compressibility, high flash point, oxidation stability. Grades by ISO VG (viscosity in cSt at 40 °C, e.g. VG 32, 46, 68). Fire-resistant fluids: water-glycol, phosphate esters.
- Hydraulic pumps: **gear** (cheapest, fixed displacement, moderate pressure), **vane** (quiet), **piston – axial (swash plate / bent axis) & radial** (**highest pressure and efficiency**, variable displacement possible by changing swash-plate angle).

Valve	Function / key point
Direction control valve (DCV)	Routes flow; designated **ports/positions**, e.g. **4/3 valve = 4 ports, 3 positions**; spool type; centre configurations – closed, open, tandem, float
Check (non-return) valve	Flow in **one direction** only; pilot-operated check holds load
Pressure relief valve	**Normally closed**; limits **maximum system pressure**; safety valve of the circuit, placed after pump
Pressure reducing valve	**Normally open**; maintains a **lower downstream** pressure in a branch
Sequence valve	Operates actuators in a **set sequence** (opens at preset pressure)
Counterbalance valve	Prevents **load from falling freely** (vertical cylinders)
Unloading valve	Diverts pump flow to tank at low pressure when accumulator/circuit is satisfied
Flow control valve	Controls **actuator speed**; needle/throttle; **pressure-compensated** type keeps Q constant despite load; meter-in / meter-out / bleed-off circuits

- **Actuators**: **linear** – cylinders: **single-acting** (spring/gravity return), **double-acting**, double-rod (equal speeds both ways), **telescopic** (long stroke from short body – dump trucks), tandem; **rotary** – hydraulic motors (gear, vane, piston); **semi-rotary** (limited angle) actuators.
- Cylinder: extension force F = pA; retraction force = p(A – a). Same Q → **retraction is faster** (smaller annulus area); velocity v = Q/A. **Cushioning** decelerates piston near stroke end.
- **Regenerative circuit**: rod-end oil fed back to cap end → faster extension, reduced force.

KEY FORMULAE

- Press: W = F·A/a; Accumulator capacity = pAL; Intensifier $p_{out} = p_{in}A_{large}/A_{small}$
- Fluid coupling $\eta = N_2/N_1 = 1 - \text{slip}$; torque converter $\eta = (T_2N_2)/(T_1N_1)$
- Hydraulic power = pQ (W, with p in Pa, Q in m^3/s)

MEMORY TRICK

"**Accumulator Accumulates, Intensifier Intensifies, Converter Converts torque**" – the coupling only couples ($T_{out} = T_{in}$), the converter has the extra **stator**. Relief valve "**Relieves – Normally Closed**"; Reducing valve "**Reduces – Normally Open**".

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Device used to store energy of pressurised liquid? → **Hydraulic accumulator**.
- Device that increases pressure intensity of a liquid? → **Hydraulic intensifier**.
- Torque converter differs from fluid coupling by having? → **A stator (reactor)**.
- Valve that limits maximum pressure in a hydraulic circuit? → **Pressure relief valve**.
- Typical working pressure of industrial pneumatic systems? → **About 6–7 bar**.

Nandyal Banking Coaching Center

5. Thermodynamics & Power Cycles

> **NOTE**
> Thermodynamics is the backbone of every mechanical paper: expect **8–12 MCQs** from laws, entropy, steam properties, air-standard cycles, Rankine cycle, boilers and turbines. Questions are mostly direct – a statement of a law, a standard value (triple point, 2257 kJ/kg, 0.528), an efficiency formula, a "which increases/decreases" effect, or a 1-step numerical (Carnot, Otto, COP). Memorise the tables in this chapter.

5.1 Basic Concepts & Zeroth Law ★★★

- **Thermodynamics** = science of energy and its transformation; **classical (macroscopic)** approach treats matter as a continuum, **statistical (microscopic)** approach deals with molecules.
- **System** = quantity of matter or region in space chosen for study; separated from **surroundings** by a **boundary** (real or imaginary, fixed or movable). System + surroundings = **universe**.

System	Mass transfer	Energy transfer	Example
Closed (control mass)	No	Yes	Gas in piston–cylinder
Open (control volume)	Yes	Yes	Turbine, compressor, nozzle, IC engine (actual)
Isolated	No	No	**Universe**; ideal thermos flask
Adiabatic	—	No heat (work allowed)	Perfectly insulated cylinder

- **Property** = any observable characteristic of a system that depends only on its **state**, not on how the state was reached. Its differential is **exact** ($\oint dX = 0$).
- **Intensive** property – independent of mass: **p, T, ρ, specific volume, viscosity, velocity**, all specific (per-kg) properties. **Extensive** – depends on mass: **V, U, H, S, mass, energy**.
- Test: divide system into two halves – intensive property is unchanged, extensive is halved. Ratio of two extensive properties is intensive (e.g. v = V/m).
- **Point function** (state function) = property: p, V, T, U, H, S, G, F. **Path function** = **heat and work** – inexact differentials (δQ, δW); they are **not properties**, they are energy in transit (boundary phenomena).
- **State** = condition described by properties. **State postulate**: state of a **simple compressible system** is fixed by **two independent intensive properties**. (In the wet region p and T are **not** independent – use p and x.)
- **Process** = change of state; **path** = series of states passed through. **Cycle** = initial and final states identical; net change of every property = **0**.
- **Quasi-static (quasi-equilibrium) process** = infinitely slow process, system remains infinitesimally close to equilibrium at every instant; it can be drawn on a property diagram. A quasi-static process with no dissipative effects is **reversible**.
- **Thermodynamic equilibrium** = simultaneous **mechanical** (no unbalanced force/pressure), **thermal** (uniform T), **chemical** (no reaction/mass transfer) and phase equilibrium.
- Causes of irreversibility: **friction, unrestrained (free) expansion, heat transfer through finite ΔT, mixing of different fluids, throttling, electric resistance, inelastic deformation, combustion**.
- **Zeroth law** (named by **R.H. Fowler**): if two bodies are each in thermal equilibrium with a third body, they are in thermal equilibrium with each other. It is the **basis of temperature measurement (thermometry)**; formulated after the first and second laws but logically precedes them.
- **Thermometric property** examples: length of mercury column (liquid-in-glass), pressure at constant volume (**constant-volume gas thermometer – most accurate, used as standard**), resistance (RTD – platinum), emf (thermocouple – Seebeck effect), radiation intensity (pyrometer).
- **Thermodynamic (Kelvin) temperature scale** is independent of the working substance (based on Carnot engine: $Q_1/Q_2 = T_1/T_2$).

Scale	Ice point	Steam point	Relation
Celsius	0 °C	100 °C	—
Fahrenheit	32 °F	212 °F	°F = **1.8 °C + 32**
Kelvin	273.15 K	373.15 K	K = **°C + 273.15**
Rankine	491.67 °R	671.67 °R	°R = °F + 459.67 = 1.8 K

- Celsius and Fahrenheit readings are equal at **−40°**. A temperature **difference** of 1 K = 1 °C = 1.8 °F.
- Triple point of water = **273.16 K** (the old defining fixed point of the kelvin).
- **Work** (thermodynamic definition): work is done by a system if the sole effect on the surroundings could be the raising of a weight. **Heat** = energy transfer due to temperature difference only.
- **Sign convention** (engineering/Rajput, Nag): **work done BY system = +ve**, work done ON system = −ve; **heat added TO system = +ve**, heat rejected = −ve. (Cengel/IUPAC convention takes work done on system as +ve – note the difference.)
- Displacement (pdV) work applies only to a **quasi-static** process; area under the curve on **p–V diagram = work**. **Free (unrestrained) expansion** into vacuum: **W = 0** (and Q = 0, ΔU = 0).
- Other work forms: shaft work = 2πNT, electrical = VIt, spring = $\frac{1}{2}k(x_2^2 - x_1^2)$, stretched wire = $-\int F\,dL$, surface tension = $-\int \sigma\,dA$, paddle-wheel/stirring work is always done **on** the system (negative) and needs no volume change.
- Unit of energy: joule; 1 kWh = **3600 kJ**; 1 cal = 4.187 J; 1 bar = **10^5 Pa**; 1 atm = 101.325 kPa = 760 mm Hg.

Process ($pV^n = C$)	Work $W_{1-2} = \int p\,dV$	Heat Q_{1-2} (ideal gas)
Isochoric (V = C, n = ∞)	**0**	$mc_v(T_2 - T_1)$
Isobaric (p = C, n = 0)	$\mathbf{p(V_2 - V_1)} = mR(T_2 - T_1)$	$mc_p(T_2 - T_1)$
Isothermal (pV = C, n = 1)	$\mathbf{p_1V_1 \ln(V_2/V_1)} = mRT \ln(p_1/p_2)$	Q = W (ΔU = 0)
Adiabatic reversible (n = γ)	$\mathbf{(p_1V_1 - p_2V_2)/(\gamma - 1)}$	0

Nandyal Banking Coaching Center

Polytropic (n)	$(p_1V_1 - p_2V_2)/(n-1) = mR(T_1-T_2)/(n-1)$	$W(\gamma-n)/(\gamma-1)$
Free expansion	0	0

KEY FORMULAE

- Polytropic relations: $T_2/T_1 = (p_2/p_1)^{(n-1)/n} = (V_1/V_2)^{n-1}$
- Polytropic specific heat $c_n = c_v(n-\gamma)/(n-1)$ → **negative for 1 < n < γ** (heat added yet temperature falls)
- Slope on p–V: isothermal = −p/V; adiabatic = −γp/V → **adiabatic is γ times steeper**
- Work in a cycle = $\oint \delta W = \oint \delta Q$ = enclosed area on p–V diagram

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Which of these is a path function – heat, enthalpy, entropy, internal energy? → **Heat**.
- Zeroth law forms the basis of? → **Temperature measurement**.
- Specific volume is an ___ property? → **Intensive**.
- Work done in free expansion is? → **Zero**.
- Work in isothermal process of an ideal gas? → $\mathbf{p_1V_1\ln(V_2/V_1)}$.
- Temperature at which °C and °F readings coincide? → **−40**.

5.2 First Law & Applications ★★★

- **First law** = law of **conservation of energy**. For a **cycle**: $\oint \delta Q = \oint \delta W$ (Joule's paddle-wheel experiment; J = 4.187 J/cal).
- For a **process** (closed system): **Q − W = ΔE**; neglecting KE & PE, **δQ = dU + δW**. The first law defines **internal energy** as a property.
- **Internal energy (U)** = sum of molecular KE + PE (sensible, latent, chemical, nuclear). For an ideal gas U = f(T) only (**Joule's law**): $dU = mc_v dT$ for **any** process.
- **Enthalpy H = U + pV** (property, kJ); for ideal gas also a function of T only: $dH = mc_p dT$. Heat at constant pressure = ΔH; heat at constant volume = ΔU.
- Energy of an **isolated system is constant** (corollary). Q is not a property but (Q − W) is.
- **PMM-1** (perpetual motion machine of first kind) = device producing work continuously **without any energy input** – violates the **first law**; impossible.
- **Specific heats**: $c_v = (\partial u/\partial T)_v$, $c_p = (\partial h/\partial T)_p$. **Mayer's relation: $c_p - c_v = R$**; $\gamma = c_p/c_v > 1$.
- Air: **c_p = 1.005, c_v = 0.718, R = 0.287 kJ/kg·K, γ = 1.4**. Universal gas constant **$\bar{R}$ = 8.314 kJ/kmol·K**; $R = \bar{R}/M$.
- γ by atomicity (kinetic theory): **monatomic 1.67** (He, Ar), **diatomic 1.4** (air, N_2, O_2), **triatomic ≈ 1.33** (CO_2, steam ≈ 1.3). Degrees of freedom f: $\gamma = 1 + 2/f$.
- Ideal gas equation: pV = mRT = n$\bar{R}$T; molar volume at NTP (0 °C, 1 atm) = **22.4 m³/kmol** (Avogadro).

Process	n	p-V-T relation	ΔU	Q	W (closed)
Constant pressure	**0**	V/T = C (Charles)	$mc_v\Delta T$	$mc_p\Delta T$	p ΔV
Isothermal	**1**	pV = C (Boyle)	0	= W	$p_1V_1\ln(V_2/V_1)$
Isentropic	**γ**	pV^γ = C	$mc_v\Delta T$	0	−ΔU
Polytropic	**n**	pV^n = C	$mc_v\Delta T$	$W(\gamma-n)/(\gamma-1)$	$(p_1V_1-p_2V_2)/(n-1)$
Constant volume	**∞**	p/T = C (Gay-Lussac)	$mc_v\Delta T$	$mc_v\Delta T$	0
Throttling	—	h = C, T = C (ideal gas)	0 (ideal gas)	0	0

- For constant-pressure heating of an ideal gas, fraction of heat converted to work = R/c_p = **$(\gamma-1)/\gamma$** (= 0.286 for air); fraction raising U = **$1/\gamma$** (= 0.714).

Steady Flow Energy Equation (SFEE)

- Conditions for steady flow: mass flow rate and properties at every point **constant with time**; mass in = mass out (continuity $\dot{m} = \rho AV$).
- **Flow work** (flow energy) = **pv** per kg; it is why enthalpy (u + pv) appears for open systems.
- Shaft work of a reversible steady-flow device = **$-\int v\,dp$** (area projected on the **p**-axis); closed system work = $\int p\,dv$ (area projected on the v-axis).

KEY FORMULAE

- SFEE (per kg): $\mathbf{h_1 + V_1^2/2 + gz_1 + q = h_2 + V_2^2/2 + gz_2 + w}$
- Nozzle exit velocity: $\mathbf{V_2 = \sqrt{[2(h_1-h_2) + V_1^2]}}$; with Δh in kJ/kg and $V_1 \approx 0$, $V_2 = \mathbf{44.72\sqrt{\Delta h}}$ m/s
- Ideal gas isothermal steady-flow work = isothermal closed work; isentropic steady-flow work = **γ** × isentropic closed work
- Tank filling (initially evacuated, insulated, from a line at T_{line}): $\mathbf{u_2 = h_{line}}$ → $T_2 = \mathbf{\gamma T_{line}}$ (e.g. 300 K line → 420 K in tank)
- Tank emptying/discharge (adiabatic): gas remaining expands isentropically

Device	Assumptions	SFEE reduces to
Nozzle (accelerates fluid)	q = 0, w = 0, Δz = 0	Δh converted to KE: $V_2^2/2 - V_1^2/2 = h_1 - h_2$
Diffuser	q = 0, w = 0	KE converted to enthalpy (pressure rise)
Turbine	q ≈ 0, ΔKE, ΔPE ≈ 0	$\mathbf{w = h_1 - h_2}$
Compressor / pump	adiabatic	$w = -(h_2 - h_1)$ (work input); pump $w = v\,\Delta p$
Throttling valve (porous plug, partially open valve, capillary)	q = 0, w = 0, ΔKE ≈ 0	**$h_1 = h_2$ (isenthalpic)**; irreversible, entropy increases, pressure drops
Boiler	w = 0, ΔKE, ΔPE ≈ 0	$\mathbf{q = h_2 - h_1}$
Condenser / heat exchanger	w = 0, overall adiabatic	$\dot{m}_h\Delta h_h = \dot{m}_c\Delta h_c$ (heat lost = heat gained)

- Throttling is **not** isentropic and **not** reversible; enthalpy is equal at inlet and outlet but the intermediate states are not equilibrium states (shown by a dotted line).
- Throttling of an **ideal gas** → **no change in temperature**; for a real gas temperature may fall or rise (Joule–Thomson effect, §5.4).

MEMORY TRICK

"**0-1-γ-∞**" → **P-T-S-V**: n = 0 constant **P**ressure, n = 1 constant **T**emperature, n = γ constant **S** (isentropic), n = ∞ constant **V**olume.

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Throttling process is? → **Isenthalpic (constant enthalpy)**.
- $c_p - c_v$ for an ideal gas equals? → **R (characteristic gas constant)**.
- Internal energy of an ideal gas depends on? → **Temperature only**.
- Index n for an isothermal process? → **1**.
- Evacuated rigid insulated tank filled from air main at 27 °C – final temperature? → **γT = 420 K (147 °C)**.
- PMM-1 violates? → **First law of thermodynamics**.

5.3 Second Law, Entropy & Availability ★★★

- First law gives quantity; **second law gives direction and quality** of energy: heat (low-grade) cannot be fully converted into work (high-grade).
- **Kelvin–Planck statement**: it is impossible to construct a device operating in a cycle that produces net work while exchanging heat with a **single reservoir** → no heat engine can have **100% efficiency**.
- **Clausius statement**: it is impossible to construct a device operating in a cycle whose **sole effect** is transfer of heat from a colder to a hotter body → refrigerator needs work input (COP ≠ ∞).
- The two statements are **equivalent** – violation of one implies violation of the other.
- **PMM-2** = engine converting all heat from one reservoir into work – violates **Kelvin–Planck** statement (not the first law).
- **Thermal reservoir** = body of infinite heat capacity; source (high T) and sink (low T).

Device	Performance index	Carnot (reversible) value
Heat engine	$\eta = W/Q_1 = 1 - Q_2/Q_1$	$1 - T_L/T_H$
Refrigerator	$COP_R = Q_L/W$	$T_L/(T_H - T_L)$
Heat pump	$COP_{HP} = Q_H/W$	$T_H/(T_H - T_L)$

- **$COP_{HP} = COP_R + 1$** (same temperature limits); COP_{HP} always > 1; COP_R may be < or > 1. Also $COP_{HP} = 1/\eta_{Carnot\ engine}$.
- **Carnot cycle** (Sadi Carnot, 1824) = two **reversible isothermal** + two **reversible adiabatic (isentropic)** processes; on T–s it is a **rectangle**. Efficiency depends only on reservoir temperatures, **independent of working fluid**.
- **Carnot theorem**: (i) no engine working between two given reservoirs can be more efficient than a reversible engine; (ii) **all reversible engines** between the same two reservoirs have the **same efficiency**.
- Carnot cycle is impractical: isothermal processes need infinitely slow piston, adiabatic processes need very fast piston; very low work ratio (mep) for gas.
- To raise η_{Carnot}, **lowering T_L** is more effective than raising T_H by the same amount ($\partial\eta/\partial T_L = -1/T_H$; $\partial\eta/\partial T_H = T_L/T_H^2$). η = 100% only if T_L = 0 K (impossible – third law).
- Reversible engines in **series** with equal efficiencies: intermediate $T = \sqrt{(T_1 T_3)}$; for equal work output: $T_2 = (T_1 + T_3)/2$.
- **Clausius inequality**: $\oint \delta Q/T \le 0$ (= 0 reversible cycle, < 0 irreversible, > 0 impossible).
- **Entropy** (Clausius): $dS = (\delta Q/T)_{rev}$; property (point function), extensive, unit **kJ/K** (specific: kJ/kg·K). Measure of molecular disorder (Boltzmann S = k ln W).
- For any process: $dS \ge \delta Q/T$; $S_{gen} \ge 0$. Entropy can be transferred by **heat and mass**, but not by work.
- **Principle of increase of entropy**: $\Delta S_{isolated} = \Delta S_{universe} \ge 0$ – entropy of the universe tends to a maximum.
- **Reversible adiabatic = isentropic**. Irreversible adiabatic → entropy **increases**. Entropy of a closed system can decrease only if heat is removed from it.
- **T–s diagram**: area under the curve = **heat transferred** (reversible). Area enclosed by a cycle = net work = net heat.
- On T–s, slope of constant-volume line = T/c_v > slope of constant-pressure line = T/c_p → **isochore is steeper than isobar**. Isotherm is horizontal; isentrope is vertical.

KEY FORMULAE

- Tds relations: **T ds = du + p dv**; **T ds = dh − v dp** (valid for any process between equilibrium states)
- Ideal gas: $s_2 - s_1 = c_v \ln(T_2/T_1) + R \ln(v_2/v_1) = c_p \ln(T_2/T_1) - R \ln(p_2/p_1)$
- Isothermal: $\Delta s = R \ln(v_2/v_1)$; free expansion to double volume: $\Delta S = mR \ln 2$ (> 0 though Q = 0)
- Solid/liquid heated: $\Delta S = mc \ln(T_2/T_1)$; phase change: ΔS = mL/T (e.g. melting ice 333.5/273 ≈ 1.22 kJ/kg·K)
- Polytropic: $\Delta s = c_n \ln(T_2/T_1)$
- Heat transfer Q from T_1 to T_2: $\Delta S_{univ} = Q/T_2 - Q/T_1 > 0$

Available Energy, Exergy & Irreversibility

- **Available energy (AE)** = maximum work obtainable from heat Q supplied at T with environment at **T_0**: $AE = Q(1 - T_0/T)$. **Unavailable energy (UE) = $T_0\Delta S$**.
- When heat flows through a finite ΔT, unavailable energy increases by $T_0\Delta S_{univ}$ – **energy is degraded** ("law of degradation of energy").
- **Dead state** = system in equilibrium with environment (p_0, T_0, ≈ 1 atm, 25 °C); exergy = 0 there.
- **Exergy (availability)** = maximum useful work obtainable as system comes to dead state. Exergy is **not conserved** – it is destroyed by irreversibility.
- Closed (non-flow) availability: **$\varphi = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$**.
- Flow (stream) availability: **$\psi = (h - h_0) - T_0(s - s_0) + V^2/2 + gz$**.
- **Gibbs function G = H − TS** (useful for steady flow / chemical reactions, constant p & T); **Helmholtz function F = U − TS** (constant V & T). Max work at constant T & V = −ΔF; max useful work at constant T & p = −ΔG.
- **Irreversibility $I = W_{rev} - W_{actual}$** = exergy destroyed.
- **Gouy–Stodola theorem**: **$I = T_0 S_{gen} = T_0\Delta S_{universe}$**.
- **Second-law (exergetic) efficiency** $\eta_{II} = \eta_{th}/\eta_{rev}$ = exergy recovered/exergy supplied.
- **Third law** (Nernst heat theorem, Planck): entropy of a **pure perfect crystalline substance at 0 K is zero**; absolute zero cannot be reached in a finite number of steps. It allows absolute entropy values.

Law	Introduces property	Violation

Nandyal Banking Coaching Center

Zeroth	**Temperature**	—
First	**Internal energy**	PMM-1
Second	**Entropy**	PMM-2
Third	Absolute entropy (S = 0 at 0 K)	Reaching 0 K

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Carnot engine between 727 °C and 27 °C – efficiency? → 1 – 300/1000 = **70%**.
- Refrigerator COP = 4; same machine as heat pump COP? → **5**.
- Entropy of universe in a real process? → **Increases**.
- Irreversibility = $T_0 S_{gen}$ is known as? → **Gouy–Stodola theorem**.
- Which law defines entropy as a property? → **Second law**.
- Kelvin–Planck statement refers to? → **Heat engine (no 100% conversion with one reservoir)**.

5.4 Properties of Pure Substances & Steam ★★★

- **Pure substance** = homogeneous and **invariable chemical composition** in all phases: water, water + ice + steam, air (if no phase change). Air with liquid air is **not** a pure substance (composition of phases differs).
- **Phase diagram (p–T)**: sublimation, fusion (melting) and vaporisation curves meet at the **triple point**. Fusion curve of water has **negative slope** (water expands on freezing) – pressure lowers the melting point.
- **Triple point** = all three phases coexist; on p–T a point, on p–v a **line**. Water: **0.01 °C (273.16 K), 611 Pa (0.6113 kPa)**. CO_2 triple point ≈ 5.17 bar → dry ice **sublimes** at 1 atm.
- **Critical point** = liquid and vapour indistinguishable; $\mathbf{h_{fg} = 0}$, $v_f = v_g$. Water: **374.15 °C, 221.2 bar** (modern IAPWS: 373.95 °C, 220.64 bar), $v_c \approx$ **0.00317 m³/kg**.
- Above critical pressure → **supercritical**: water turns to steam without boiling (no distinct latent heat) – basis of **Benson boiler**.
- Latent heat h_{fg} **decreases** with increasing pressure: **2257 kJ/kg at 1.01325 bar (100 °C)**, zero at critical point. At 1 atm h_f = 419 kJ/kg, h_g = 2676 kJ/kg.
- Subcooled (compressed) liquid → saturated liquid → wet steam → dry saturated steam → superheated steam. **Degree of superheat** = $T_{sup} - T_{sat}$; degree of subcooling = $T_{sat} - T$.
- **Dryness fraction x** = mass of dry vapour / total mass of wet steam (x = 0 saturated liquid, x = 1 dry saturated). **Wetness = 1 – x**. Quality is defined only in the wet region.
- **Steam tables**: saturation tables (by T and by p) give v_f, v_g, h_f, h_{fg}, h_g, s_f, s_{fg}, s_g; superheat tables need both p and T. Reference: **u = 0 and s = 0 for saturated liquid at the triple point** (0.01 °C).
- **Mollier chart (h–s)**: best for steam turbine/nozzle work (isentropic Δh = vertical distance). In the wet region **constant-pressure lines are straight** (slope = T_{sat}) and coincide with isotherms; constant-pressure lines **diverge** upward; in superheat region isotherms become nearly horizontal (ideal-gas behaviour at low p). Usually covers only x > ~0.6 and superheat.
- On T–s, slope of constant-pressure line = $(\partial T/\partial s)_p = T/c_p$; on h–s, slope = $(\partial h/\partial s)_p = \mathbf{T}$.

KEY FORMULAE

- Wet steam: $\mathbf{h = h_f + x\, h_{fg}}$; $s = s_f + x\, s_{fg}$; $v = v_f + x\, v_{fg} \approx \mathbf{x\, v_g}$ (at low p)
- Superheated: $h = h_g + c_{ps}(T_{sup} - T_{sat})$ ($c_{ps} \approx 2.1$ kJ/kg·K); $s = s_g + c_{ps} \ln(T_{sup}/T_{sat})$
- Internal energy u = h – pv
- **Clapeyron equation: $dp/dT = h_{fg}/(T\, v_{fg})$**; Clausius–Clapeyron (low p, ideal vapour): $\ln(p_2/p_1) = (h_{fg}/R)(1/T_1 - 1/T_2)$

Steam Calorimeters (measurement of x)

Calorimeter	Principle	Remarks
Barrel (bucket)	Steam condensed in water; heat balance	Crude, inaccurate (radiation losses)
Separating	Mechanical separation of water droplets; **x = M/(M + m)** (M = dry steam condensed, m = water separated)	Gives approximate x (separation incomplete); suited to very wet steam
Throttling	Throttle to low p until superheated; $\mathbf{h_{f1} + x\, h_{fg1} = h_{g2} + c_p(T_{sup} - T_{sat2})}$	Needs steam to become superheated after throttling → only for fairly dry steam (x ≳ 0.95)
Separating + throttling	Separator first, then throttle	$\mathbf{x = x_1 \times x_2}$; most accurate, any wetness
Electrical	Electric heating to superheat	Energy balance

Real Gases & Thermodynamic Relations

- Real gases approach ideal behaviour at **low pressure and high temperature** (relative to critical point).
- **Compressibility factor Z = pv/RT**; Z = 1 ideal gas; Z < 1 attractive forces dominate; Z > 1 repulsive forces dominate (very high p). **Generalized compressibility chart** uses reduced properties $p_r = p/p_c$, $T_r = T/T_c$ – **law of corresponding states**.
- **Van der Waals equation: $(p + a/v^2)(v - b) = RT$**; a/v^2 = correction for intermolecular attraction, b = co-volume (finite molecular volume).
- Van der Waals critical constants: $\mathbf{v_c = 3b}$, $\mathbf{p_c = a/(27b^2)}$, $\mathbf{T_c = 8a/(27Rb)}$ → $\mathbf{Z_c = 3/8 = 0.375}$ (actual gases 0.23–0.31). Critical isotherm has a point of inflection: $(\partial p/\partial v)_T = 0$ and $(\partial^2 p/\partial v^2)_T = 0$.
- Other equations of state: Beattie–Bridgeman (5 constants), Benedict–Webb–Rubin (8 constants), Redlich–Kwong, virial equation ($Z = 1 + B/v + C/v^2 \ldots$).
- **Joule–Thomson coefficient $\mu_{JT} = (\partial T/\partial p)_h$. μ = 0 for an ideal gas.** μ > 0 → gas **cools** on throttling (most gases at room T); μ < 0 → gas heats.
- **Inversion curve** = locus of μ = 0 on T–p plot; above the **maximum inversion temperature** throttling always heats. H_2 (~200 K) and He (~40 K) have low inversion temperatures → they **heat up** on throttling at room temperature (must be precooled for liquefaction – Linde–Hampson process). Van der Waals max inversion temperature = 2a/(Rb) = **6.75 T_c**.

Nandyal Banking Coaching Center

KEY FORMULAE

- Maxwell relations: $(\partial T/\partial v)_s = -(\partial p/\partial s)_v$; $(\partial T/\partial p)_s = (\partial v/\partial s)_p$; $(\partial p/\partial T)_v = (\partial s/\partial v)_T$; $(\partial v/\partial T)_p = -(\partial s/\partial p)_T$
- Derived from du = Tds − pdv, dh = Tds + vdp, dF = −sdT − pdv, dG = −sdT + vdp ("Gibbs relations")
- $c_p - c_v = T v \beta^2/\kappa_T \geq 0$ (β = volume expansivity, κ_T = isothermal compressibility); for liquids/solids $c_p \approx c_v$; $c_p = c_v$ for water at 4 °C (β = 0)
- κ_s (isentropic compressibility) = κ_T/γ
- $\mu_{JT} = [T(\partial v/\partial T)_p - v]/c_p$

MEMORY TRICK

Water's two magic points: **Triple = 0.01 °C & 0.611 kPa** ("zero point zero one, six-one-one"); **Critical = 374 °C & 221 bar** ("3-7-4, 2-2-1").

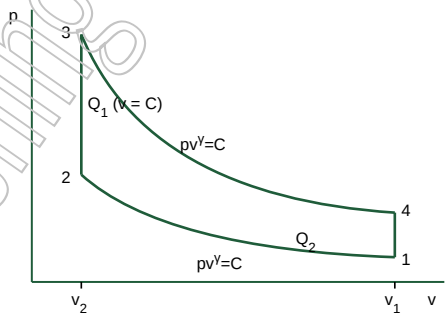
PREVIOUSLY ASKED / LIKELY QUESTIONS

- Latent heat of vaporisation at critical point? → **Zero**.
- Critical pressure of water? → **221.2 bar (≈ 22.1 MPa)**.
- Dryness fraction of very wet steam is best measured by? → **Combined separating and throttling calorimeter**.
- Joule–Thomson coefficient of an ideal gas? → **Zero**.
- Critical compressibility factor of a van der Waals gas? → **0.375**.
- Equation relating h_{fg} to slope of saturation curve? → **Clapeyron equation**.

5.5 Gas Power Cycles ★★★

- **Air-standard assumptions**: working fluid is air (ideal gas, constant c_p, c_v), closed cycle, combustion replaced by heat addition from external source, exhaust by heat rejection, all processes internally reversible. **Cold air-standard** = specific heats at 25 °C.
- **Air-standard efficiency** = ideal efficiency of the cycle; actual engine indicated efficiency ≈ 50–60% of it (relative efficiency).
- **Compression ratio r** = $V_1/V_2 = (V_s + V_c)/V_c$. Swept volume $V_s = (\pi/4)D^2L$.
- **Mean effective pressure (mep)** = **net work per cycle / swept volume**; the constant pressure that, acting over one stroke, gives the same net work. Higher mep → smaller engine for same power.
- **Work ratio** = net work / gross (positive) work.

Cycle	Processes	Used in / feature
Carnot	2 isothermal + 2 isentropic	Max possible η; impractical, very low mep
Otto (Beau de Rochas 1862, Otto 1876)	2 isentropic + 2 **constant volume**	SI (petrol) engines
Diesel (1892)	2 isentropic + heat addition at **constant pressure** + rejection at constant volume	Slow-speed CI engines
Dual / mixed / limited pressure (Sabathe)	2 isentropic + heat addition partly at constant V, partly at constant p + rejection at constant V	Modern high-speed CI engines
Stirling (1816)	2 **isothermal** + 2 **constant volume** (regenerative)	Stirling engine, cryocoolers (reversed)
Ericsson	2 **isothermal** + 2 **constant pressure** (regenerative)	Ideal GT with infinite intercooling/reheat/regeneration
Brayton / Joule	2 isentropic + 2 **constant pressure**	Gas turbines
Atkinson	2 isentropic + constant-V heat addition + **constant-p heat rejection**; expansion ratio > compression ratio	Hybrid-vehicle engines (Atkinson/Miller valve timing)
Lenoir	**No compression**: constant-V heat addition, isentropic expansion, constant-p rejection	Pulse-jet; early gas engines



Air-standard Otto cycle on p–v: 1-2 isentropic compression, 2-3 constant-volume heat addition, 3-4 isentropic expansion, 4-1 constant-volume heat rejection; r = v_1/v_2.

KEY FORMULAE

- **Otto: $\eta = 1 - 1/r^{\gamma-1}$** – depends **only on r** (and γ); r = 8, γ = 1.4 → η = **56.5%**; r = 10 → 60.2%
- Otto: $T_2/T_1 = T_3/T_4 = r^{\gamma-1}$; mep = $p_1 r(r^{\gamma-1} - 1)(\alpha - 1)/[(\gamma-1)(r - 1)]$, $\alpha = p_3/p_2$ (pressure ratio)
- Otto: for maximum work between T_1 and T_3, $T_2 = T_4 = \sqrt{(T_1T_3)}$
- **Diesel: $\eta = 1 - (1/r^{\gamma-1})\cdot[(\rho^\gamma - 1)/(\gamma(\rho - 1))]$**, cut-off ratio ρ = V_3/V_2; bracket term > 1 → $\eta_{Diesel} < \eta_{Otto}$ for same r; η **decreases as ρ (load) increases**; as ρ → 1, $\eta_{Diesel} \to \eta_{Otto}$
- Expansion ratio of Diesel = r/ρ
- **Dual: $\eta = 1 - (1/r^{\gamma-1})\cdot[(\alpha\rho^\gamma - 1)/((\alpha - 1) + \gamma\alpha(\rho - 1))]$**; α = 1 → Diesel; ρ = 1 → Otto
- **Stirling / Ericsson with perfect regenerator: $\eta = 1 - T_L/T_H$ (= Carnot)**; without regenerator η is lower
- **Brayton: $\eta = 1 - 1/r_p^{(\gamma-1)/\gamma} = 1 - T_1/T_2$**; depends only on pressure ratio r_p
- Brayton optimum pressure ratio for **maximum net work: $r_{p,opt} = (T_{max}/T_{min})^{\gamma/[2(\gamma-1)]} = [\sqrt{(T_3/T_1)}]^{\gamma/(\gamma-1)}$**; then $T_2 = T_4 = \sqrt{(T_1T_3)}$
- At that optimum: $W_{max} = c_p(\sqrt{T_3} - \sqrt{T_1})^2$, $\eta = 1 - \sqrt{(T_1/T_3)}$
- Brayton r_p for **zero work** (max efficiency = Carnot): $r_p = (T_3/T_1)^{\gamma/(\gamma-1)}$
- Brayton work ratio = $1 - (T_1/T_3)r_p^{(\gamma-1)/\gamma}$ → low (sensitive to component efficiencies)

Comparison condition	Order of efficiency

Nandyal Banking Coaching Center

Same compression ratio and same heat input	**Otto > Dual > Diesel**
Same maximum pressure and same maximum temperature (same heat rejection)	**Diesel > Dual > Otto**
Same maximum pressure and same heat input	**Diesel > Dual > Otto**
Same compression ratio and same heat rejection	Otto > Dual > Diesel

- Logic: $\eta = 1 - Q_{rejected}/Q_{supplied}$; whichever cycle needs to reject less heat for the given constraint is more efficient. Diesel engines are more efficient in practice because they run at much higher r (14–22 vs 6–11 for SI; SI is limited by **knock**).
- Otto efficiency increases with r but rate of increase falls at high r; also increases with γ (air better than richer mixtures).
- **Regenerator** in Stirling/Ericsson stores heat during one constant-V (or constant-p) process and returns it in the other – external heat is received only at T_H, hence Carnot efficiency.
- Atkinson cycle has **higher efficiency than Otto** for the same compression ratio (extra expansion) but lower power density (lower mep).

MEMORY TRICK

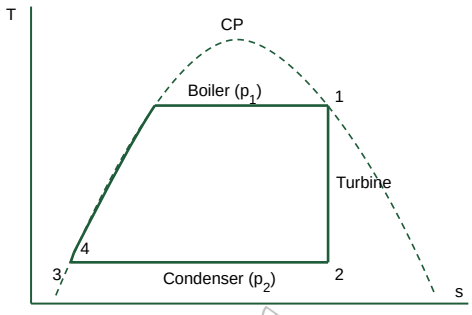
"**Same r → ODD**" (Otto, Dual, Diesel – descending). "**Same peak → reverse it, DDO**" (Diesel, Dual, Otto). Stirling = **S**tirling has **V** (isochoric); Ericsson = **E**ricsson has **P** (isobaric) – both with two isotherms.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Efficiency of Otto cycle depends on? → **Compression ratio only** (for given γ).
- For the same compression ratio and heat input, the most efficient cycle? → **Otto**.
- Stirling cycle consists of? → **Two isothermal and two constant-volume processes**.
- Brayton cycle with $r_p = 8$ (γ = 1.4) – efficiency? → $1 - 1/8^{0.286} \approx$ **44.8%**.
- Diesel efficiency as cut-off ratio increases? → **Decreases**.
- Cycle with no compression process? → **Lenoir cycle**.

5.6 Vapour Power Cycles & Steam Generators ★★★

- **Rankine cycle** (ideal cycle for steam power plants): 1-2 **isentropic expansion in turbine**, 2-3 **constant-pressure condensation** in condenser, 3-4 **isentropic pumping**, 4-1 **constant-pressure heat addition** in boiler.
- **Why not Carnot with steam?** (i) compressing wet mixture (two-phase) to saturated liquid is very difficult – pump cavitation; (ii) heat addition at constant T is limited to the wet region (T below critical 374 °C) – superheating is not isothermal; (iii) expansion of very wet steam erodes turbine blades. Rankine condenses fully to liquid, so **pump work is tiny** (≈ 1% of turbine work).
- Rankine η < Carnot η between same temperature limits (mean temperature of heat addition is lower), but **work ratio is high (≈ 0.99)** and specific steam consumption is low.



Simple Rankine cycle on T–s (dry saturated steam at turbine inlet; dashed = saturation dome, CP = critical point). 3-4 pump (exaggerated), 4-1 boiler, 1-2 turbine, 2-3 condenser.

KEY FORMULAE

- Turbine work $W_T = h_1 - h_2$; pump work $\mathbf{W_P = v_f(p_1 - p_2)} = h_4 - h_3$
- $\eta_{Rankine} = [(h_1 - h_2) - (h_4 - h_3)]/(h_1 - h_4) \approx (h_1 - h_2)/(h_1 - h_{f3})$ neglecting pump work
- **Specific steam consumption (SSC) = 3600/W_{net}** kg/kWh (W_{net} in kJ/kg)
- **Heat rate = 3600/η** kJ/kWh (lower is better; 36% η → 10,000 kJ/kWh)
- Mean temperature of heat addition $T_{m1} = (h_1 - h_4)/(s_1 - s_4)$; $\eta = 1 - T_2/T_{m1}$
- Combined (binary/topping-bottoming) cycle: $\eta = \eta_1 + \eta_2 - \eta_1\eta_2$

Change	Efficiency	Net work	Dryness at turbine exit
Increase boiler pressure (same T_{max})	**Increases**	≈ same / slight decrease	**Decreases** (moisture ↑)
Superheating	**Increases**	Increases	**Increases**
Lower condenser pressure	**Increases**	Increases	**Decreases**; air leakage problem
Reheating	Small increase (~4–5%, if T_m rises)	Increases	**Increases (main purpose)**
Regeneration (feed heating)	**Increases**	**Decreases** per kg (SSC ↑)	— (reduces exhaust flow, smaller condenser)

- Moisture at turbine exhaust should generally not exceed **10–12%** (x ≥ 0.88–0.90) to avoid blade erosion.
- **Reheat**: steam partially expanded in HP turbine is reheated (usually to initial temperature) at constant pressure; optimum reheat pressure ≈ **20–25% of boiler pressure**. Reheat permits higher boiler pressures.
- **Regeneration**: steam bled from turbine heats feed water → raises mean temperature of heat addition. Ideal regenerative cycle with infinite heaters → **Carnot efficiency**. Typical plants use 5–8 heaters.
- **Open feed-water heater** (direct contact/mixing) – bled steam mixes with feed water; exit = saturated liquid at heater pressure; a pump needed after each heater; also acts as **deaerator** (removes O_2, CO_2). Simple, cheap, better heat transfer.
- **Closed feed-water heater** (surface/shell-and-tube) – no mixing; drains cascaded backward (via trap) or pumped forward; can operate at different pressures; costlier.

Nandyal Banking Coaching Center

- Optimum feed water temperature with one heater ≈ **mean of boiler saturation temperature and condenser temperature** (equal enthalpy rise per stage).
- **Binary vapour cycle**: **mercury** (topping, high T at low p) + **steam** (bottoming); mercury condenser = steam boiler. Ideal fluid: high critical T, low saturation p at high T, triple point below ambient.
- **Cogeneration (CHP)** = simultaneous production of power and process heat; back-pressure or pass-out (extraction) turbines; utilisation factor = $(W + Q_{process})/Q_{in}$ (can be 70–85%).
- **Combined cycle (CCGT)**: Brayton topping (gas turbine exhaust ~500–600 °C) + Rankine bottoming via **HRSG** (heat recovery steam generator); η ≈ **50–62%** (vs ~33–38% subcritical coal, ~40–45% supercritical).

Boilers (Steam Generators)

Feature	Fire-tube	Water-tube
Inside tubes	Hot flue gases	Water / steam
Pressure	Low (≤ ~25 bar)	**High** (up to supercritical)
Steaming rate	Lower; large water content	Higher; quicker raising of steam
Explosion risk	Greater damage (large water volume)	Less severe
Water quality	Less critical	Needs treated water
Examples	**Cochran, Lancashire, Cornish, Locomotive, Scotch marine**	**Babcock & Wilcox, Stirling, La Mont, Benson, Loeffler, Velox**

Boiler	Key identification
Cochran	**Vertical**, multitubular, fire-tube, internally fired; compact, small capacity
Lancashire	**Horizontal, two large flue tubes**, internally fired, natural draught, stationary; ~8 m long; uses brickwork flues
Cornish	Like Lancashire but **single flue tube**
Locomotive	Horizontal multitubular fire-tube; **artificial (induced) draught by exhaust steam blast**; high steaming rate for its size
Babcock & Wilcox	**Horizontal straight-tube water-tube**, tubes inclined **~15°**, externally fired, natural circulation, with drum
La Mont	**Forced circulation** (pump circulates ~8–10 × evaporation), water-tube, with drum
Benson	**Supercritical** (above 221 bar), **once-through, drumless**, forced flow; no bubble formation
Loeffler	Forced circulation: **superheated steam is used to evaporate feed water** in drum (avoids salt deposition in tubes)
Velox	Pressurised combustion; gas velocity above sonic → very high heat transfer rate
Schmidt–Hartmann	Indirect heating – two-fluid circuits (impure water allowed)

- **Mountings** = fittings mounted on boiler for **safety and control** – compulsory by law (IBR). **Accessories** = auxiliaries to **increase efficiency** – optional.

Mountings (safety/control)	Accessories (efficiency)
Safety valve (dead-weight, lever, spring-loaded, high-steam low-water) – at least two per boiler	**Economiser** – heats **feed water** using flue gases
Water level indicator (gauge glass) – usually two	**Superheater** – superheats steam (dryness/temperature ↑)
Pressure gauge (Bourdon tube)	**Air preheater** – heats **combustion air** using flue gases (last in gas path)
Fusible plug – low-melting alloy (Sn–Pb) plug over furnace crown; melts on low water level and steam/water extinguishes fire	Feed pump, **injector** (feeds water using steam jet; also used on locomotives)
Steam stop valve (junction valve)	Steam separator, steam trap
Feed check valve (non-return)	Reheater, soot blower
Blow-off cock (removes sludge, empties boiler); manhole	Draught fans (FD/ID)

- Flue gas path in a modern utility boiler: furnace → **superheater → reheater → economiser → air preheater** → ESP → ID fan → chimney.
- **Boiler performance: actual evaporation** = kg steam per kg fuel; **factor of evaporation F = $(h - h_{f,feed})/2257$**.
- **Equivalent evaporation "from and at 100 °C"** = mass of water evaporated from feed water at 100 °C into dry saturated steam at 100 °C, absorbing the same heat: $\mathbf{m_e = m_a(h - h_f)/2257}$ (2257 kJ/kg = h_{fg} at 1.01325 bar). Used to compare boilers.
- **Boiler efficiency $\eta = m_s(h - h_f)/(m_f \times CV)$**; typical 80–90% (pulverised coal ~85–90%). Heat balance sheet: largest loss is **dry flue gas loss**.
- **Draught** = pressure difference causing flow of air/flue gas; measured in **mm of water** by U-tube manometer (draught gauge). Natural (chimney) draught is small, ~10–20 mm water.
- Artificial draught: **forced** (FD fan before furnace, positive pressure), **induced** (ID fan before chimney, handles hot gases, larger power), **balanced** (FD + ID, furnace slightly below atmospheric), steam-jet (locomotives).
- **Chimney efficiency** is extremely low (< 1%); natural draught does not depend on fan power but is affected by atmospheric temperature.

KEY FORMULAE

- Natural draught: $h_w = 353\,H\,[1/T_a - (m+1)/(m\,T_g)]$ mm of water (H = chimney height in m, T in K, m = kg air per kg fuel)
- Draught in metres of hot gas column: $H_g = H[(T_g/T_a)\cdot m/(m+1) - 1]$
- Condition for **maximum discharge** through chimney: $T_g/T_a = 2(m+1)/m$ (draught = height of one hot gas column)
- Gas velocity in chimney (theoretical): $V = \sqrt{(2gH_g)}$

- **Indian Boiler Regulations (IBR), 1950** framed under the **Indian Boilers Act, 1923** (now replaced by the Boilers Act, 2025 – see box below; administered by the Central Boilers Board). A "boiler" = closed pressure vessel of capacity **≥ 25 litres** (raised from 22.75 L by the 2007 amendment) used for generating steam under pressure.
- IBR: new boiler **hydraulic test at 1.5 × maximum working pressure**; registration and certificate of fitness from Chief Inspector of Boilers; every boiler must carry at least **two safety valves**, two water-level indicators, a pressure gauge, feed check

Nandyal Banking Coaching Center

valve, stop valve, blow-down cock and fusible plug (where applicable).

CODE UPDATE

The **Boilers Bill, 2024** (passed by Rajya Sabha Dec 2024, Lok Sabha 25 Mar 2025) became the **Boilers Act, 2025 (Act No. 12 of 2025)** on presidential assent, **4 April 2025**; it came into force on **1 May 2025** and **repeals the Boilers Act, 1923** (decriminalisation of minor offences, uniform registration/inspection). Boiler = vessel of **≥ 25 litres** (unchanged). Under its savings clause (s. 45), **IBR 1950 continues** for technical requirements until new regulations are notified – check the latest notification.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Main purpose of reheating in a Rankine cycle? → **To increase dryness fraction of steam at turbine exhaust**.
- Benson boiler operates at? → **Supercritical pressure (drumless, once-through)**.
- Which is a boiler accessory – fusible plug, safety valve, economiser, pressure gauge? → **Economiser**.
- Equivalent evaporation is referred to? → **From and at 100 °C (h_{fg} = 2257 kJ/kg)**.
- Babcock & Wilcox boiler is? → **Water-tube, externally fired, horizontal inclined tubes**.
- Regeneration in a Rankine cycle ___ the specific steam consumption? → **Increases** (but improves efficiency).

5.7 Steam & Gas Turbines, Nozzles ★★☆

Nozzles

- **Nozzle** converts enthalpy (pressure energy) into kinetic energy; **diffuser** does the reverse. Flow is ideally **isentropic (adiabatic)**.
- Subsonic flow (M < 1): converging section accelerates; supersonic flow (M > 1): diverging section accelerates. Maximum velocity in a convergent nozzle = **sonic (M = 1) at the throat**.
- **Critical pressure** = throat pressure at which mass flow is maximum; **choked flow**: once $p_{exit} \le p_{critical}$, further lowering of back pressure does **not** increase mass flow.
- **Convergent–divergent (de Laval) nozzle** needed for supersonic exit velocity (back pressure < critical pressure).
- **Supersaturated (metastable) flow**: steam expands too fast to condense at saturation line – remains dry vapour below saturation temperature; limit = **Wilson line** (≈ 0.96–0.97 dryness on Mollier chart). Effects: **mass flow increases** (2–5%), enthalpy drop slightly reduces, entropy and specific volume change.
- Nozzle efficiency = actual enthalpy drop / isentropic drop (typically 0.90–0.97); friction mostly in divergent part. Velocity coefficient = √(nozzle efficiency).
- Over-expanded nozzle → shock waves inside/at exit; under-expanded → expansion outside nozzle.

KEY FORMULAE

- **Critical pressure ratio $p_c/p_1 = [2/(n+1)]^{n/(n-1)}$**
- Exit velocity $V_2 = \sqrt{[2n/(n-1)]\, p_1 v_1 [1 - (p_2/p_1)^{(n-1)/n}]}$
- Critical (throat) velocity = **$\sqrt{(n\, p_c\, v_c)}$** = local sonic velocity $\sqrt{(\gamma R T_c)}$ for gas; $T_c/T_1 = 2/(\gamma+1) = 0.833$ for air
- Mass flow $\dot{m}$ = A V/v; max at throat when $p_t = p_c$

Fluid	Index n	Critical pressure ratio
Air / diatomic gas	1.4	**0.528**
Superheated steam	1.3	**0.546**
Dry saturated steam	1.135	**0.577**
Wet steam (Zeuner, x)	1.035 + 0.1x	≈ 0.58

Steam Turbines

Feature	Impulse turbine	Reaction turbine
Pressure drop	**Only in nozzles**; pressure constant across moving blades	In **both fixed and moving blades**
Blade profile	Symmetrical, constant-area passage	Aerofoil, converging passage
Relative velocity	$V_{r2} = V_{r1}$ (no friction) or kV_{r1}	**$V_{r2} > V_{r1}$**
Admission	Partial admission possible	**Full admission** only
Speed / stages	High speed per stage, fewer stages	Lower blade speed, many stages, larger size
Tip leakage	Small (no Δp across blades)	Significant; balance piston needed for axial thrust
Examples	**De Laval** (simple), **Curtis**, **Rateau**	**Parsons** (50% reaction)

- **De Laval turbine**: single-stage impulse; very high rotor speed (~30,000 rpm) → needs reduction gearing; high leaving loss. Hence **compounding** to reduce rotor speed.
- **Velocity compounding (Curtis)**: one set of nozzles + two or more rows of moving blades with **fixed guide blades** between them; whole pressure drop in nozzle. Optimum u/V_1 = **cos α/(2n)** for n rows (cos α/4 for two rows); work in 2-row Curtis is divided **3:1** between first and second rows. Less efficient but compact – used in HP stage/small turbines.
- **Pressure compounding (Rateau)**: series of nozzle + moving-blade stages; pressure drops in steps in each nozzle ring; lower velocities, higher efficiency, but larger and costlier.
- **Pressure–velocity compounding**: combination (e.g. Curtis stages in series).
- **Degree of reaction R = Δh (moving blades) / Δh (stage)**. R = 0 → pure impulse; **R = 0.5 → Parsons** (symmetrical blades: $\alpha_1 = \beta_2$, $\beta_1 = \alpha_2$; velocity triangles symmetrical).
- **Blade (diagram) efficiency** = work done on blades / KE supplied = $2u\,\Delta V_w/V_1^2$. **Stage efficiency** = blade efficiency × nozzle efficiency.
- Axial thrust in impulse turbine = $\dot{m}(V_{f1} - V_{f2})$; in reaction turbine also due to pressure difference.
- Governing: throttle governing, nozzle control governing, by-pass governing. Losses: nozzle friction, blade friction, disc friction, leaving (carry-over) loss, leakage, partial admission, wetness loss.

Nandyal Banking Coaching Center

KEY FORMULAE

- Single-stage impulse (symmetrical blades, no friction): optimum blade speed ratio $\mathbf{u/V_1 = \cos\alpha/2}$; $\mathbf{\eta_{b,max} = \cos^2\alpha}$
- With blade friction k and $\beta_1 \neq \beta_2$: $\eta_{b,max} = (\cos^2\alpha/2)(1 + k\cos\beta_2/\cos\beta_1)$
- 50% reaction (Parsons): optimum $\mathbf{u/V_1 = \cos\alpha}$; $\mathbf{\eta_{b,max} = 2\cos^2\alpha/(1 + \cos^2\alpha)}$
- Power = $\dot{m}\, u\, \Delta V_w$ (W); for impulse $\Delta V_w = V_{w1} + V_{w2}$ (when V_{w2} is opposite)

Gas Turbines

- **Open cycle GT**: air drawn from atmosphere, internal combustion, exhaust to atmosphere – light, quick starting, used in aircraft and peaking plants. **Closed cycle**: working fluid recirculated (air, helium), external heating and cooler; can use any fuel, high pressure → compact, no erosion of blades, but needs heat exchangers.
- Compressor in GT consumes **50–70%** (≈ 2/3) of turbine work → low work ratio; component efficiencies critical.
- Turbine inlet temperature (TIT) limited by blade material (~1100–1600 °C with blade cooling, thermal barrier coatings). Overall air–fuel ratio very lean (**≈ 50:1 or leaner**; excess air cools turbine inlet gases).
- Isentropic efficiencies: $\eta_C = (T_{2s} - T_1)/(T_2 - T_1)$; $\eta_T = (T_3 - T_4)/(T_3 - T_{4s})$.

Modification	Net work	Thermal efficiency
Regeneration (heat exchanger: exhaust preheats compressed air)	Unchanged	**Increases** (only if $T_4 > T_2$, i.e. at **low r_p**); benefit falls as r_p rises
Intercooling (between compressor stages)	**Increases** (compressor work ↓)	**Decreases** without regeneration; increases with regeneration
Reheating (between turbine stages)	**Increases**	**Decreases** without regeneration; increases with regeneration
Intercooling + reheating + regeneration (infinite stages)	Max	→ **Ericsson = Carnot**

KEY FORMULAE

- Ideal regenerative Brayton: $\mathbf{\eta = 1 - (T_1/T_3)\, r_p^{(\gamma-1)/\gamma}}$ (decreases with r_p, opposite to simple cycle)
- Regenerator effectiveness $\varepsilon = (T_x - T_2)/(T_4 - T_2)$
- Two-stage compression with perfect intercooling: optimum intermediate pressure $\mathbf{p_i = \sqrt{(p_1 p_2)}}$ (equal pressure ratio per stage; same for reheat)
- Jet thrust $\mathbf{F = \dot{m}(V_j - u)} + (p_e - p_a)A_e$); thrust power = F·u
- **Propulsive efficiency** $\mathbf{\eta_p = 2u/(V_j + u)} = 2/(1 + V_j/u)$; maximum (100%) when $V_j = u$, but then thrust = 0

Jet propulsion device	Key facts
Turbojet	Diffuser → compressor → combustor → turbine (drives compressor only) → nozzle; all thrust from jet; best at high subsonic/supersonic speeds; afterburner raises thrust
Turbofan (bypass)	Fan bypasses part of air; bypass ratio up to ~10–12 in modern airliners; higher propulsive efficiency, quieter – most common airliner engine
Turboprop	Turbine drives a **propeller** via reduction gear; **~80–90%** thrust from propeller; best fuel economy at low speed (< ~700 km/h)
Ramjet	**No compressor or turbine** – ram compression in diffuser; **cannot start from rest** (needs initial speed, efficient at M ≈ 2–4); simplest air-breathing engine
Scramjet	Supersonic combustion ramjet; hypersonic (M > 5)
Pulse jet	Intermittent combustion with flap valves (V-1 flying bomb); Lenoir-type cycle; can start at rest
Rocket	Carries **both fuel and oxidiser** → works in vacuum/space; thrust independent of flight speed; solid, liquid (e.g. LH_2–LOX, cryogenic) and hybrid propellants; specific impulse $I_{sp} = F/(\dot{m}g)$ (s)

MEMORY TRICK

Critical ratios "**528 – 546 – 577**": the **wetter** the fluid (lower n), the **higher** the ratio. Impulse optimum "**half cos**" (u/V = cosα/2, η = $\cos^2\alpha$); Parsons optimum "**full cos**" (u/V = cosα).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Critical pressure ratio for superheated steam? → **0.546**.
- Maximum blade efficiency of a single-stage impulse turbine? → **$\cos^2\alpha$** (at $u/V_1 = \cos\alpha/2$).
- Parsons turbine has degree of reaction? → **50%**.
- Curtis turbine is an example of? → **Velocity compounding**.
- Effect of regeneration on GT efficiency at high pressure ratio? → **Benefit reduces; may decrease when $T_2 > T_4$**.
- Jet engine that cannot start from rest? → **Ramjet**.

Nandyal Banking Coaching Center

6. I.C. Engines, Air Compressors & Compressed Air Systems

NOTE

IC engines give 3–5 sure-shot MCQs in every mechanical PSU/SSC JE paper: SI vs CI, 2-stroke vs 4-stroke, Morse test/Willan's line, knock (octane/cetane and opposite remedies), BS-VI and emission control. Compressors add 2–3 more: minimum work (isothermal), volumetric efficiency with clearance, intermediate pressure $\sqrt{(p_1 p_3)}$, surging. For a warehousing corporation, compressed-air system practice (leakage, dryers, FRL, ring main) and DG-set engine basics are also directly relevant.

6.1 IC Engine Basics & Classification ★★★

- **Heat engine** = converts heat into work. **IC engine**: combustion takes place **inside** the cylinder (petrol, diesel, gas engines, gas turbine). **EC engine**: combustion outside (steam engine, steam turbine, Stirling engine).
- Classification bases: cycle (Otto/Diesel/Dual), strokes (2/4), ignition (spark/compression), fuel (petrol, diesel, gas, dual-fuel), cooling (air/water), cylinder arrangement (in-line, V, radial, opposed, W), valve location (L-, I-, F-, T-head), speed, application.
- **Otto cycle** (constant-volume heat addition) → SI engine. **Diesel cycle** (constant-pressure heat addition) → slow-speed CI engine. **Dual / mixed / limited-pressure cycle** (part at constant V, part at constant p) → modern high-speed CI engines.
- For the **same compression ratio and heat input**: $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$. For the **same maximum pressure and temperature**: $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$.
- Air-standard Otto efficiency depends **only on compression ratio** (and γ); Diesel efficiency also depends on **cut-off ratio** – efficiency falls as cut-off ratio (load) increases.

Feature	SI (petrol) engine	CI (diesel) engine
Basic cycle	**Otto** (constant volume)	**Diesel / Dual**
Fuel	Petrol, CNG, LPG (high self-ignition temperature, volatile)	Diesel (low self-ignition temperature, less volatile)
Charge drawn in suction	**Air–fuel mixture** (premixed; carburettor/port injection)	**Air only**; fuel injected near end of compression
Ignition	**Spark plug**	**Heat of compression** (self-ignition)
Compression ratio	**6–12** (limited by knock)	**14–22** (limited by weight / peak pressure)
Fuel feed device	Carburettor / MPFI / GDI	Fuel injection pump + injector
Load control (governing)	**Quantity** governing (throttle)	**Quality** governing (fuel quantity, air un-throttled)
Speed	**High** (up to ~6,000 rpm and more)	Lower (large engines a few hundred rpm)
Weight per kW	**Lighter**	**Heavier** (higher pressures)
Thermal efficiency	Lower (brake ≈ 25–30%)	**Higher** (brake ≈ 30–40%+) due to higher CR
Initial cost / running cost	Lower / higher	Higher / lower
Noise & vibration	Less	More (diesel knock)
Main pollutants	CO, HC, NO_x	**NO_x, PM (smoke)**; low CO
Use	Cars, two-wheelers, small gensets	Trucks, buses, tractors, locomotives, ships, DG sets

Feature	Four-stroke engine	Two-stroke engine
Power strokes	**One per two revolutions** of crankshaft	**One per revolution**
Cycle completed in	720° of crank rotation	360° of crank rotation
Valves / ports	Poppet **valves** + camshaft (runs at **half** crank speed)	**Ports** covered/uncovered by piston (no valves in simple engines)
Flywheel	Heavier (non-uniform torque)	Lighter (more uniform torque)
Power for same size & speed	Less	Theoretically double; practically ≈ **1.5–1.8 times**
Scavenging	Positive (by piston exhaust stroke)	By fresh charge – poor; **short-circuiting** loss of fresh charge
Volumetric efficiency	Higher	Lower
Thermal efficiency	Higher	Lower (charge loss, especially SI)
Lubrication	Wet sump: **splash / pressure feed**	Petrol 2-stroke: **mist (petroil)** – oil mixed with fuel; crankcase is used for pre-compression
Fuel & lube oil consumption	Less	More; higher HC emission
Wear	Less	More (higher heat load)
Use	Cars, trucks, buses	Mopeds, chain-saws, very large **marine diesels** (uniflow)

- **Scavenging** = clearing burnt gases with fresh charge/air. Types: **cross-flow** (deflector piston), **loop / reverse** (Schnürle), **uniflow** (exhaust valve in head, ports at bottom) – **uniflow is most efficient**. Scavenging blower or crankcase compression supplies the air.
- **Bore (D)** = inner diameter of cylinder. **Stroke (L)** = distance between TDC and BDC = **2 × crank radius**.
- **TDC** (top/inner dead centre) and **BDC** (bottom/outer dead centre) = extreme piston positions where piston velocity is zero.
- **Swept (displacement) volume** $V_s = (\pi/4)D^2L$ per cylinder. **Engine capacity (cc)** = V_s × number of cylinders.
- **Clearance volume** V_c = volume above piston at TDC. **Total volume** = $V_s + V_c$.
- **Compression ratio** $r = (V_s + V_c)/V_c = 1 + V_s/V_c$. Clearance ratio $C = V_c/V_s \rightarrow r = 1 + 1/C$.
- **Square engine**: L/D = 1; **over-square (short-stroke)**: L/D < 1 (high-speed petrol engines); **under-square (long-stroke)**: L/D > 1 (slow diesels).
- **Mean piston speed** = 2LN/60 m/s (N in rpm); usually **8–15 m/s**; limited by inertia and friction.

Nandyal Banking Coaching Center

KEY FORMULAE

- $V_s = (\pi/4)\, D^2\, L$; $r = (V_s + V_c)/V_c$
- $\eta_{Otto} = 1 - 1/r^{\gamma-1}$; $\eta_{Diesel} = 1 - (1/r^{\gamma-1})\cdot[(\rho^{\gamma} - 1)/(\gamma(\rho - 1))]$, ρ = cut-off ratio
- Mean piston speed $V_p = 2LN/60$

- **Valve timing diagram** (4-stroke): valves do not open/close exactly at dead centres because of valve inertia, gas inertia and to improve breathing at high speed.
- **Inlet valve opens (IVO) before TDC** (≈ 10–20° bTDC) and **closes after BDC** (≈ 30–50° aBDC) – ram effect of incoming charge improves volumetric efficiency.
- **Exhaust valve opens before BDC** (≈ 40–55° bBDC) – blow-down; **closes after TDC** (≈ 10–20° aTDC) – kinetic energy of exhaust gases helps scavenging.
- **Valve overlap** = period when **both inlet and exhaust valves are open**, near **TDC** (end of exhaust/start of suction). Typical 15–30° in petrol engines; larger in diesels and racing engines (no fuel loss as only air is inducted in CI).
- Spark occurs ≈ 20–35° bTDC (SI); fuel injection starts ≈ 15–25° bTDC (CI) – the **ignition/injection advance**.
- **Port timing diagram** (2-stroke): piston-controlled ports open and close **symmetrically** about BDC (transfer & exhaust ports) or TDC (crankcase inlet port). **Exhaust port opens first and closes last**; transfer port opens after exhaust port. E.g. EPO ≈ 45° bBDC, TPO ≈ 35° bBDC, TPC ≈ 35° aBDC, EPC ≈ 45° aBDC.
- **Firing order** = sequence in which cylinders fire; chosen for engine balance, uniform torque and even crankshaft/bearing loading. 4-cylinder in-line: **1-3-4-2** (or 1-2-4-3). 6-cylinder in-line: **1-5-3-6-2-4**. In a 4-stroke n-cylinder engine firing interval = **720°/n** (180° for 4-cyl, 120° for 6-cyl).

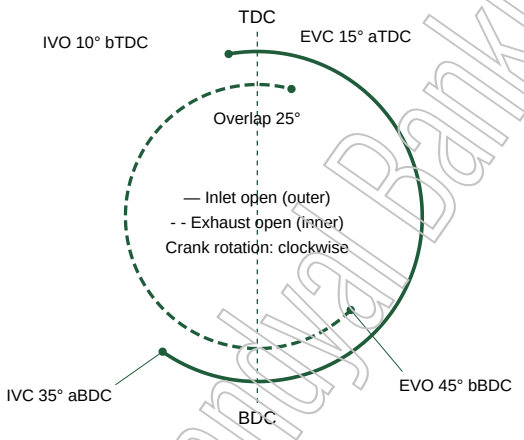


Fig. 6.1 Typical valve timing diagram of a 4-stroke SI engine (angles are typical, vary with engine). Overlap = 10° + 15° = 25° about TDC.

MEMORY TRICK

"**Inlet Early, Exhaust Late** at TDC" – inlet opens before TDC, exhaust closes after TDC → both open together = **overlap at TDC**. Each valve opens **before** and closes **after** its dead centre.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Compression ratio range of a CI engine? → **14–22** (SI: 6–12).
- In a 4-stroke engine the camshaft rotates at? → **Half the crankshaft speed**.
- Valve overlap occurs near? → **TDC** (end of exhaust, beginning of suction).
- Most efficient scavenging method? → **Uniflow**.
- Usual firing order of a 4-cylinder in-line engine? → **1-3-4-2**.
- Lubrication in a 2-stroke petrol engine? → **Mist (petroil) lubrication**.

6.2 Engine Performance & Testing ★★★

- **Indicated power (IP)** = power developed inside the cylinder by the gases; found from the **indicator diagram** (p–V) using an engine indicator / pressure transducer.
- **Brake power (BP)** = power available at the crankshaft; measured with a **dynamometer**. BP = $2\pi NT/60$ (W).
- **Friction power (FP)** = IP − BP (piston-ring friction is the largest part, plus bearings, valve train, pumping and auxiliaries).
- **Mechanical efficiency** η_m = BP/IP; typically **75–90%**; **maximum at full load** and **zero at no load** (BP = 0); decreases with increase of speed (FP rises).
- **Indicated thermal efficiency** η_{ith} = IP/($\dot{m}_f \times$ CV). **Brake thermal efficiency** η_{bth} = BP/($\dot{m}_f \times$ CV) = $\eta_m \times \eta_{ith}$.
- **Relative (efficiency) ratio** = actual (indicated) thermal efficiency ÷ air-standard efficiency.
- **Specific fuel consumption**: BSFC = $\dot{m}_f$/BP (kg/kWh). Typical: petrol ≈ **0.25–0.35**, diesel ≈ **0.20–0.25 kg/kWh**. Lower BSFC = better economy. η_{bth} = 3600/(BSFC × CV) with BSFC in kg/kWh, CV in kJ/kg.
- **Volumetric efficiency** η_v = actual mass (volume) of air inducted ÷ mass (volume) of air corresponding to swept volume at **ambient (intake) conditions**. Naturally aspirated engines ≈ **75–90%**; supercharged engines can exceed 100%.
- η_v decreases with: higher speed (flow friction, choking at inlet valve), hot intake air, hot cylinder walls, high exhaust back-pressure, small valve area, throttling. Improved by: larger valves, ram/tuned intake, multi-valve heads, **supercharging**.
- **Mean effective pressure (mep)** = constant pressure which, acting over the full stroke, gives the same work per cycle. **imep** from IP; **bmep** from BP; bmep = $\eta_m \times$ imep. bmep is the best parameter to **compare engines of different sizes**.
- imep from indicator card = (area of diagram × spring constant)/length of diagram.

KEY FORMULAE

- P (kW) = $p_m\, L\, A\, N'\, n$ / 60000 ; p_m in N/m², L in m, A in m², n = no. of cylinders
- N′ = N (rpm) for **2-stroke**; N′ = **N/2** for **4-stroke** (power strokes per minute)
- BP = $2\pi NT/60000$ kW ; FP = IP − BP ; η_m = BP/IP = bmep/imep
- η_{bth} = BP/($\dot{m}_f$·CV) ; η_{ith} = IP/($\dot{m}_f$·CV)
- $\eta_v = \dot{m}_{a,actual} / (\rho_a\, V_s \times N' \times n)$
- Morse test: $IP_k = BP_{all} - BP_{(k\ cut)}$; $IP_{total} = \Sigma\, IP_k$

Nandyal Banking Coaching Center

Method to find FP / IP	Principle	Applicable to
Morse test	Each cylinder cut off in turn (spark shorted / injector cut); speed restored by **reducing load**; fall in BP = IP of that cylinder	**Multi-cylinder engines** (classically **SI**)
Willan's line (fuel-rate extrapolation)	Fuel consumption vs BP plotted at **constant speed**; straight line extrapolated to **zero fuel** – negative BP intercept = **FP**	**CI engines** (fuel rate ∝ load, quality governed); not SI (throttling)
Motoring test	Engine run at same speed by an electric (swinging-field) dynamometer with fuel cut; power absorbed = FP (+ pumping)	Any engine; usually slightly over-estimates FP (cooler, more viscous oil; absence of gas pressure partly offsets this)
Retardation test	Fuel cut; time for speed to fall measured at no-load and with known load	Any engine
Indicator diagram	IP directly; FP = IP – BP	Slow-speed engines (accurate pressure–crank-angle data needed)

- **Dynamometers**: absorption – **prony brake, rope brake**, **hydraulic (Froude)**, **eddy-current**, swinging-field DC/AC; transmission – torsion, belt. Eddy-current and hydraulic are most common for engine test beds; swinging-field electric can also **motor** the engine.
- **Heat balance sheet** (on a % of fuel energy supplied basis, per minute or per hour): (i) heat to **BP**, (ii) heat to **cooling water**, (iii) heat in **exhaust gases**, (iv) **unaccounted** losses (radiation, friction etc., found by difference).

Heat balance item (approx. at full load)	SI engine	CI engine
Brake power	≈ 21–28%	≈ 29–42%
Cooling water	≈ 12–27%	≈ 15–35%
Exhaust gases	≈ 30–55%	≈ 25–45%
Unaccounted	balance (few %)	balance (few %)

- Thumb rule: fuel energy splits roughly into **one-third each** to useful work, coolant and exhaust (textbook values vary).
- **Air–fuel ratio** (by mass): **stoichiometric petrol ≈ 14.7:1** (often rounded to 15:1); **diesel ≈ 14.5:1**; CNG (methane) ≈ 17.2:1; ethanol ≈ 9:1; hydrogen ≈ 34:1.
- SI mixture requirements: cold start – very rich (≈ 9:1 or richer, **choke**); idling – rich (≈ 10–12:1); **maximum power** – slightly rich (≈ 12.5–13:1); **maximum economy** – slightly lean (≈ 15–16:1, cruising); acceleration – momentarily rich (accelerating pump). Combustible range ≈ 7:1 to 20:1.
- CI engines always run **lean overall** (≈ 18:1 at full load up to 80:1+ at idling) – heterogeneous mixture; smoke limits full-load fuelling.
- **Equivalence ratio** $\varphi = (F/A)_{actual} \div (F/A)_{stoich}$. $\varphi = 1$ stoichiometric; **$\varphi > 1$ rich**; **$\varphi < 1$ lean**. **Relative air–fuel ratio $\lambda = 1/\varphi$** ($\lambda > 1$ lean).
- **Supercharging** = supplying intake air/charge at a pressure (density) higher than ambient to increase power output from the same engine size; compressor is **mechanically driven** from the crankshaft (Roots blower, vane, centrifugal) – consumes part of engine power, no lag.
- **Turbocharging** = compressor (centrifugal) driven by an **exhaust-gas turbine** – uses waste exhaust energy, so overall efficiency improves; drawback **turbo lag**; boost controlled by **waste-gate** or **variable geometry turbine (VGT)**.
- **Intercooler / charge-air cooler** after the compressor lowers charge temperature → higher density, lower knock (SI) and lower NO_x.
- Supercharging suits **CI engines** best (reduces ignition delay and diesel knock); in SI engines it is **limited by detonation** (CR often reduced). Aircraft engines are supercharged to compensate for altitude.

MEMORY TRICK

"**Morse for Many, Willan for Wheels of Diesel**" – Morse test needs **many** (multi) cylinders; Willan's line is for **diesel** (quality governed) engines.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Morse test is used to find? → **IP (and hence FP) of a multi-cylinder engine**.
- Willan's line method is applicable to? → **CI engines** (fuel consumption vs BP at constant speed).
- Mechanical efficiency at no load is? → **Zero**.
- Stoichiometric A/F ratio for petrol? → **≈ 14.7:1**.
- For a 4-stroke engine, number of power strokes per minute = ? → **N/2**.
- Turbocharger turbine is driven by? → **Exhaust gases**.

6.3 Combustion in SI & CI Engines ★★★

- **SI combustion** (Ricardo) – three stages: (i) **Ignition lag / preparation phase** – chemical process of growth of flame kernel after spark (≈ 1–2 ms; nearly constant in time, so spark must be **advanced** at higher speeds); (ii) **Flame propagation** – main combustion, rapid pressure rise, flame front travels across chamber; (iii) **After-burning** – combustion continues behind flame front during expansion.
- Flame speed increases with **turbulence** (most important), slightly rich mixture, higher CR, higher intake temperature/pressure and higher engine speed (turbulence ∝ speed). Turbulence is created by squish, swirl, tumble.
- **Detonation / knock (SI)** = **auto-ignition of the end-gas** (unburnt charge ahead of the flame front) before the normal flame reaches it → very rapid pressure rise, pressure waves, metallic **"pinking"** sound.
- **Effects of detonation**: noise, mechanical damage (piston crown erosion, broken rings), increased heat transfer → **overheating**, loss of power and efficiency, possible **pre-ignition**, carbon deposits.
- **Pre-ignition** = ignition of the charge **before the spark** by hot spots (spark plug electrode, exhaust valve, carbon deposits). Different from detonation, though detonation can cause it.
- **Factors increasing SI knock**: high **compression ratio**, high intake temperature/pressure (supercharging), **spark advance**, low-octane fuel, large bore (long flame travel), low turbulence, low engine speed, hot coolant, deposits.

Nandyal Banking Coaching Center

- **Control of SI knock**: use **higher-octane fuel**, **retard the spark**, **lower CR**, cool the intake charge, increase turbulence, central spark plug location (short flame path), smaller bore, **knock sensor** with ECU spark retard, EGR, water/methanol injection.
- **Highest Useful Compression Ratio (HUCR)** – Ricardo variable-compression engine; the CR at which knock just begins. **CFR engine** (Cooperative Fuel Research) – variable-CR single-cylinder engine for octane/cetane rating.
- **Octane number** = percentage by volume of **iso-octane** (2,2,4-trimethylpentane, **ON = 100**) in a mixture with **n-heptane** (**ON = 0**) that matches the knock intensity of the test fuel in the CFR engine.
- **RON** (Research, 600 rpm, milder) is **higher** than **MON** (Motor, 900 rpm, severe); **sensitivity** = RON − MON; anti-knock index (AKI, US pumps) = (RON + MON)/2.
- For fuels above 100: **Performance Number** PN; ON = 100 + (PN − 100)/3.
- Hydrocarbon knock resistance (SI): **aromatics** > iso-paraffins (branched) > olefins > naphthenes > **normal paraffins** (least). Straight-chain paraffins are best **diesel** fuels.
- **Anti-knock additives**: **TEL (tetra-ethyl lead)** – very effective but toxic; it **poisons catalytic converters**; phased out – leaded petrol banned in India from **2000**. Present octane boosters: **MTBE**, ETBE, **ethanol** (RON ≈ 108), aromatics, MMT.
- Indian petrol (BS-VI): regular petrol **RON ≥ 91**; premium grades RON 95/100 also marketed. BS-VI diesel: **cetane number ≥ 51**; sulphur **≤ 10 ppm** in both.
- **CI combustion** – four stages: (i) **Ignition delay** (from start of injection to start of combustion) = **physical delay** (atomisation, vaporisation, mixing) + **chemical delay** (pre-flame reactions); (ii) **Rapid / uncontrolled combustion** – fuel accumulated during delay burns at once → steep pressure rise; (iii) **Controlled (mixing-controlled) combustion** – fuel burns as fast as it is injected; (iv) **After-burning** – during expansion.
- **Diesel knock** = caused by a **long ignition delay** → large quantity of fuel accumulates and burns suddenly → very high rate of pressure rise (dp/dθ). Knock in CI occurs at the **beginning** of combustion; in SI at the **end** of combustion (end-gas).
- **Control of diesel knock**: **reduce ignition delay** – **high-cetane fuel**, higher CR, higher intake temperature & pressure (supercharging), hot cylinder walls, good atomisation & swirl, reduced injection rate at start (pilot injection, pintaux nozzle), cetane improvers (amyl nitrate, ethyl nitrate).
- **Cetane number** = % by volume of **cetane (n-hexadecane, CN = 100)** in a mixture with **α-methyl naphthalene (CN = 0)** giving the same ignition delay as the test fuel. (Modern secondary reference: heptamethylnonane, CN = 15.)
- High-octane fuels have **low** cetane number and vice versa – a good petrol is a bad diesel. **Diesel index** = (aniline point in °F × API gravity)/100 – correlates with cetane number.

Factor (to reduce knock)	SI engine	CI engine
Self-ignition temperature of fuel	**High**	**Low**
Ignition delay of fuel	**Long**	**Short**
Compression ratio	**Low**	**High**
Inlet temperature	Low	High
Inlet pressure	Low	High
Cylinder wall / coolant temperature	Low	High
Engine speed	High	Low
Cylinder size	Small	Large
Fuel rating	High **octane**	High **cetane**
Knock occurs at	End of combustion (end-gas)	Start of combustion

MEMORY TRICK

Whatever **prevents** knock in SI **causes** it in CI – the remedies are **opposite** because SI wants to **delay** self-ignition of the end-gas, while CI wants to **hasten** self-ignition. Only turbulence helps both.

Combustion chamber	Features
SI: **T-head**	Valves on opposite sides; long flame path, knock-prone; obsolete
SI: **L-head (side-valve)**	Simple, cheap; poor breathing, low CR
SI: **I-head (overhead valve)**	Compact; better η_v, higher CR; most common type today; wedge, bath-tub, hemispherical (pent-roof, 4 valves) shapes
SI: **F-head**	One valve in head, one in block – compromise
CI: **Open / direct-injection (quiescent)**	Fuel sprayed into a cavity in piston crown; multi-hole nozzle, high injection pressure; good cold starting, lower heat loss, **highest efficiency**; used in large and modern CRDI engines
CI: **Swirl chamber** (e.g. Ricardo Comet)	Indirect injection (IDI); strong compression swirl; pintle nozzle, low injection pressure
CI: **Pre-combustion chamber**	IDI; partial combustion in pre-chamber blows fuel into main chamber; smooth, multi-fuel
CI: **Air-cell / energy cell** (Lanova)	IDI; separate air-cell
CI: **M-type (MAN)**	Fuel sprayed on hot spherical piston-cavity wall – wall evaporation, quiet

- IDI engines need **glow plugs** for cold starting (high heat loss through throat); DI engines have better fuel economy (≈ 10–15% better) and have replaced IDI in vehicles.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Detonation in SI engine is due to? → **Auto-ignition of the end-gas**.
- Octane number of n-heptane? → **Zero** (iso-octane = 100).
- Reference fuel with cetane number zero? → **α-methyl naphthalene**.
- Diesel knock is caused by? → **Long ignition delay**.
- Effect of increasing CR on knock? → **Increases SI knock, reduces CI knock**.
- Anti-knock agent poisoning catalytic converters? → **TEL (lead)**.

Nandyal Banking Coaching Center

6.4 Fuel, Ignition, Cooling & Lubrication Systems ★★☆

- **Carburetion** = formation of a combustible air–fuel mixture by mixing air with atomised and vaporised fuel outside the cylinder (SI).
- **Simple carburettor**: **float chamber** (constant fuel level, needle valve), **venturi** (throat – maximum air velocity, minimum pressure; pressure difference draws fuel through the main jet), **fuel nozzle/jet**, **throttle valve** (butterfly, after venturi – controls quantity of mixture), **choke valve** (before venturi – closed for **rich mixture in cold starting**).
- Drawback of simple carburettor: mixture becomes **richer as air flow (speed) increases** (fuel density constant, air density falls). Remedies: **compensating jet**, air-bleed, emulsion tube, metering-pin, auxiliary air valve. Other systems: idling system, accelerating pump, economiser/power-enrichment.
- Types: **up-draught, down-draught (most common in cars), cross-draught (side-draught)**; constant-choke (Solex, Zenith, Carter) and constant-vacuum / variable-venturi (SU, constant-depression).
- **Petrol injection**: **TBI** (throttle body / single-point), **MPFI** (multi-point – one injector per cylinder in the **intake port**, low pressure ≈ 3–4 bar, ECU controlled with O_2/λ sensor feedback), **GDI** (gasoline direct injection – into the cylinder at ≈ 100–200+ bar; permits stratified lean charge and higher CR; but produces **particulates**, hence PM/PN limits).
- **Diesel injection** requirements: metering exact quantity, timing, rate of injection, atomisation, penetration and distribution. Systems: **air injection** (obsolete), **solid (airless) injection** – individual pump, **distributor pump**, unit injector, and **common rail**.
- **Fuel injection pump** (Bosch jerk type): plunger with **helix**; rotating the plunger (by rack connected to governor) changes the effective stroke → fuel quantity. Delivery valve prevents dribbling and maintains residual line pressure.
- **Injector / nozzle**: spring-loaded needle opened by fuel pressure. Nozzle types: **single-hole, multi-hole** (DI engines), **pintle** (IDI, self-cleaning), **pintaux** (pintle + auxiliary hole, better cold start).
- **CRDI (common rail direct injection)**: high-pressure pump keeps fuel in a common rail (accumulator) at very high pressure (≈ **1,300–2,500 bar** in modern systems) independent of engine speed; **solenoid/piezo injectors** controlled by ECU; **multiple injections** (pilot, main, post) → lower noise, NO_x and smoke. Conventional pump–nozzle systems ≈ 150–300 bar opening pressure.

Ignition system	Features
Battery (coil) ignition	12 V battery, ignition coil (step-up transformer, primary few hundred turns, secondary ~20,000 turns), **contact breaker** (cam-operated), **condenser (capacitor)** across CB points to prevent arcing and speed up primary current collapse, distributor; secondary voltage ≈ 15–30 kV. Good spark at **low speed / starting**; spark weakens at high speed. Used in cars.
Magneto ignition	Own generator (rotating magnet), **no battery**; spark intensity **improves with speed**, poor at starting. Used in two-wheelers (older), racing, aircraft piston engines, small engines.
Electronic – transistorised (TCI)	Transistor switches coil primary; CB replaced by magnetic/Hall-effect pick-up; no point wear.
Capacitor discharge ignition (CDI)	Capacitor charged to ≈ 300–400 V and discharged through coil by thyristor (SCR); very fast voltage rise, good for high-speed and two-wheeler engines; short spark duration.
Distributor-less / coil-on-plug	ECU-controlled; one coil per plug (or pair – wasted-spark).

- **Spark advance mechanisms**: **centrifugal advance** – advances timing with increase of **speed**; **vacuum advance** – advances at **part load** (high manifold vacuum). Modern ECUs use maps + knock sensor.
- **Spark plug gap** ≈ 0.6–1.1 mm; "hot" plug (long insulator nose) for low-speed/cold engines, "cold" plug for high-power engines. **Firing order** is set by the distributor/ECU.
- **Cooling** needed because peak gas temperature ≈ 2,000–2,500 °C; about 25–35% of fuel energy goes to coolant. Over-cooling is also bad (higher wear, sludge, fuel consumption, CI knock).
- **Air cooling**: **fins** on cylinder & head, fan/blower (cowling) – simple, light, no leakage/freezing; noisy, less uniform; used in two-wheelers, small engines, aircraft piston engines.
- **Water (liquid) cooling**: water jackets, **radiator** (cross-flow/down-flow, tube-and-fin), **pump** (forced circulation; centrifugal) or **thermosyphon** (natural circulation by density difference – old), fan, **thermostat**, pressure cap.
- **Thermostat** (wax-pellet / bellows type) – closed when cold for quick warm-up; begins to open at ≈ **80–90 °C**. **Pressurised radiator cap** (≈ 0.9–1.1 bar gauge) raises boiling point to ≈ 115–125 °C.
- **Coolant**: water + **ethylene glycol** (anti-freeze, raises boiling point) + corrosion inhibitors.
- **Lubrication functions**: reduce friction & wear, cool, seal (piston rings), clean (detergent), prevent corrosion, absorb shock.

Lubrication system	Description / use
Splash	Dipper on connecting rod big-end splashes oil from sump; small, low-speed engines
Pressure (forced) feed	Gear pump (≈ 2–4 bar) forces oil through **drilled crankshaft** to main & big-end bearings, cam shaft, rockers; most automobile and large engines
Semi-pressure	Combination of splash and pressure
Mist / petroil	2-stroke petrol engines: oil mixed with fuel ≈ **2–5%** (1:50 to 1:20); crankcase cannot hold oil since it is used for charge compression
Wet sump vs dry sump	Wet sump: oil stored in crankcase sump. Dry sump: oil stored in separate tank, scavenge pump – racing, aircraft, off-road (no starvation on tilting)

- **Viscosity** – most important property of a lubricant. **Viscosity index (VI)** = measure of variation of viscosity with temperature; **high VI = less variation** (desirable). Reference oils: Pennsylvanian (paraffinic) **VI = 100**, Gulf-coast (naphthenic) **VI = 0**. Polymer **VI improvers** make multigrade oils.
- **SAE grades**: monograde SAE 30, 40 etc. (viscosity at **100 °C**); "W" = **winter** grade (cold-cranking/pumping viscosity). **Multigrade 20W-40** behaves like SAE 20W when cold and like SAE 40 at 100 °C.

Nandyal Banking Coaching Center

- Other properties: **flash point** (vapour flashes momentarily) < **fire point** (burns continuously ≥ 5 s); **cloud point** (wax crystals appear) > **pour point** (lowest temperature at which oil flows); oiliness; TBN (alkalinity – neutralises acids); additives – detergent-dispersant, anti-oxidant, anti-wear (ZDDP), EP, anti-foam, pour-point depressant.
- Engine oils are classified also by API service categories (S- for petrol, C- for diesel).

Governing method	How load is controlled	Used in
Quantity governing	Throttle varies **quantity of mixture**; A/F ratio nearly constant	**SI (petrol) engines**
Quality governing	Air quantity constant; **fuel quantity varied** → mixture strength (quality) changes	**CI (diesel) engines**
Hit-and-miss governing	Fuel supply **cut off completely for some cycles** when speed exceeds limit; full-charge firing otherwise	Small **gas engines**, old stationary engines; efficient at part load but uneven
Combination	Quality + quantity	Some large gas/dual-fuel engines

- Diesel generator sets use a **speed governor** (mechanical centrifugal / electronic) acting on the fuel-pump rack to hold **constant speed (frequency)**: 1,500 rpm for a 4-pole 50 Hz alternator.

MEMORY TRICK

"**Petrol Quantity, Diesel Quality**" – P-Q-D-Q. "Choke = Cold, Compensating jet = Correct richness at high speed." "**Magneto Moves**" – its spark improves as speed rises; battery ignition is best at starting.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Purpose of the choke in a carburettor? → **Rich mixture for cold starting**.
- Governing used in diesel engines? → **Quality governing**.
- In a battery ignition system, the condenser is used to? → **Prevent arcing at contact-breaker points**.
- Thermostat in the cooling system opens at about? → **80–90 °C**.
- In SAE 20W-40, "W" stands for? → **Winter**.
- Viscosity index of Pennsylvanian oil? → **100**.

6.5 Emissions & Alternative Fuels ★★☆

- **Main pollutants**: **CO** (carbon monoxide), **HC** (unburnt hydrocarbons), **NO_x** ($NO + NO_2$; NO is ≈ 90%+ of engine NO_x), **PM** (particulate matter / soot), **smoke**, SO_2 (fuel sulphur), CO_2 (greenhouse gas, not "toxic" pollutant), aldehydes.
- **CO** – due to **rich mixture** (lack of O_2); minimum at lean mixtures; **low in diesel** (lean). Most significant pollutant from SI engines by mass.
- **HC** – rich mixture, misfire, **flame quenching** at cold walls, **crevice** volumes (piston-ring crevice is the largest source), oil-film absorption, valve overlap/short-circuiting (2-stroke – very high HC).
- **NO_x** – formed at **high combustion temperature** (thermal / Zeldovich mechanism) with excess O_2; peak near slightly lean mixture (φ ≈ 0.9); increased by high CR, spark/injection advance, high load.
- **PM / smoke** – mainly **diesel**: black smoke (soot – rich zones, overload), blue smoke (lube-oil burning), white smoke (unburnt fuel at cold start). Diesel has a **NO_x–PM trade-off** (measures that cut one tend to raise the other).

Control device	Removes / reduces	Key facts
Three-way catalytic converter (TWC)	**CO, HC and NO_x** simultaneously	SI engines; **Pt, Pd** (oxidation of CO, HC) + **Rh** (reduction of NO_x) on ceramic honeycomb (cordierite) with alumina wash-coat; works only in a narrow window near **stoichiometric (λ ≈ 1)** – needs closed-loop control with **O_2 (λ) sensor** (zirconia); light-off ≈ 250–300 °C; needs **unleaded** fuel
Oxidation catalyst (two-way, DOC)	CO, HC (and SOF of PM)	Diesel engines (lean exhaust, so NO_x cannot be reduced by TWC)
EGR (exhaust gas recirculation)	**NO_x**	Recirculated inert gas dilutes charge, lowers peak flame temperature; cooled EGR in diesels
DPF (diesel particulate filter)	**PM** (≈ 90%+ by mass)	Wall-flow ceramic (cordierite / SiC); periodic **regeneration** by burning soot
SCR (selective catalytic reduction)	**NO_x**	Injects **urea solution (AdBlue / DEF, 32.5% urea)** → NH_3 reduces NO_x to $N_2 + H_2O$; heavy-duty BS-VI diesels
LNT (lean NO_x trap)	NO_x	Stores NO_x and reduces it during brief rich operation; small diesels
PCV valve (positive crankcase ventilation)	Crankcase blow-by HC	Returns blow-by gases to intake
Evaporative (charcoal canister)	Fuel-tank evaporative HC	Activated-carbon canister, purged into intake

- **Bharat Stage (BS) emission norms** – India's vehicle emission standards, modelled on **Euro norms**, notified under the **Central Motor Vehicles Rules, 1989** (MoRTH); timelines set by Government (CPCB is the pollution watchdog).
- **BS-IV** nationwide from **1 April 2017**. India **skipped BS-V** and moved directly from BS-IV to **BS-VI** nationwide from **1 April 2020** (Supreme Court barred sale/registration of BS-IV vehicles from that date). BS-VI ≈ **Euro 6**.
- BS-VI fuel: **sulphur ≤ 10 ppm** (BS-IV: 50 ppm; BS-III: 150/350 ppm). BS-VI fuel was introduced first in **Delhi in April 2018**.
- Approx. BS-IV → BS-VI reduction for diesel cars: NO_x **0.25 → 0.08 g/km** (≈ 68% cut); PM limit **4.5 mg/km** (diesel and direct-injection petrol); particle-number (PN) limit added. Petrol car NO_x 0.08 → 0.06 g/km.
- **BS-VI Phase 2 (Stage II)** from **1 April 2023** – **Real Driving Emissions (RDE)** testing with PEMS (portable emission measurement system) and stricter **OBD-II** on-board diagnostics, in addition to laboratory cycle tests.
- Laboratory test cycle for Indian cars: **MIDC** (Modified Indian Driving Cycle, adapted from NEDC); type approval by ARAI (Pune), ICAT (Manesar) etc.

Nandyal Banking Coaching Center

CODE UPDATE

BS-VI (1 April 2020, BS-V skipped) → BS-VI Phase 2 with RDE & OBD-II (1 April 2023). Ethanol blending: **E10** was achieved in **June 2022** (ahead of the Nov 2022 target); the National Policy on Biofuels (amended 2022) advanced the **E20** target from 2030 to ethanol supply year **2025–26**; E20 petrol retail sale began in **February 2023**, and the **20% blending level was reached in July 2025** (≈ 5 years ahead of the original 2030 target).

Alternative fuel	Key facts
CNG (compressed natural gas)	Mainly **methane (CH_4, ~85–95%)**; stored at ≈ **200–250 bar** in cylinders; high octane (RON ≈ 120+) → suits **SI engines** (higher CR possible); very low PM, low CO; lower volumetric efficiency (gaseous fuel displaces air) → ≈ 10–15% power loss in converted petrol engines; lighter than air (safer dispersion)
LPG / Auto-LPG	Mainly **propane + butane**; stored as **liquid under moderate pressure** (≈ 8–10 bar); RON ≈ 105–110; **heavier than air** – leaks collect at low level (ventilation needed); ethyl mercaptan added for smell
Ethanol (C_2H_5OH)	From sugarcane molasses, cane juice, damaged grain, maize; RON ≈ 108; oxygenated fuel (≈ 35% O by mass) → lower CO/HC; **CV ≈ 27 MJ/kg** (petrol ≈ 44 MJ/kg) → higher volumetric fuel consumption; hygroscopic; corrosive to some rubbers/metals; stoichiometric A/F ≈ 9:1. **E20** = 20% ethanol + 80% petrol; flex-fuel vehicles can run on higher blends
Methanol (CH_3OH)	CV ≈ 20 MJ/kg; toxic; M15 blend trials
Biodiesel	Fatty-acid methyl esters made by **transesterification** of vegetable oils/animal fats with **methanol** + catalyst (NaOH/KOH); by-product **glycerol**; non-edible sources in India: **Jatropha, Karanja (Pongamia)**, used cooking oil; higher cetane number & flash point than diesel, ≈ 10–11% oxygen, lower CO/HC/PM, slightly higher NO_x, ≈ 10% lower CV; B20 = 20% blend; can be used in unmodified CI engines
Hydrogen	Highest CV per kg (≈ **120 MJ/kg LCV**), very wide flammability limits (≈ **4–75%** by volume in air), very low ignition energy, high flame speed; zero CO_2/CO/HC – only NO_x concern in IC engines; backfire and pre-ignition tendency; used in **fuel cells (PEMFC)** and H_2-ICE; **National Green Hydrogen Mission** (Jan 2023) – target **5 MMT/yr** green hydrogen by 2030
Biogas / CBG	Biogas ≈ 55–65% CH_4 + CO_2; upgraded to **compressed biogas (CBG)** – SATAT scheme; used in dual-fuel diesel engines
Electric vehicles	Battery EV (Li-ion), hybrid (HEV), plug-in hybrid (PHEV), fuel-cell EV; zero tail-pipe emission; regenerative braking; schemes: FAME-II (ended March 2024), followed by PM E-DRIVE

- **Dual-fuel engine**: CI engine inducts gaseous fuel (CNG/biogas) with air and ignites it by a **pilot diesel injection**.
- **Calorific values** (LCV, approx.): diesel ≈ 42–43 MJ/kg; petrol ≈ 43–44 MJ/kg; CNG (methane) ≈ 50 MJ/kg; LPG ≈ 46 MJ/kg; ethanol ≈ 27 MJ/kg; methanol ≈ 20 MJ/kg; hydrogen ≈ 120 MJ/kg.

MEMORY TRICK

"**Rich gives CO-HC, Hot gives NO_x.**" Three-way catalyst = **Pt-Pd oxidise**, **Rh reduces** ("**R**h → **R**eduction"). EGR and SCR both fight NO_x; DPF fights PM.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- A three-way catalytic converter controls? → **CO, HC and NO_x.**
- EGR is mainly used to reduce? → **NO_x.**
- Reductant used in SCR systems? → **Urea solution (AdBlue/DEF).**
- BS-VI norms were implemented across India from? → **1 April 2020** (BS-V skipped).
- Maximum sulphur content in BS-VI fuel? → **10 ppm.**
- Biodiesel is produced by? → **Transesterification** (by-product glycerol).

6.6 Reciprocating Air Compressors ★★★

- **Compressor** = machine that raises the pressure of air/gas. Classification: **positive displacement** (reciprocating, rotary vane, screw, Roots, scroll) and **dynamic / roto-dynamic** (centrifugal, axial).
- Terminology: **fan** – pressure ratio up to ≈ 1.1; **blower** – ≈ 1.1 to 2–2.5; **compressor** – above that (common textbook ranges).
- **Reciprocating compressors** – suited to **high pressure ratio, low flow**; single-acting or double-acting; single/multi-stage; lubricated or oil-free. Valves are **automatic (spring-loaded plate)** valves opened by pressure difference.
- **Free air** = air at atmospheric (intake) conditions. **Free Air Delivery (FAD)** = actual volume of air delivered, **reduced to intake (free air) conditions**, per minute (m^3/min or cfm). Tested per **ISO 1217**. FAD is the true capacity – **always less than displacement**.
- **Piston displacement** = $(\pi/4)D^2L\,N$ (single-acting); double-acting ≈ 2× (minus piston-rod area on crank side).
- **Work of compression** (without clearance) – order for the same pressure ratio: **isothermal (minimum) < polytropic (1 < n < γ) < isentropic/adiabatic (maximum)**. Hence aim: compression close to isothermal → cooling (water jackets, fins, intercoolers, water spray).
- For air compressors with cooling, typically **n ≈ 1.2–1.35**; γ = 1.4.
- **Isothermal efficiency** = isothermal work ÷ actual work (indicated or shaft) – the usual measure of compressor performance for **reciprocating** compressors. **Isentropic (adiabatic) efficiency** = isentropic work ÷ actual work – used for **rotary/centrifugal** compressors.
- **Clearance** (≈ **4–10%** of swept volume) is necessary to avoid piston striking the head and to accommodate valves. Clearance air **re-expands** on suction stroke, so the **effective suction stroke reduces**.
- Clearance **does not change the theoretical work per kg** of air delivered (work of compression of clearance air is recovered on re-expansion) but **reduces volumetric efficiency** and capacity; a bigger cylinder is needed for the same FAD.
- **Volumetric efficiency** η_v = effective suction volume ÷ swept volume. Decreases with **increase in clearance ratio** and **increase in pressure ratio**; increases with n. Typical ≈ 70–85%.
- **Limiting pressure ratio** when $\eta_v = 0$ (no delivery): $p_2/p_1 = [(1 + C)/C]^n$.

Nandyal Banking Coaching Center

- Actual η_v is also lowered by intake heating, valve pressure drops, leakage and valve inertia; **overall volumetric efficiency** referred to free-air conditions = FAD/swept volume.

KEY FORMULAE

- Polytropic work/cycle: $W = [n/(n-1)]\, p_1V_1\, [(p_2/p_1)^{(n-1)/n} - 1] = [n/(n-1)]\, mR(T_2 - T_1)$
- Isentropic work: replace n by γ ; Isothermal work: $W = p_1V_1 \ln(p_2/p_1) = mRT_1 \ln(p_2/p_1)$
- Delivery temperature: $T_2/T_1 = (p_2/p_1)^{(n-1)/n}$
- $\eta_{iso} = W_{isothermal}/W_{actual}$
- With clearance: $\eta_v = 1 + C - C\,(p_2/p_1)^{1/n}$; $C = V_c/V_s$
- FAD (reduced to free air p_a, T_a): $V_a = p_1V_1T_a/(T_1p_a)$ per cycle × cycles/min
- Heat rejected during polytropic compression: $Q = W_{poly,closed} \times (\gamma - n)/(\gamma - 1)$

- **Multi-stage compression** with **intercooling** – air compressed in LP cylinder, cooled in an intercooler, then compressed in HP cylinder.
- **Benefits of multi-staging**: (i) **saving in work** (compression approaches isothermal; saving = area between stages on p–V), (ii) **higher volumetric efficiency** (lower pressure ratio per stage → less re-expansion), (iii) **lower delivery temperature** → better lubrication, less risk of oil-vapour ignition/explosion, (iv) better mechanical balance and more uniform torque (lighter flywheel), (v) lighter LP-cylinder construction, less leakage, (vi) moisture separated in intercooler. Drawback: higher initial cost, complexity.
- **Perfect (complete) intercooling** = air cooled in the intercooler back to the **initial suction temperature T_1** (at constant intermediate pressure). Incomplete intercooling: cooled to a temperature above T_1.
- **Condition for minimum work** (two-stage, perfect intercooling, same n): intermediate pressure $\mathbf{p_2 = \sqrt{(p_1\, p_3)}}$, i.e. equal pressure ratio in each stage → **equal work**, equal delivery temperature and equal heat rejected in each stage.
- For z stages: pressure ratio per stage = $(p_{z+1}/p_1)^{1/z}$.
- Thumb rules: single-stage reciprocating for low pressures (up to ≈ 6–8 bar in industrial plant air), two-stage for ≈ 7–30 bar; three or more stages for higher pressures. Stage pressure ratio usually kept ≤ ≈ 4–5.
- **Aftercooler** = cools compressed air after the last stage – removes moisture (condensate) and reduces volume before receiver.
- Capacity control of reciprocating compressors: **load/unload** (suction valve unloader – inlet valve held open), clearance pockets, speed control, start/stop for small units.

KEY FORMULAE – MULTI-STAGE

- Two-stage optimum: $p_2 = \sqrt{(p_1p_3)}$; $p_2/p_1 = p_3/p_2$
- Minimum total work (z stages): $W = z\,[n/(n-1)]\, p_1V_1\, [(p_{z+1}/p_1)^{(n-1)/(zn)} - 1]$
- Example: p_1 = 1 bar, p_3 = 16 bar → p_2 = **4 bar** (ratio 4 per stage)

Quantity	Effect of increasing pressure ratio (single stage)
Volumetric efficiency	**Decreases**
Delivery temperature	**Increases**
Work per kg	**Increases**
Isothermal efficiency	Decreases
Effect of clearance on work per kg (theory)	**None**

MEMORY TRICK

"**I-P-A: Isothermal < Polytropic < Adiabatic**" for work (Isothermal is the Ideal, least). Intermediate pressure is the **geometric mean** – "**root of product**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Work done on air is minimum when compression is? → **Isothermal**.
- Volumetric efficiency of a reciprocating compressor as pressure ratio increases? → **Decreases**.
- Ideal intermediate pressure in two-stage compression from 1 bar to 9 bar? → **3 bar** ($\sqrt{(1 \times 9)}$).
- Perfect intercooling means air is cooled to? → **Initial (suction) temperature**.
- FAD is expressed at? → **Intake (free air/ambient) conditions**.
- Effect of clearance on theoretical work per kg? → **No effect** (only η_v falls).

6.7 Rotary Compressors & Compressed Air Systems ★★☆

- **Rotary positive-displacement** compressors – air trapped between rotating elements and casing; continuous, pulsation-free flow; no valves, smaller foundation.
- **Roots blower** – two (or three) lobe rotors (8-shaped/figure-of-eight lobes) rotating in opposite directions; **no internal compression** – air carried at constant volume and compressed by **back-flow** of high-pressure air when the pocket opens to delivery (irreversible). Work/rev = $(p_2 - p_1)$ V. Low pressure ratio (≈ up to 2); used as **supercharger**, scavenging blower in 2-stroke diesels, pneumatic conveying, aeration.
- **Roots efficiency** = isentropic work ÷ actual work; it **decreases as pressure ratio increases**.
- **Vane (sliding vane) compressor** – eccentric rotor with radial vanes in slots; vanes held out by centrifugal force; **partly internal compression** (pocket volume decreases) + partly by back-flow → **more efficient than Roots** for the same pressure ratio. Pressure up to ≈ 8–10 bar (oil-flooded).
- **Screw (helical lobe) compressor** – male (driving, convex lobes) and female rotor; **full internal compression** (built-in volume ratio); **oil-injected** (oil seals, cools, lubricates; ≈ 7–13 bar single stage) or **oil-free** (timing gears, usually two-stage); most common **industrial plant-air** compressor; low vibration, compact; poorer part-load efficiency unless **VSD**.
- **Scroll** – orbiting spiral; small, quiet, oil-free (also refrigeration/AC).
- **Centrifugal compressor** – air enters the **eye** of the **impeller** axially, flows radially outward; KE given by impeller converted to pressure in the **diffuser** (vaned/vaneless) and **volute/scroll** casing. Pressure rise roughly half in impeller and half in diffuser (for radial vanes). Stage pressure ratio ≈ **up to 4–5** (higher in special designs).

Nandyal Banking Coaching Center

- **Impeller vane shapes**: **radial** ($\beta_2 = 90°$) – commonest for compressors (strong, simple, no bending stress); **backward-curved** – stable characteristic, higher efficiency; forward-curved – unstable, rarely used for compressors.
- **Slip factor** $\sigma = V_{w2}/V_{w2,ideal}$ ($\approx V_{w2}/U_2$ for radial vanes) – accounts for **relative eddy** (fluid inertia) causing whirl at outlet to be less than blade speed; typical **≈ 0.9**; increases with number of vanes (Stanitz: $\sigma = 1 - 0.63\pi/Z$).
- **Power input factor** ψ – accounts for disc friction, windage, losses; ≈ **1.035–1.04**. Work per kg = $\psi \sigma U_2^2$.
- **Inlet guide vanes / pre-whirl** reduce the relative Mach number at the eye tip.
- **Surging** = unstable, **cyclic flow reversal** at **low flow rate (high pressure ratio)** when the delivery system pressure exceeds what the compressor can maintain on the positive-slope part of the characteristic; causes noise, vibration, damage. **Surge line** on the performance map; prevented by anti-surge (blow-off/recycle) valves, IGVs.
- **Choking (stonewall)** = at **maximum flow**, Mach number reaches 1 at some section → flow cannot increase further.
- **Rotating stall** = flow separation in a few blade passages which propagates from passage to passage around the annulus; causes blade vibration and can lead to surge.
- **Axial-flow compressor** – alternate rows of **rotor** (moving) and **stator** (fixed) blades; flow parallel to axis; small pressure rise per stage (≈ **1.1–1.3**) but many stages → high overall ratio; **highest isentropic efficiency (≈ 85–90%)**, large mass flow, small frontal area → used in **gas turbines, jet engines**. Annulus area **decreases** from inlet to outlet (density increases).
- **Degree of reaction** R = enthalpy (pressure) rise in rotor ÷ enthalpy rise in stage. **R = 0.5 (50%)** → symmetric velocity triangles ($\alpha_1 = \beta_2$, $\alpha_2 = \beta_1$), equal pressure rise in rotor and stator – common design.
- **Work done factor** (axial) – accounts for non-uniform axial velocity across the annulus (boundary layer growth); < 1 (≈ 0.85–0.98), decreases with number of stages.
- Axial compressors have a **narrower stable operating range** (steeper characteristic) – more prone to stall/surge than centrifugal.

Feature	Reciprocating	Screw / vane (rotary PD)	Centrifugal	Axial
Flow	Pulsating	Continuous	Continuous	Continuous
Best suited for	**High pressure, low flow**	Medium pressure (≈ 7–13 bar), medium flow	Medium pressure, **large flow**	Low ratio per stage, **very large flow**
Pressure ratio per stage	High (≈ 4–8)	≈ up to 13 (oil-injected)	≈ **4–5**	≈ **1.1–1.3**
Efficiency (isentropic)	Good at high ratio	Good at full load	≈ 75–85%	**≈ 85–90% (highest)**
Frontal area / weight	Bulky, heavy foundation	Compact	Large frontal area	**Small frontal area**
Instability	None (PD)	None (PD)	Surging (wide stable range)	Stalling & surging (narrow range)
Oil-free air	Possible (non-lube)	Oil-free versions	**Inherently oil-free**	Oil-free
Applications	Workshops, garages, high-pressure gas	**Industrial plant air**	Large plants, turbochargers, small gas turbines	**Aircraft gas turbines**, power-plant GTs

- **Compressed air system** sequence: **intake filter → compressor → aftercooler → moisture separator → air receiver → dryer → main-line filters → distribution piping → FRL → end use**.
- Compressed air is one of the **most expensive utilities**: only ≈ **10–30%** of the electrical input reaches the point of use as useful energy; ≈ 80–90% becomes heat (recoverable for water/space heating).
- **Air intake**: cool, clean, dry air from outside/shaded location; **every 4 °C rise in inlet air temperature increases power consumption by ≈ 1%** (BEE). Clogged intake filters also raise power.
- **Air receiver** – storage to meet peak demand, damp pulsations, reduce load/unload cycling, allow condensate to separate; fitted with **safety (relief) valve**, pressure gauge, **drain valve** at bottom, inspection opening. Designed as an unfired pressure vessel (IS 2825 / ASME Sec VIII).
- **Aftercooler** – air- or water-cooled heat exchanger after compressor; removes the bulk of the water vapour as condensate (fitted with separator and automatic trap).
- **Dryers**: **refrigerated** – cools air to ≈ **+3 °C pressure dew point** (2–10 °C); low running cost; general plant air. **Desiccant (adsorption)** – activated alumina, silica gel, molecular sieve; dew point ≈ **−40 °C** (down to −70 °C); **heatless** type uses ≈ 15–20% purge air; heated/blower-purge types use less air. **Deliquescent** (absorption, salt tablets) – small.
- **Filters**: particulate (general), **coalescing** (oil aerosols/water), activated-carbon (oil vapour/odour). Quality classes as per **ISO 8573-1** (particles, water, oil).
- **FRL unit** = **Filter → Regulator → Lubricator** (in this order, in flow direction) at the point of use for pneumatic tools/cylinders. Filter removes dirt/water; **regulator** maintains constant downstream pressure; **lubricator** adds oil mist for tools (omitted where oil-free air required – food, instruments, paint shop).
- **Piping layout**: **ring main (loop)** preferred – air reaches any point from two directions → lower pressure drop and more uniform pressure; main header with **slope ≈ 1:100 (≈ 1%) in direction of flow**; **drain legs/automatic drain traps** at low points and ends; **branch take-offs from the top** of the header (goose-neck) so condensate does not enter; isolation valves for sections; adequately sized pipes (velocity ≈ 6–10 m/s in mains).
- **Pressure drop**: distribution loss should be kept small (≈ ≤ 0.3 bar from compressor to farthest point; ≤ 10% of operating pressure as an outer limit). Every extra bar of generation pressure costs power.
- **Leakage** – the **single largest energy waste** in compressed-air systems; typically **20–30%** (even more) of compressor output in poorly maintained plants; target ≤ **5–10%**. Common leak points: couplings, hoses, quick-connects, FRLs, condensate traps,

Nandyal Banking Coaching Center

threaded joints, valves. A 1 mm hole at 7 bar wastes ≈ 1 L/s. Leaks occur even when production is idle.

- **Leak test (load–unload method)** with no consumption: Leakage % = **T/(T + t) × 100**, where T = on-load time, t = unload time; leakage quantity = Q × T/(T + t), Q = FAD. Ultrasonic leak detectors locate leaks.
- **Energy conservation**: **reduce generation pressure** to the minimum required (≈ **6–10% power saving per 1 bar reduction**, BEE); arrest leaks; cool intake air; avoid misuse (cleaning, body cooling, open blowing → use blowers/nozzles); use **VSD** screw compressor for variable demand; sequence multiple compressors; correct receiver size; low-pressure-drop dryers/filters; zero-loss (level-sensing) drains; waste-heat recovery; isolate idle sections; use pressure boosters for few high-pressure users.
- **Specific power consumption** (kW per m^3/min of FAD) – the key performance indicator; checked by **FAD test** (receiver pump-up / nozzle method).
- **Safety**: never point compressed air at the body (air embolism, eye injury); for cleaning use safety nozzles limiting dead-end pressure to ≈ 2 bar (30 psi, OSHA practice); depressurise before maintenance (lock-out/tag-out); test safety valves; drain receivers daily; use whip-check restraints on hoses; hearing protection; avoid oil carry-over (fire/explosion risk in discharge lines – high delivery temperature + carbon deposits); periodic hydraulic testing of receivers.

KEY FORMULAE

- Roots blower work/rev = $(p_2 - p_1)\,V$
- Centrifugal work per kg = $\psi\,\sigma\,U_2^2$; $U_2 = \pi D_2 N/60$
- Euler work (axial stage) = $U\,(V_{w2} - V_{w1})$; $R = \Delta h_{rotor}/\Delta h_{stage}$
- Leakage % = $T/(T + t) \times 100$; leakage (m^3/min) = FAD × T/(T + t)

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Compressor with no internal compression? → **Roots blower**.
- Surging in a centrifugal compressor occurs at? → **Low flow rate (high pressure ratio)**.
- Degree of reaction commonly used in axial compressors? → **50%** (symmetric blading).
- Correct order of an FRL unit? → **Filter – Regulator – Lubricator**.
- Largest energy waste in compressed-air systems? → **Leakage**.
- Compressor used in aircraft gas turbines? → **Axial-flow**.

Nandyal Banking Coaching Center

7. Refrigeration, Air Conditioning & Psychrometry

NOTE

RAC usually gives **5–8 MCQs** in PSU/GATE-type papers, and CWC runs its own **cold storages**, so ammonia plants, cold-room temperatures, insulation, refrigerant phase-out (Montreal/Kigali) and psychrometric processes are especially important. Most-asked items: COP formulae, TR conversion, effect of sub-cooling/superheating, refrigerant numbering, expansion devices, bypass factor, SHF and the direction of each process on the psychrometric chart.

7.1 Refrigeration Basics & Air Refrigeration ★★★

- **Refrigeration** = removing heat from a space or substance and keeping it **below the surroundings' temperature**. By the Clausius statement of the second law, this needs **external work or heat input**.
- **Ton of refrigeration (TR)** = the rate of heat removal needed to freeze **1 short ton (2000 lb ≈ 907 kg)** of water at 0 °C into ice at 0 °C in **24 hours**. The latent heat used is 144 Btu/lb.
- Exact relation: **1 TR = 12,000 Btu/h = 200 Btu/min**. In SI units: **1 TR = 3.5167 kW ≈ 211 kJ/min ≈ 12,660 kJ/h**. Many Indian textbooks round this to **210 kJ/min = 3.5 kW** for numericals.
- "Metric" or "European" TR (1000 kg of ice, 335 kJ/kg, 24 h) ≈ **3.88 kW**. This is rarely used; take 3.5167 kW unless the question says otherwise.
- **Refrigerating effect (RE)** = heat absorbed from the cold space. **Capacity** = RE per unit time (kW or TR).
- COP_R = refrigerating effect ÷ work input. It is usually **greater than 1**. $COP_{HP} = COP_R + 1$ for the same cycle and temperatures.
- **Relative COP** = actual COP ÷ theoretical COP. **Energy Efficiency Ratio (EER)** = Btu/h of cooling ÷ W of input = **3.412 × COP**.
- Power per TR: **kW/TR = 3.517 ÷ COP**. For example, a chiller at 0.6 kW/TR has COP ≈ 5.86. A lower kW/TR is better.
- **Reversed Carnot cycle** has 2 isentropic and 2 isothermal processes. It gives the **maximum COP** between two temperatures but is **not practical**: isothermal heat transfer with a gas is impractical, and wet compression is a problem.
- Carnot COP **increases** when T_L is raised or T_H is lowered. Raising T_L is more effective than lowering T_H by the same amount.

KEY FORMULAE

- $COP_{R,\,Carnot} = T_L/(T_H - T_L)$; $COP_{HP,\,Carnot} = T_H/(T_H - T_L)$ (T in kelvin)
- Example: T_L = −23 °C (250 K), T_H = 27 °C (300 K) → COP_R = 250/50 = **5**; COP_{HP} = **6**
- Bell-Coleman COP = $T_1/(T_2 - T_1)$ = $\mathbf{1/(r_p^{(\gamma-1)/\gamma} - 1)}$, where 1 = compressor inlet, 2 = compressor outlet and $r_p = p_2/p_1$
- Power required (kW) = capacity (kW) ÷ COP ; capacity (TR) = Q (kW) ÷ 3.5167

- **Bell-Coleman cycle** = **reversed Brayton (Joule) cycle**. Its processes are: isentropic compression → constant-pressure cooling (cooler) → isentropic expansion (expander) → constant-pressure heat absorption (refrigerator).
- The working fluid is **air**, which **does not change phase**, so heat is absorbed only as **sensible heat**. The mass flow per TR is therefore large and the COP is **low (≈ 0.4–0.7 actual)**.
- Bell-Coleman COP **falls as the pressure ratio rises**. An expander (rather than a throttle) is essential, because throttling an ideal gas causes **no temperature drop**.
- Open air cycle: air goes straight into the cold space, so moisture freezes in valves and lines. A **dense-air (closed) system** runs at higher pressure, so the machine is smaller and there is no moisture trouble.
- Advantages: air is **free, non-toxic, non-flammable**, and the equipment has a **low weight per TR**. These make it the standard choice for **aircraft cooling**.

Aircraft refrigeration system	Key feature	Used for
Simple (cooling turbine) system	Ram air plus engine-compressor bleed air → heat exchanger (ram-air cooled) → **cooling turbine** → cabin. The turbine drives a fan.	Low-speed aircraft; ground cooling with an evaporative attachment
Bootstrap system	Two heat exchangers plus a **secondary compressor driven by the cooling turbine**. Air is compressed again before final cooling.	High-speed / transport aircraft (ram air temperature too high for the simple system)
Regenerative system	Part of the cold turbine exhaust cools the main stream in a **regenerative heat exchanger**, then is dumped overboard.	High-speed aircraft (high ram-air temperature)
Reduced ambient system	Two cooling turbines; the second (fan-driving) turbine expands ram air to cool the first heat exchanger.	Supersonic aircraft
Bootstrap-evaporative	Bootstrap plus a water/alcohol evaporator between the heat exchangers	Very high heat loads

- **DART (Dry Air Rated Temperature)** = the turbine-exit temperature with dry air. It is used to **compare aircraft refrigeration systems**.
- **Ram effect**: the forward speed of the aircraft compresses the incoming air, raising both its **pressure and temperature**. $T_{ram} = T_a(1 + (\gamma-1)M^2/2)$.

MEMORY TRICK

"**12-3.5-211**": 1 TR = 12,000 Btu/h = 3.5 kW = 211 kJ/min. COP of a heat pump is always **one more** than COP of the fridge.

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- 1 TR is equivalent to? → **3.5167 kW (≈ 211 kJ/min)**.
- Bell-Coleman cycle is the reverse of? → **Brayton (Joule) cycle**.
- COP of a Carnot refrigerator working between 250 K and 300 K? → **5**.
- Air refrigeration is preferred in aircraft because? → **Low weight of equipment per TR**, not COP.
- Secondary compressor driven by the cooling turbine is a feature of? → **Bootstrap system**.
- For the same temperatures, $COP_{HP} - COP_R$ = ? → **1**.

7.2 Vapour Compression Refrigeration (VCR) ★★★

- Four components in flow order: **compressor → condenser → expansion device → evaporator**. The **compressor** separates the low and high pressure sides, and so does the **expansion valve**.
- The refrigerant **changes phase** and absorbs **latent heat**, so the mass flow is small and the COP high. A practical VCR has COP of about **2.5–5**.
- The **p-h (Mollier) chart** is the most used chart for refrigeration. It has **enthalpy on a linear x-axis** and **pressure on a logarithmic y-axis**. Throttling appears as a **vertical line**, and evaporation and condensation as **horizontal lines**.
- On the T-s diagram, the **area under 4–1** = refrigerating effect. Throttling 3–4 is irreversible and drawn dotted, with **entropy increasing**.

Process	Component	Nature	Energy per kg
1–2	Compressor	**Isentropic** compression, dry saturated vapour → superheated vapour	$W = h_2 - h_1$
2–3	Condenser	**Constant pressure** heat rejection (desuperheat + condense)	$Q_c = h_2 - h_3$
3–4	Expansion valve	**Isenthalpic** throttling ($h_3 = h_4$), irreversible; produces flash gas	0 (no work, no heat)
4–1	Evaporator	**Constant pressure & temperature** evaporation	$RE = h_1 - h_4$

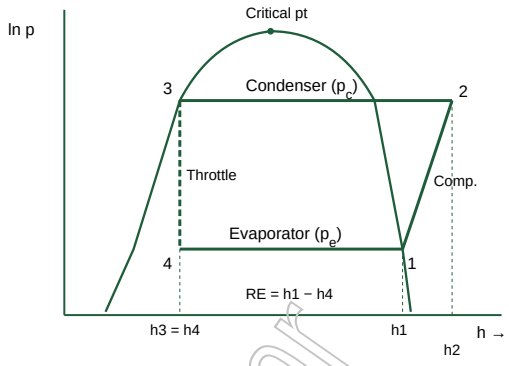


Fig. 7.1 Simple saturated VCR cycle on the p-h chart. Throttling 3–4 is vertical (constant h); compression 1–2 follows a constant-entropy line.

KEY FORMULAE

- COP = RE/W = $(h_1 - h_4)/(h_2 - h_1)$; heat rejected per kg = $h_2 - h_3 = RE + W$
- Mass flow for 1 TR: $\dot{m}$ = **$211/(h_1 - h_4)$ kg/min** (or $3.5167/(h_1 - h_4)$ kg/s)
- Heat rejection ratio = $Q_c/RE = 1 + 1/COP$ (≈ **1.2–1.3** for AC duty)
- Dryness after throttling: $x_4 = (h_4 - h_{f4})/h_{fg4}$
- Volumetric efficiency (clearance): $\eta_v = 1 + C - C(p_2/p_1)^{1/n}$; C = clearance volume/swept volume
- Theoretical piston displacement = $\dot{m} \times v_1/\eta_v$ (v_1 = specific volume at suction)
- Two-stage optimum intermediate pressure: $p_i = \sqrt{(p_1 p_2)}$

Change	RE/kg	Work/kg	COP	Remark
Lower evaporator pressure (lower T_e)	↓	↑	↓	Suction specific volume ↑ → mass flow and capacity ↓
Higher condenser pressure (higher T_c)	↓	↑	↓	Discharge temperature ↑. A dirty condenser or air in the system does this.
Sub-cooling of liquid	↑	No change	↑ **(always)**	Also less flash gas after throttling
Superheating (useful, in evaporator)	↑	↑	↑ or ↓ (depends on refrigerant)	COP ↑ slightly for R-12/R-134a, ↓ for NH_3. Protects the compressor from liquid.
Superheating outside the cold space (useless)	No change	↑	↓	Uninsulated suction line

- **Liquid–suction heat exchanger**: liquid leaving the condenser is **sub-cooled** by suction vapour, which is **superheated**. It is used with **halocarbons** (R-134a, R-22), **not with NH_3**, because NH_3 gives a very high discharge temperature.
- **Wet compression** (state 1 inside the dome) risks **liquid slugging** and valve damage. **Dry compression** is preferred.
- **Actual cycle deviations**:
  - pressure drops in the condenser, evaporator and valves;
  - non-isentropic, polytropic compression with heat transfer;

Nandyal Banking Coaching Center

- superheat at suction and sub-cooling at the condenser exit;
- suction and discharge valve throttling.

The actual cycle therefore needs more work and gives a lower COP.

- **Volumetric efficiency** of a reciprocating compressor **decreases** as the **pressure ratio and clearance increase**. It becomes **zero** at $(p_2/p_1)_{max} = (1 + 1/C)^n$. Typical values are **0.65–0.85**.
- **Flash gas** = vapour formed during throttling. It produces **no useful cooling**. Sub-cooling or a flash chamber reduces it.
- **Multi-stage compression** is used when the **pressure ratio is high (> ~8–10)** or the evaporator temperature is very low (below about −30 °C). It uses a **flash intercooler** (flash chamber) and cuts compressor work and discharge temperature.
- **Cascade system**: two or more separate circuits using **different refrigerants**. The **condenser of the low-temperature circuit** is the **evaporator of the high-temperature circuit**; this shared unit is the cascade condenser. It is used for **−70 to −100 °C** (e.g. R-23 or R-744 low stage with R-404A or NH_3 high stage).
- **Multi-evaporator systems** use back-pressure valves on the higher-temperature evaporators, for example a domestic fridge-freezer or a cold store with different chambers.
- **Accessories**:
  - **receiver**: stores liquid after the condenser;
  - **filter-drier**: silica gel or molecular sieve; removes moisture so ice does not choke the expansion valve;
  - **oil separator**: on the discharge line;
  - **accumulator**: protects the suction side from liquid;
  - **sight glass**;
  - **solenoid valve**.
- The **refrigerating effect per kg** is highest for **NH_3** (~1100 kJ/kg at normal conditions) among common refrigerants. R-134a gives about 150 kJ/kg, so it needs a much larger mass flow per TR.

MEMORY TRICK

"**Sub-cool = Sure COP gain**; Super-heat = Suspense (depends on refrigerant)." Anything that **widens the pressure gap** (lower p_e or higher p_c) **lowers COP**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Process in the expansion valve of a VCR? → **isenthalpic (h constant)**.
- Effect of sub-cooling on COP? → **Increases** (RE ↑, work unchanged).
- COP of a VCR in terms of enthalpies? → **$(h_1 - h_4)/(h_2 - h_1)$**.
- Volumetric efficiency of a compressor increases with? → **Decrease in pressure ratio / clearance**.
- Cascade refrigeration is used for? → **Very low temperatures with two different refrigerants**.
- Optimum intermediate pressure for 2-stage compression? → **$\sqrt{(p_1 p_2)}$**.

7.3 Refrigerants ★★★

- **Primary refrigerant** = the working fluid of the cycle (changes phase). **Secondary refrigerant** = an intermediate coolant that carries cold but does not run the cycle. Examples: **brine** (NaCl eutectic ≈ **−21 °C**; $CaCl_2$ eutectic ≈ **−55 °C**), glycol solutions, chilled water.
- **Numbering rule for halocarbons (ASHRAE 34)**: R-**xyz**, where **x = C − 1** (dropped if zero), **y = H + 1**, **z = number of F atoms**. The remaining bonds are Cl. A suffix a, b marks isomers; for example **R-134a** is the asymmetric isomer.
- Check: CCl_2F_2 → C = 1 (x = 0), H = 0 (y = 1), F = 2 → **R-12**. $CHClF_2$ → **R-22**. $C_2H_2F_4$ → **R-134a**. CH_2F_2 → **R-32**. C_3H_8 → **R-290**.
- **Series**:
  - **400** = zeotropic blends (R-404A, R-407C, R-410A);
  - **500** = azeotropic blends (R-500, R-502);
  - **600** = miscellaneous organics (R-600 n-butane, **R-600a isobutane**);
  - **700** = inorganics, numbered **700 + molecular mass**: NH_3 (17) → **R-717**, H_2O (18) → **R-718**, air (29) → R-729, CO_2 (44) → **R-744**;
  - **1000 series** = unsaturated compounds / **HFOs** (R-1234yf, R-1234ze).
- **CFC** (Cl, F, C, no H; e.g. R-11, R-12) has the highest ODP and is banned. **HCFC** (contains H; e.g. R-22, R-123) has low ODP and is being phased out. **HFC** (no Cl; e.g. R-134a, R-32, R-410A) has **zero ODP but high GWP** and is under Kigali phase-down. **HFO** has zero ODP and very low GWP.
- **ODP reference = R-11 (ODP = 1)**. **GWP reference = CO_2 (GWP = 1)**, on a 100-year horizon.
- **Azeotrope** behaves like a pure substance: constant boiling temperature, no glide. A **zeotrope** has a **temperature glide** and fractionates on leaking, so it must be **charged in liquid phase**. R-410A is a near-azeotrope with negligible glide.

Refrigerant	Type / formula	NBP (°C)	ODP	GWP (100-yr, AR4)	Typical use
R-11	CFC, CCl_3F	+23.8	**1.0**	4750	Old centrifugal chillers (banned)
R-12	CFC, CCl_2F_2	−29.8	1.0	10,900	Old domestic refrigerators, car AC (banned)
R-22	HCFC, $CHClF_2$	−40.8	0.055	1810	Window/split AC, cold storage (phasing out)
R-134a	HFC, CH_2FCF_3	−26.1	0	**1430**	Domestic refrigerators, car AC, chillers
R-410A	HFC blend R-32/R-125 (50/50)	≈ −51.4	0	**2088**	Split AC, VRF (high pressure)
R-32	HFC, CH_2F_2 (A2L)	−51.7	0	**675**	Modern split ACs in India
R-404A	HFC blend	≈ −46	0	3922	Commercial freezers (being replaced)

Nandyal Banking Coaching Center

R-717	Ammonia NH_3 (B2L)	−33.3	0	**0**	**Cold storages, ice plants**, food processing
R-744	CO_2 (A1)	−78.5 (sublimes)	0	**1**	Supermarkets, heat-pump water heaters, cascade low stage
R-290	Propane C_3H_8 (A3)	−42.1	0	~3	Room AC (some Indian makes), commercial units
R-600a	Isobutane (A3)	−11.7	0	~3	**Domestic refrigerators** (most new models)
R-718	Water	100	0	0	LiBr absorption chillers, steam-jet systems
R-1234yf	HFO	−29.5	0	< 1–4	Car AC (R-134a replacement)

- **ASHRAE safety groups**: the letter is toxicity (**A = lower, B = higher**). The number is flammability: **1 = no flame propagation, 2L = mildly flammable, 2 = flammable, 3 = highly flammable**. NH_3 = **B2L**; R-32 = **A2L**; R-290 and R-600a = **A3**; R-134a, R-410A and CO_2 = **A1**.
- **CO_2 (R-744)**: critical temperature is only **31.1 °C** (73.8 bar), so the high side usually works **transcritically**, with a **gas cooler** in place of the condenser and pressures around 90–130 bar. Its triple point is −56.6 °C at 5.18 bar.
- **Lubricants**: mineral oil suits CFC/HCFC. **POE (polyol ester)** oil suits HFCs (R-134a, R-410A, R-32). **PAG** is used in car AC with R-134a/R-1234yf. Ammonia is **immiscible** with mineral oil, so oil settles out and is drained from the low points.

Desirable property	Reason
Low boiling point; evaporator pressure **slightly above atmospheric**	Avoids air/moisture leaking inwards
Moderate condensing pressure	Light, cheap equipment
High latent heat of vaporisation	Less mass flow per TR
Low specific volume of vapour (reciprocating); high specific volume suits **centrifugal** compressors	Small compressor displacement
High critical temperature, low freezing point	Condensation possible with ambient cooling; no freezing in the evaporator
High thermal conductivity, low viscosity	Good heat transfer, low pressure drop
High dielectric strength	Needed in **hermetic** compressors (refrigerant cools the motor)
Non-toxic, non-flammable, chemically stable, non-corrosive, zero ODP, low GWP, cheap, easy leak detection	Safety and environment

- **Ammonia (R-717)** is the workhorse of **cold storages and ice plants**:
  - **highest refrigerating effect per kg** and excellent heat-transfer properties;
  - cheap, with **zero ODP and zero GWP**;
  - its **pungent smell** gives early leak warning;
  - lighter than air.
- Ammonia drawbacks:
  - **toxic**;
  - mildly **flammable**, about **15–28% by volume** in air;
  - **attacks copper and its alloys (brass, bronze)** when moisture is present, so **steel pipes and fittings** are used and **open-type compressors** (not hermetic) are required;
  - very high discharge temperature, so compressor heads are water-cooled.

Leak detection method	Refrigerant	Indication
Burning sulphur stick / candle	**NH_3**	**Dense white smoke** (ammonium sulphite)
Moist litmus / phenolphthalein paper; HCl swab	NH_3	Litmus turns blue / phenolphthalein turns pink; white fumes of NH_4Cl
Halide torch (copper element)	Chlorine-bearing halocarbons (CFC, HCFC)	Flame turns **green** for a small leak, **purple/violet** for a large leak. It does **not** detect chlorine-free HFCs.
Soap (bubble) solution	All refrigerants	Bubbles at the leak; the simplest method
Electronic (heated diode / IR) detector, UV dye	All, especially **HFCs**	Alarm or glow

Nandyal Banking Coaching Center

CODE UPDATE

Montreal Protocol on ozone-depleting substances:

- Signed **16 Sept 1987** (World Ozone Day) under the 1985 **Vienna Convention**; in force from **1 Jan 1989**.
- India joined in **1992**.
- CFCs were phased out in developed countries by 1996 and in developing (Article 5) countries by **1 Jan 2010**. India stopped CFC production in 2008, ahead of schedule.
- HCFC phase-out for developing countries: freeze 2013 → 10% cut 2015 → 35% 2020 → 67.5% 2025 → **97.5% by 2030**, with a 2.5% servicing tail to 2030–2040 (100% by 2040). India phased out HCFC-141b (foam blowing) on **1 Jan 2020**.

Kigali Amendment:

- Adopted **15 Oct 2016** at Kigali, Rwanda; in force **1 Jan 2019**. It brings **HFCs** under the Montreal Protocol for **phase-down**, not phase-out.
- India ratified it in **2021**.
- India is in the **Article 5 Group 2** schedule: **baseline 2024–2026**, **freeze 2028**, then cuts of 10% (2032), 20% (2037), 30% (2042) and **85% (2047)**.
- China and most other developing countries (Group 1) have baseline 2020–22, freeze 2024 and an 80% cut by 2045. Developed countries began in 2019 and reach an 85% cut by 2036.
- Expected benefit: avoiding about **0.4 °C** of warming by 2100.

India Cooling Action Plan (ICAP):

- Launched **March 2019** by MoEFCC (Ozone Cell). India was **one of the first countries** with a comprehensive cooling plan.
- Covers a **20-year horizon** (2017-18 to 2037-38).
- Targets by 2037-38: reduce cooling demand by **20–25%**, refrigerant demand by **25–30%** and cooling energy by **25–40%**.
- Also targets training and certifying **1,00,000 servicing technicians** by 2022-23.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Chemical formula of R-22? → **$CHClF_2$** (HCFC).
- Designation of ammonia and CO_2? → **R-717 and R-744**.
- Refrigerant used in large cold storages? → **Ammonia (R-717)**.
- Pipe material that must NOT be used with ammonia? → **Copper / brass**.
- Ammonia leak is detected by? → **Burning sulphur stick (white fumes)**.
- Kigali Amendment deals with? → **Phase-down of HFCs** (India freeze 2028).

7.4 Vapour Absorption & Other Refrigeration Systems ★★☆

- In a **vapour absorption system (VAS)**, the compressor is replaced by a "**thermal compressor**" made of the **absorber + pump + generator**. The input is mainly **heat** (steam, waste heat, solar, gas). Pump work is very small (≈ 1% of the heat input).
- The **condenser, expansion valve and evaporator** are the same as in a VCR.
- Absorption of refrigerant in the absorber is **exothermic**, so the absorber must be **cooled**. A cold absorbent absorbs more refrigerant.
- The **generator** is heated to drive refrigerant vapour out of the strong solution.
- The **heat exchanger** between the weak and strong solutions saves generator heat and improves COP.
- Aqua-ammonia only: the **analyser** (on top of the generator) and the **rectifier** (a water-cooled dephlegmator) remove the **water vapour** carried along with the NH_3 vapour. The drained water goes back to the generator. Without them, water would freeze at the expansion valve and in the evaporator.

Feature	Aqua-ammonia (NH_3–H_2O)	Lithium bromide–water (LiBr–H_2O)
Refrigerant	**Ammonia**	**Water**
Absorbent	**Water**	**LiBr** solution
Lowest temperature	**Below 0 °C** (to about −40 °C)	**Not below 0 °C**; chilled water ≈ 5–7 °C
Rectifier / analyser	**Required** (water is volatile)	**Not required** (LiBr is non-volatile)
Operating pressure	Above atmospheric	**High vacuum** (evaporator ≈ 0.8–1 kPa)
Special problem	Toxicity of NH_3	**Crystallisation** of LiBr; air leakage; corrosion
Typical COP	≈ 0.5–0.6	Single effect ≈ **0.6–0.7**; double effect ≈ **1.0–1.2**
Application	Industrial / cold storage refrigeration	**Large central AC chillers**

KEY FORMULAE

- COP_{VAS} = refrigerating effect ÷ (heat in generator + pump work) ≈ Q_e/Q_g
- Ideal (maximum) COP: $\mathbf{COP_{max} = [(T_g - T_o)/T_g] \times [T_e/(T_o - T_e)] = (T_e/T_g)\cdot(T_g - T_o)/(T_o - T_e)}$
- This is a Carnot engine (between T_g and T_o) driving a Carnot refrigerator (between T_o and T_e). T_g = generator, T_o = ambient (condenser/absorber), T_e = evaporator.
- Example: T_g = 400 K, T_o = 300 K, T_e = 250 K → COP_{max} = (100/400)(250/50) = **1.25**
- Energy balance: $Q_g + Q_e + W_p = Q_c + Q_a$

- **Electrolux (Platen–Munters) refrigerator** is a **three-fluid** system:
  - **ammonia** = refrigerant, **water** = absorbent, **hydrogen** = inert gas;
  - the **total pressure is the same everywhere**; H_2 lowers the **partial pressure of NH_3** in the evaporator so it can evaporate at low temperature;
  - **no pump, no moving parts**: circulation is by a **bubble pump** (thermosiphon) and gravity;
  - heated by kerosene, gas or an electric heater; silent;
  - used in **hotels, caravans and remote areas**;
  - very low COP.
- **VAS vs VCR**: VAS runs on low-grade heat, has few moving parts, is quiet and holds capacity well at part load. Its COP is **low** and it is bulky. VCR runs on high-grade work (electricity) and has a high COP.
- **Steam-jet (ejector) refrigeration**:
  - refrigerant is **water**; a high-velocity steam jet through a **nozzle/ejector** creates a **vacuum** in the flash chamber, and

Nandyal Banking Coaching Center

water flashes and chills;
  - not below about **2–4 °C**, because water freezes at 0 °C;
  - no moving parts; low COP;
  - used where cheap steam is available (process plants, AC water chilling).
- **Thermoelectric refrigeration (Peltier effect, 1834)**:
  - DC current through a junction of two dissimilar materials causes **heating at one junction and cooling at the other**;
  - uses **p- and n-type semiconductors (bismuth telluride, Bi_2Te_3)**;
  - performance depends on the figure of merit **$Z = \alpha^2/(\rho k)$**: high Seebeck coefficient, low resistivity, low thermal conductivity;
  - no moving parts or refrigerant, but very **low COP**;
  - used in electronics cooling, portable coolers and medical kits.

  The **Seebeck effect** is the reverse (temperature difference → EMF, as in a thermocouple). The **Thomson effect** is heat in a single conductor carrying current along a temperature gradient.
- **Vortex tube (Ranque–Hilsch)**:
  - compressed air enters **tangentially**;
  - **hot air** leaves at the periphery through the throttle (control) valve at the far end;
  - **cold air** leaves through the central orifice near the inlet;
  - no moving parts; very low COP;
  - used for **spot cooling** of cutting tools and electronic cabinets, and for miners' suits.
- Other methods include **adiabatic demagnetisation** of a paramagnetic salt (temperatures near **0 K**, cryogenics), cooling by evaporation, and the **Linde–Hampson** and Claude cycles (gas liquefaction using the **Joule–Thomson effect**, valid only below the inversion temperature).

MEMORY TRICK

"**LiBr = Water is the Refrigerant** → AC only (above 0 °C), no rectifier." In aqua-ammonia, **water is the absorbent**, and because it is volatile we need an **Analyser + Rectifier** ("A-R for A-A").

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In a LiBr–water absorption system, the refrigerant is? → **Water**.
- Function of the rectifier in an aqua-ammonia system? → **Remove water vapour from ammonia vapour**.
- Role of hydrogen in an Electrolux refrigerator? → **Reduces partial pressure of NH_3; equalises total pressure**.
- Thermoelectric refrigeration works on? → **Peltier effect**.
- Maximum COP of an absorption system with T_g = 400 K, T_o = 300 K, T_e = 250 K? → **1.25**.

7.5 Refrigeration Equipment & Cold Storage Practice ★★☆

Compressor	Principle	Typical capacity range*	Application / remark
Reciprocating	Positive displacement	Fractional TR to ~150 TR	Domestic, commercial, **ammonia cold stores**. Most versatile.
Rotary (rolling piston / vane)	Positive displacement	Up to ~5 TR	Room AC, refrigerators. Small and compact.
Scroll	Positive displacement (orbiting scroll)	~1–50 TR	Split/packaged AC. Few parts, **quiet, high volumetric efficiency**, no valves.
Screw (twin / single)	Positive displacement (rotary)	~20–1500 TR	Chillers, large NH_3 plants. Handles some liquid; slide-valve capacity control.
Centrifugal	**Dynamic** (rotodynamic)	~200–10,000 TR	**Large central AC chillers**. Uses high-volume, low-pressure refrigerants (R-123, R-134a, R-1233zd). Surge at low load.

- *Ranges are indicative; they vary between textbooks and manufacturers.
- **Hermetic** compressor: motor and compressor sealed in a welded shell, with no shaft seal and no leakage. The **refrigerant cools the motor**, so high dielectric strength is needed. Used in domestic refrigerators and room ACs.
- **Semi-hermetic** compressor: bolted shell that can be opened for service.
- **Open type** compressor: driven through a shaft seal. It is required for **ammonia**, because NH_3 attacks the copper windings.
- Capacity control methods: cylinder unloading, hot-gas bypass, **variable speed (inverter)**, slide valve (screw), inlet guide vanes (centrifugal).

Condenser	Features
Air-cooled, natural convection	Wire-and-tube at the back of domestic refrigerators
Air-cooled, forced convection	Finned coil with fan: window/split AC and packaged units. No water needed, but condensing temperature is higher.
Water-cooled: double tube, shell & coil	Small to medium capacity
Water-cooled shell & tube	**Water in tubes, refrigerant in the shell**. Large plants and chillers. Works with a cooling tower.
Evaporative condenser	Water is sprayed over the refrigerant coil while air is blown across. Cooling comes mainly from **evaporation**. The condensing temperature approaches the **ambient WBT**, giving the **lowest condensing temperature** and least water use. Widely used in **large ammonia cold storages**.

- **Cooling tower**: **range** = hot water in − cold water out. **Approach** = cold water out − **ambient WBT**. Effectiveness = range/(range + approach). The water cannot be cooled below the ambient WBT.

Expansion device	Controls / maintains	Application
Capillary tube (ID ≈ 0.5–2 mm, length ≈ 1–6 m)	Fixed restriction; no control. Pressures **equalise during the off-cycle**, so a **low starting torque** motor will do. Critical charge; no receiver.	**Small hermetic units**: domestic refrigerators, window/split AC (up to about 3 TR)

Nandyal Banking Coaching Center

Automatic (constant pressure) expansion valve, AEV	**Constant evaporator pressure**. Poor response to load changes: it overfeeds at low load and starves at high load.	Small, constant-load units (water coolers)
Thermostatic expansion valve, TEV/TXV	**Constant degree of superheat** (≈ 5 °C) at the evaporator exit. A **feeler bulb** sits on the suction line.	**Most widely used** for DX evaporators: AC, commercial, cold rooms
Low-side float valve	Constant **liquid level in the evaporator**	**Flooded evaporators** (large NH_3 plants)
High-side float valve	Liquid level in the condenser/receiver; passes whatever liquid condenses	Single-evaporator flooded systems, centrifugal chillers
Electronic expansion valve (EEV)	Superheat via a microprocessor and stepper motor	Inverter AC, VRF, modern chillers
Hand expansion valve	Manual setting	Large plants with an attendant, bypass

- **Evaporator types**:
  - **Flooded evaporator**: mostly full of liquid refrigerant, giving **high heat-transfer rate**. It needs a larger charge, a **surge drum**/accumulator and a float control. Used in large NH_3 and chiller plants.
  - **Dry-expansion (DX) evaporator**: fed only as much liquid as will evaporate (fed by a TXV), with superheated vapour at the outlet. Smaller charge and simple oil return.
  - Constructions: bare tube, **plate surface** (domestic freezer), **finned tube** (air coolers), shell & tube (water chillers).
- **Defrosting**: frost on the evaporator acts as **insulation** and blocks airflow, so capacity and COP fall. Methods:
  - manual, by shutdown;
  - off-cycle (for coils above 0 °C);
  - electric heater;
  - **hot-gas defrost**, the fastest and most efficient, used in commercial and cold stores;
  - water spray;
  - reverse cycle.

  "Frost-free" fridges use automatic electric defrost with a timer.

Commodity (cold storage)	Storage temperature	RH
Potato (seed, long-term)	**2–4 °C**	**90–95%**
Potato (table/processing)	8–12 °C (avoids sugar build-up; sprout inhibitor CIPC used)	90–95%
Onion	0–2 °C	**65–70%** (low RH, unlike most produce)
Apple, grapes	−1 to 2 °C	90–95%
Banana	**13–14 °C** (chilling injury below ~12 °C)	85–95%
Mango	10–13 °C	85–90%
Frozen meat / fish / frozen foods	**−18 °C or below** (fish often −20 to −30 °C)	—

- **Insulation**:
  - **PUF (polyurethane foam)**: k ≈ **0.020–0.025 W/m·K**, the best common insulant; used as sandwich **PUF panels** about 80–150 mm thick.
  - **EPS (expanded polystyrene, "thermocole")**: k ≈ 0.033–0.037 W/m·K.
  - Others: PIR, mineral wool, cork (old practice, k ≈ 0.04).
- The **vapour barrier** goes on the **warm (outer) side** of the insulation. Moisture moves from warm to cold, so a barrier there stops it condensing or freezing inside the insulation, which would destroy its k-value.
- Freezer rooms need **under-floor heating** or ventilation to prevent **frost heave** of the floor. Cold stores use **air curtains / strip curtains** and anterooms to cut infiltration.
- Controlled-atmosphere (CA) storage reduces O_2 (2–5%) and raises CO_2 to slow respiration, for example in apples. Ethylene scrubbers are used for climacteric fruit.
- **Cold-store refrigeration load** = product sensible heat (above freezing) + latent heat of freezing + sensible heat below freezing + **heat of respiration** (fruit and vegetables) + wall/roof transmission + infiltration (door openings) + lights, people, fans and forklifts. A safety factor of about 10% is usually added.
- **Can-ice plant**:
  - ammonia system with the evaporator coils in a **brine tank** (NaCl or $CaCl_2$) at about **−10 °C**, plus an agitator;
  - galvanised **ice cans** are dipped in the brine;
  - **freezing time ∝ (thickness)²**;
  - **low-pressure air agitation** in the cans gives **clear (transparent) ice**;
  - cans are thawed in a dip tank to release the blocks.
- For ice from 30 °C water: cooling water to 0 °C ≈ 126 kJ/kg, freezing ≈ 335 kJ/kg, plus sub-cooling of the ice. So **1 TR produces less than 1 tonne of ice per day** (well below the "1 short ton" in the TR definition).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Thermostatic expansion valve maintains constant? → **Degree of superheat at evaporator exit**.
- Expansion device in a domestic refrigerator? → **Capillary tube**.
- Compressor used for very large (> 1000 TR) AC plants? → **Centrifugal**.
- Vapour barrier in a cold store is placed on? → **Warm (outer) side of insulation**.
- Condenser giving the lowest condensing temperature (approaches WBT)? → **Evaporative condenser**.
- Ideal storage condition for seed potatoes? → **2–4 °C, 90–95% RH**.

7.6 Psychrometry ★★★

- **Psychrometry** = study of **moist air**, a mixture of **dry air and water vapour**. Both are treated as ideal gases. Standard atmospheric pressure = **101.325 kPa**.
- **Dalton's law**: $p = p_a + p_v$. Each gas fills the whole volume at its own partial pressure and the mixture temperature.

Nandyal Banking Coaching Center

- **Saturated air** = air holding the maximum vapour at a given DBT ($p_v = p_s$, the saturation pressure at DBT). In **unsaturated air**, the vapour is **superheated**.
- **Specific (absolute) humidity / humidity ratio ω** = kg of vapour per kg of **dry air**.
- **Relative humidity φ** = p_v/p_s at the same DBT; equivalently, the ratio of the actual vapour mass to the saturated vapour mass at that DBT.
- **Degree of saturation μ** = ω/ω_s. μ is **slightly less than φ**; the two are equal only at 0 and 100%.
- **Dry-bulb temperature (DBT)** = temperature read by an ordinary thermometer.
- **Wet-bulb temperature (WBT)** = reading of a thermometer with a **wetted wick** in moving air (≥ about 3–5 m/s).
  - Evaporation cools the bulb, so **WBT < DBT** in unsaturated air.
  - For air–water, **WBT ≈ thermodynamic WBT (adiabatic saturation temperature)**, because the **Lewis number ≈ 1**.
- **Dew-point temperature (DPT)** = temperature at which vapour starts to condense when air is cooled at **constant pressure and constant ω**. It is the saturation temperature at p_v.
- **DBT > WBT > DPT** for unsaturated air. **At saturation (φ = 100%), DBT = WBT = DPT**.
- **Wet-bulb depression (WBD)** = DBT − WBT; it is **zero at saturation** and large for dry air. **Dew-point depression** = DBT − DPT.
- Instruments:
  - **sling (whirling) psychrometer**;
  - Assmann (aspirated) psychrometer;
  - **hair hygrometer** (hair length changes with RH);
  - dew-point hygrometer;
  - electronic capacitive sensors.

KEY FORMULAE

- Specific humidity: **$\omega = 0.622\ p_v/(p - p_v)$** kg/kg d.a. (0.622 = M_{water}/M_{air} = 18.015/28.97)
- Relative humidity: **$\varphi = p_v/p_s$** ; degree of saturation: **$\mu = \omega/\omega_s = \varphi(p - p_s)/(p - p_v)$**
- Enthalpy of moist air: **$h = 1.005\ t + \omega(2500 + 1.88\ t)$** kJ/kg d.a. (t in °C; 2500 ≈ h_{fg} at 0 °C; 1.88 = c_p of vapour)
- Humid specific heat: $c_{pm} = 1.005 + 1.88\omega \approx$ **1.0216 kJ/kg·K**
- Specific volume: $v = 0.287\ T/(p - p_v)$ m^3/kg d.a. (p in kPa, T in K)
- Sensible heat (kW) = **0.0204 × cmm × Δt** ; latent heat (kW) = **50 × cmm × Δω** ; total heat (kW) = **0.02 × cmm × Δh** (cmm = m^3/min, ρ ≈ 1.2 kg/m^3)
- Example: p_v = 2 kPa at 101.325 kPa → ω = 0.622 × 2/99.325 = **0.0125 kg/kg d.a.**

- **Psychrometric chart** layout (at 101.325 kPa):
  - **DBT** on the horizontal x-axis; constant-DBT lines are **vertical**.
  - **ω (and DPT)** on the vertical axis, plotted on the right; constant-ω and constant-DPT lines are **horizontal**.
  - The **saturation curve (φ = 100%)** forms the upper-left boundary, and the RH curves sit below it.
  - **Constant WBT lines** slope downward to the right and nearly **coincide with constant-enthalpy lines**.
  - **Constant specific-volume lines** are steeper than the WBT lines.
- For a given ω, a **higher DBT means lower RH**. Heating air at constant ω therefore **dries it relatively**: RH falls, but DPT and ω are unchanged.

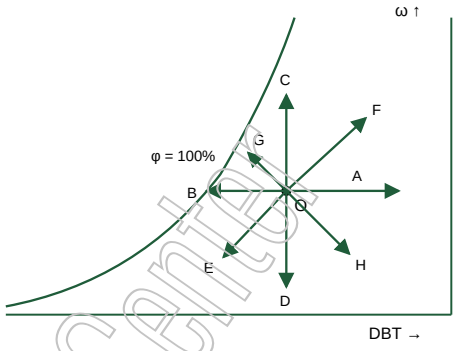


Fig. 7.2 Psychrometric processes from state O: A sensible heating, B sensible cooling, C humidification, D dehumidification, E cooling & dehumidification, F heating & humidification, G evaporative cooling (along constant WBT), H chemical dehumidification.

Process	Line on chart	DBT	ω / DPT	RH	WBT / h
Sensible heating	Horizontal →	↑	Constant	↓	↑
Sensible cooling	Horizontal ←	↓	Constant	↑	↓
Humidification (pure)	Vertical ↑	Constant	↑	↑	↑
Dehumidification (pure)	Vertical ↓	Constant	↓	↓	↓
Cooling & dehumidification	Down-left (toward ADP)	↓	↓	↑ (usually)	↓
Heating & humidification	Up-right	↑	↑	Varies	↑
Evaporative cooling (adiabatic saturation)	Up-left along **constant WBT**	↓	↑	↑	**WBT and h ≈ constan**
Chemical dehumidification	Down-right (≈ constant h)	↑	↓	↓	≈ constant
Adiabatic mixing of two streams	On the straight line joining the two states, nearer the larger mass	$\omega_3 = (m_1\omega_1 + m_2\omega_2)/(m_1 + m_2)$; h_3 similarly; t_3 ≈ mass-weighted mean			

- **Apparatus dew point (ADP)** = the **effective surface temperature** of a cooling coil. On the chart, it is where the **process (SHF) line meets the saturation curve**. Air leaving an ideal coil (BPF = 0) would be at ADP.

Nandyal Banking Coaching Center

- **Bypass factor (BPF, x)** = the fraction of air that passes through the coil **without touching the surface** and so stays unchanged.
  - Contact factor (coil efficiency) = 1 − BPF.
  - BPF **decreases** with more rows, closer fin spacing and lower air velocity.
  - Typical BPF ≈ **0.05–0.35**.
  - For n identical rows, **BPF = x^n**.
- **Sensible heat factor (SHF)** = sensible heat ÷ total heat = SH/(SH + LH). SHF = 1 is a horizontal line (pure sensible). SHF = 0 is a vertical line (pure latent). The SHF protractor on the chart is used to draw it.

KEY FORMULAE

- Cooling coil: **BPF = $(t_2 - t_{ADP})/(t_1 - t_{ADP})$** ; heating coil (surface t_s): BPF = $(t_s - t_2)/(t_s - t_1)$
- Contact factor η = 1 − BPF = $(t_1 - t_2)/(t_1 - t_{ADP})$
- Example: t_1 = 30 °C, ADP = 10 °C, t_2 = 14 °C → BPF = 4/20 = **0.2**
- Evaporative (saturation) efficiency = $(t_1 - t_2)/(t_1 - t_{wb1})$
- SHF = SH/(SH + LH)

MEMORY TRICK

On the chart: "**Heat → Right, Water → Up**." Evaporative cooling rides the **WBT line** (enthalpy constant). The order **D-W-D** (DBT ≥ WBT ≥ DPT) collapses to one value at saturation.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- When air is saturated, DBT, WBT and DPT are? → **Equal**.
- During sensible heating, which remains constant? → **Specific humidity (and DPT)**.
- Evaporative cooling follows which line? → **Constant WBT (≈ constant enthalpy)**.
- Specific humidity ω = ? → **0.622 $p_v/(p - p_v)$**.
- Bypass factor of a coil with t_1 = 30 °C, t_2 = 14 °C, ADP = 10 °C? → **0.2**.
- Summer air conditioning involves? → **Cooling and dehumidification**.

7.7 Air Conditioning Systems & Cooling Load ★★★

- **Air conditioning** = simultaneous control of **temperature, humidity, air motion (velocity/distribution) and air purity (cleanliness, odour)** in a space. The basic tool for analysing it is the psychrometric chart.
- **Human comfort** depends on six factors:
  - environmental: DBT, humidity, air velocity, mean radiant temperature;
  - personal: activity level (**met**) and clothing (**clo**).

  1 met = 58.2 W/m² of body surface (seated at rest; ≈ 100 W for an adult). **1 clo = 0.155 m²·K/W**.
- **ASHRAE comfort zone** (Std 55): roughly **22–26 °C DBT** (≈ 23–26 °C in summer, 20–24 °C in winter) and **40–60% RH**.
- Typical Indian summer inside design condition: **24–25 °C DBT, 50 ± 5% RH**.
- Air velocity in the occupied zone ≈ **0.1–0.25 m/s**. Higher speeds are felt as a draught.
- **Effective temperature (ET)** = an index combining DBT, humidity and air motion. It equals the temperature of **saturated, still air** that gives the same feeling of warmth.
  - Optimum ET on the old ASHVE comfort chart: about **21–22 °C in summer** and about **20 °C in winter**; values vary slightly between textbooks.
  - The modern index is **ET***, based on 50% RH.
- Heat given out by an adult doing office work ≈ **115–130 W total**, roughly 55–60% sensible and the rest latent. A seated person at rest gives about 100 W.
- The body loses heat by **convection, radiation, evaporation (sweat)**, respiration and conduction. At high DBT (close to 37 °C), **evaporation** becomes the dominant path, which is why high RH feels oppressive.

AC system	Typical capacity	Key features
Window AC	0.75–2 TR	Self-contained single box through t Cheapest; noisy. Capillary tube; rotary/reciprocating compressor.
Split AC (wall/cassette/ductable)	0.75–3 TR (ductable up to ~ 15 TR)	Indoor fan-coil unit plus outdoor **co unit** (compressor + condenser), joi refrigerant piping. Quiet.
Packaged unit	~5–50 TR	Factory-assembled unit with all components; ducted; air- or water- Shops, small halls.
VRF / VRV (Variable Refrigerant Flow/Volume)	~ 8–100 TR per system	One outdoor unit with an **inverter compressor** serves **many indoor u** through refrigerant piping. EEVs va refrigerant flow to each unit. Heat- type can **heat and cool simultaneo** Offices and hotels; good part-load
Central plant (chiller-based)	> 50–100 TR, up to thousands	**Chillers** (reciprocating/screw/centrifugal/a produce chilled water at about **7 °C 12 °C return**. This feeds **AHUs/FCU** Condenser water enters the conde about 32 °C and leaves at about 37 cooled back in a cooling tower.

Central system type	Medium to the room	Examples / remarks
All-air	Only conditioned air through ducts	Single-duct **CAV** (constant volume, variable temperature); **VAV** (variable air volume, constant temperature, energy-efficient); dual-duct; multizone. Large ducts; good humidity control.
All-water	Only chilled/hot water piped to room units	**Fan-coil units (FCU)**. Small pipes instead of large ducts. Ventilation is poor unless provided separately.
Air-water	Both primary air (ducts) and water	**Induction units**, FCU with dedicated outdoor air, chilled beams. Used in multi-room buildings (hotels, hospitals).
Unitary / DX (direct expansion)	Refrigerant itself cools the air	Window, split, packaged, VRF

- **Summer AC** (hot-humid): **cooling + dehumidification** by a cooling coil below DPT. In very humid climates, **reheat** may be needed to bring RH down.

Nandyal Banking Coaching Center

- Hot-dry climates such as Rajasthan suit **evaporative cooling**.
- **Winter AC**: **heating + humidification**, typically preheat → humidify (air washer or steam spray) → reheat.
- **Year-round AC** combines both, with automatic controls (thermostat for temperature, **humidistat** for RH).
- **AHU (air handling unit)** contains: mixing box (fresh + return air), **filters**, cooling coil, heating coil, humidifier, fan, dampers.
- **Fresh-air (ventilation) requirement** is given in L/s or m^3/min per person plus per m^2 of floor area (ASHRAE 62.1). Indian practice traditionally uses about **0.3–0.5 m^3/min per person** for offices, more where people smoke or in hospitals.

Cooling load component	Sensible	Latent	Estimation basis
Solar gain through glass	✔	—	Area × SHGF × shading coefficient × CLF
Conduction through walls/roof (sun-lit)	✔	—	**Q = U A × CLTD** (cooling-load temperature difference; older method: ETD)
Conduction through glass, partitions	✔	—	U A Δt
Occupants	✔	✔	Per person sensible + latent × CLF for sensible
Lights	✔	—	Wattage × use factor × **ballast factor (~1.2 for fluorescent)**
Equipment / motors	✔	✔ (if steam/cooking)	Rated power ÷ efficiency, use factor
Infiltration (leakage)	✔	✔	0.0204·cmm·Δt and 50·cmm·Δω
Ventilation (outdoor) air	✔	✔	Taken on the **coil (grand) load**, not the room load
Supply duct heat gain, fan heat, leakage	✔	—	Percentage allowance (~ 5–10%)

KEY FORMULAE

- **RSHF** (room SHF) = RSH/(RSH + RLH) = RSH/RTH. The supply-air state lies on the RSHF line through the room state.
- **GSHF** (grand SHF) = TSH/(TSH + TLH) = TSH/GTH, where TSH = RSH + OASH and TLH = RLH + OALH (includes fresh-air load).
- **ERSHF** (effective room SHF) = ERSH/(ERSH + ERLH), where **ERSH = RSH + BPF × OASH** and **ERLH = RLH + BPF × OALH**. The ERSHF line meets the saturation curve at the **ADP**.
- OASH = 0.0204 × cmm_o × ($t_o - t_i$) ; OALH = 50 × cmm_o × ($\omega_o - \omega_i$) (kW)
- Dehumidified air quantity: $\mathbf{cmm_d = ERSH/[0.0204\ (1 - BPF)(t_i - t_{ADP})]}$
- Fan laws (same fan, same density): **Q ∝ N**, **Δp ∝ N^2**, **Power ∝ N^3**. With size D: Q ∝ ND^3, Δp ∝ N^2D^2, P ∝ N^3D^5.
- Equivalent circular diameter of a rectangular duct (a × b), same friction & flow: $\mathbf{D_e = 1.30\ (ab)^{0.625}/(a + b)^{0.25}}$; same velocity & friction: $D_e = 2ab/(a + b)$
- Duct friction (Darcy): $\Delta p = f\ (L/D)\ \rho V^2/2$; velocity pressure $p_v = \rho V^2/2$; total pressure = static + velocity pressure

Duct design method	Principle	Remarks
Equal friction	Same **pressure drop per unit length** (e.g. ≈ 0.8–1 Pa/m) in all sections	**Most widely used** for supply, return and exhaust ducts. Dampers needed for balancing.
Velocity reduction	Velocity is **arbitrarily reduced** in steps along the duct	Simple systems; needs experience; ducts not balanced
Static regain	Velocity is reduced at each branch so that the **static-pressure regain** offsets friction in the next section. **Static pressure is about equal** at every outlet.	Best for **high-velocity, long** supply ducts; self-balancing; larger and costlier ducts

- Duct **aspect ratio** should be kept low (ideally ≤ 4:1). A circular or square section gives the least material and friction for a given flow.
- Low-velocity systems run below about **10 m/s**. Main ducts in comfort AC typically run at about 5–8 m/s and branches at about 3–5 m/s, to limit noise.
- **Fans**:
  - **Centrifugal, forward-curved**: small and quiet, low pressure; power keeps **rising with flow (overloading)**; used in FCUs and window AC.
  - **Centrifugal, backward-curved**: **highest efficiency**, **non-overloading**; used in AHUs.
  - **Radial-blade**: for dusty or material-laden air.
  - **Axial** (propeller, tube-axial, vane-axial): high flow at low pressure; **condenser fans** and cooling towers.
- **Energy efficiency**:
  - The **BEE** (Bureau of Energy Efficiency, under the Energy Conservation Act 2001) runs a **Standards & Labelling** programme. **Star rating** goes from **1 to 5 stars**; 5 is the most efficient.
  - Room ACs are rated by **ISEER (Indian Seasonal Energy Efficiency Ratio)**, mandatory since **2018**.
  - ISEER = total annual cooling delivered ÷ total annual energy consumed, using Indian temperature bins (24–43 °C) and a reference of **1600 operating hours** per year.
  - It rewards **inverter (variable-speed)** ACs, which perform well at part load.
  - BEE made **24 °C the default temperature setting** for room ACs from 1 Jan 2020.
  - Chiller efficiency is expressed in **kW/TR** (lower is better) or COP. **IPLV** is the part-load figure.
  - Other measures: the **ECBC** (Energy Conservation Building Code) for commercial buildings, and thermal energy storage (ice/chilled water) to shift peak load.

MEMORY TRICK

"**R-G-E**": **R**oom SHF uses room loads only; **G**rand SHF adds all the **G**oing-in fresh air; **E**ffective SHF adds only the **bypassed** fraction (BPF × outdoor load) and points at the **ADP**.

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Most commonly used duct design method? → **Equal friction method**.
- Duct design method giving about equal static pressure at all outlets? → **Static regain method**.
- Effective room sensible heat (ERSH) = ? → **RSH + BPF × OASH**.
- If fan speed is doubled, power becomes? → **8 times** ($P \propto N^3$).
- Rating used by BEE for room air conditioners? → **ISEER**.
- Comfortable indoor condition for summer AC? → **≈ 24–25 °C DBT, 50% RH**.

Nandyal Banking Coaching Center

8. Heat Transfer

NOTE

Heat Transfer gives 4–6 questions in most PSU/SSC JE/GATE-type papers. The favourites are: thermal resistance of plane, cylinder and sphere; critical radius of insulation; fin formulae and effectiveness; lumped system and Biot number; meaning of Nu, Pr, Gr, Bi, Fo; the pool boiling curve; Stefan–Boltzmann, Wien and Kirchhoff laws; view factor rules; radiation shields; LMTD and ε–NTU. Learn the standard values (σ, 2898 μm·K, Nu = 3.66/4.36, Pr of air ≈ 0.7) by heart.

8.1 Conduction – Fundamentals ★★★

- **Three modes**: conduction (molecular contact, needs a medium), convection (conduction + bulk fluid motion), radiation (electromagnetic waves, **needs no medium**, fastest – travels at speed of light).
- **Fourier's law**: $Q = -kA\,(dT/dx)$. The **negative sign** shows heat flows in the direction of decreasing temperature. It is an **empirical (phenomenological) law**, valid for homogeneous, isotropic solids.
- **Thermal conductivity k** – unit **W/m·K** (W/m·°C); dimensions $\mathbf{MLT^{-3}\theta^{-1}}$. It is a property of the material.
- Heat flux q = Q/A, unit **W/m²**.
- In metals heat is carried mainly by **free electrons** (and lattice vibrations, phonons); in non-metals/insulators mainly by **lattice vibrations (phonons)**. Hence good electrical conductors are good thermal conductors – **Wiedemann–Franz law**: $k/(\sigma_e T)$ = constant (Lorenz number).
- Order of k: **pure metals > alloys > non-metallic solids > liquids > gases**. Alloying **reduces** k (e.g. pure Cu ≈ 401, brass ≈ 110, stainless steel ≈ 15 W/m·K).
- **Diamond** has the highest k of common solids (≈ 2000–2300 W/m·K); among metals **silver** is highest, then copper, gold, aluminium.

Material (≈ 300 K)	k (W/m·K)	Material	k (W/m·K)
Diamond	**≈ 2300**	Glass	≈ 0.8–1.4
Silver (pure)	**≈ 429**	Concrete / brick	≈ 0.7–1.4
Copper (pure)	**≈ 401**	Ice	≈ 2.2
Gold	≈ 317	Water (liquid)	**≈ 0.6**
Aluminium (pure)	**≈ 237**	Wood	≈ 0.1–0.2
Iron (pure)	≈ 80	Engine oil	≈ 0.15
Mild / carbon steel	≈ 45–55	Glass wool, mineral wool	**≈ 0.04**
Stainless steel	**≈ 15**	Polyurethane foam	≈ 0.02–0.03
Mercury (liquid metal)	≈ 8.5	Air	**≈ 0.026**

Material class	Effect of rise in temperature on k
Pure metals	**k decreases** (electron scattering increases)
Alloys	k generally **increases** slightly
Gases	**k increases** (k ∝ √T, kinetic theory); k nearly independent of pressure
Liquids	k **decreases** – **exception: water** (k rises up to ≈ 130–150 °C, then falls)
Insulators / non-metals	**k increases**

- Variable conductivity is usually taken as $\mathbf{k = k_0(1 + \beta T)}$; for linear variation use k at the **mean temperature**. If β > 0, the temperature profile in a plane wall is **convex (concave downward)**; if β = 0 it is linear.
- **Insulating materials** owe low k to **trapped air in pores**; moisture **increases** k of insulation (water k ≈ 23× air). Good insulator: k < ≈ 0.1 W/m·K (some books say < 0.2).
- **Thermal diffusivity** $\alpha = k/\rho c_p$, unit **m²/s** – rate at which a temperature disturbance spreads; ratio of heat conducted to heat stored. High α → body reaches equilibrium faster. Metals high α; wood, rubber low α. α of air (≈ 2.2×10^{-5}) > α of water (≈ 1.4×10^{-7}) though $k_{air} \ll k_{water}$.
- ρc_p = **heat capacity per unit volume** (thermal capacity), J/m³K.
- Steady state → temperature at each point independent of time; does not mean uniform temperature.

KEY FORMULAE – GENERAL CONDUCTION EQUATION

- Cartesian: $\partial^2T/\partial x^2 + \partial^2T/\partial y^2 + \partial^2T/\partial z^2 + q_g/k = (1/\alpha)\,\partial T/\partial t$
- Cylindrical (r, φ, z): $(1/r)\partial/\partial r(r\,\partial T/\partial r) + (1/r^2)\partial^2T/\partial\varphi^2 + \partial^2T/\partial z^2 + q_g/k = (1/\alpha)\,\partial T/\partial t$
- Spherical (r, φ, θ): $(1/r^2)\partial/\partial r(r^2\,\partial T/\partial r) + [1/(r^2\sin^2\theta)]\partial^2T/\partial\varphi^2 + [1/(r^2\sin\theta)]\partial/\partial\theta(\sin\theta\,\partial T/\partial\theta) + q_g/k = (1/\alpha)\,\partial T/\partial t$
- Steady, with generation → **Poisson equation**: $\nabla^2T + q_g/k = 0$
- Steady, no generation → **Laplace equation**: $\nabla^2T = 0$
- Unsteady, no generation → **Fourier (diffusion) equation**: $\nabla^2T = (1/\alpha)\,\partial T/\partial t$

Geometry (1-D, steady, no generation)	Thermal resistance R_{th}	Temperature profile
Plane wall, thickness L	**L/(kA)**	**Linear**
Hollow cylinder, radii r_1, r_2, length L	$\mathbf{\ln(r_2/r_1)/(2\pi kL)}$	**Logarithmic**
Hollow sphere, radii r_1, r_2	$\mathbf{(r_2 - r_1)/(4\pi k\, r_1 r_2)}$	**Hyperbolic** ($T \propto 1/r$)
Convection surface	**1/(hA)**	—
Radiation surface	$1/(h_rA)$	—

- **Electrical analogy**: ΔT ↔ voltage, Q ↔ current, R_{th} ↔ electrical resistance. Unit of R_{th} = **K/W**.
- Cylinder can be treated as a plane wall using the **logarithmic mean area** $A_m = (A_2 - A_1)/\ln(A_2/A_1)$; sphere using the **geometric mean area** $A_m = \sqrt{(A_1A_2)} = 4\pi r_1 r_2$.
- **Composite wall (layers in series)**: $R_{total} = \Sigma R = 1/h_iA + L_1/k_1A + L_2/k_2A + \ldots + 1/h_oA$. Same Q through every layer; the **largest temperature drop occurs across the layer with the largest resistance** (usually the insulation).
- **Parallel layers**: $1/R = 1/R_1 + 1/R_2$ (same ΔT across each).
- **Overall heat transfer coefficient U**: $Q = UA\Delta T_{overall}$; for a plane wall $1/U = 1/h_i + \Sigma L/k + 1/h_o$. Unit W/m²K. For a tube U must be quoted on a reference area: $U_oA_o = U_iA_i = 1/R_{total}$.
- **Thermal contact resistance** $R_{tc} = \Delta T_{interface}/(Q/A)$, unit m²K/W; caused by surface roughness – actual contact only at a few points, gaps filled with air. Reduced by **higher contact pressure,**

Nandyal Banking Coaching Center

smoother surfaces, softer materials, thermal grease/conductive paste or thin soft-metal foil (indium, lead, silver). Thermal contact conductance $h_c = 1/R_{tc}$.

- **Heat generation** (uniform q_g, surface at T_s): profile is **parabolic**; maximum temperature at the centre.

KEY FORMULAE – CONDUCTION WITH HEAT GENERATION (MAX. TEMPERATURE RISE)

- Plane wall of thickness 2L: $T_{max} - T_s = \mathbf{q_g L^2/(2k)}$
- Solid cylinder radius R: $T_{max} - T_s = \mathbf{q_g R^2/(4k)}$
- Solid sphere radius R: $T_{max} - T_s = \mathbf{q_g R^2/(6k)}$
- Surface temperature: $T_s = T_\infty + q_g(V/A)/h$ → wall $q_g L/h$, cylinder $q_g R/(2h)$, sphere $q_g R/(3h)$

MEMORY TRICK

Generation denominators "**2-4-6**" for wall–cylinder–sphere (= 2 × number of dimensions 1-2-3). Profiles without generation: "**Line–Log–Hyper**" for Plane–Cylinder–Sphere.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Unit of thermal diffusivity? → **m^2/s**.
- Temperature distribution in a hollow cylinder (steady, no generation)? → **Logarithmic**.
- Thermal conductivity of gases with rise of temperature? → **Increases**.
- Steady-state conduction with heat generation is governed by? → **Poisson equation**.
- Thermal resistance of a hollow sphere? → $\mathbf{(r_2 - r_1)/(4\pi k\, r_1 r_2)}$.
- Max. temperature rise in a cylindrical wire generating heat q_g? → $\mathbf{q_g R^2/4k}$.

8.2 Critical Insulation & Fins ★★★

- Adding insulation to a **cylinder or sphere** increases conduction resistance but also increases outer surface area (decreasing convection resistance 1/hA). So heat loss first **increases**, reaches a **maximum at the critical radius**, then decreases.
- For a **plane wall** there is **no critical thickness** – area does not change, so insulation always reduces heat loss.
- If $r_{outer} < r_c$: adding insulation **increases heat loss** (up to r_c). If $r_{outer} > r_c$: insulation always **reduces** heat loss.
- Critical radius **depends only on k of insulation and h** – not on pipe radius.
- Application: **electric cables / wires** – insulation is kept near r_c so that heat dissipation is maximum (cable runs cooler). For steam pipes, insulation thickness must take r_o well beyond r_c.
- For small-diameter tubes with low h (natural convection), r_c can exceed the bare radius: e.g. k = 0.05 W/mK, h = 5 W/m²K → r_c = 0.05/5 = **10 mm**.

KEY FORMULAE – CRITICAL RADIUS

- Cylinder: $\mathbf{r_c = k/h}$
- Sphere: $\mathbf{r_c = 2k/h}$
- (k = conductivity of insulation, h = outside convective coefficient)

Fins (extended surfaces)

- Fins increase heat transfer by **increasing surface area**. They are most effective where **h is low** – i.e. on the **gas/air side** (e.g. air-cooled IC engine cylinders, motorcycle engines, car radiators, air-side of AC condenser coils, transformer casing, electronic heat sinks).
- Fin material should have **high k** (aluminium, copper). Fins should be **thin and closely spaced** (high P/A_c), but not so close as to block the boundary layer flow.
- Assumptions in fin analysis: steady, 1-D conduction along the fin, constant k and h, no generation, uniform cross-section, negligible radiation.
- Fin parameter $\mathbf{m = \sqrt{hP/kA_c}}$, unit m^{-1}. P = perimeter, A_c = cross-sectional area. For a pin fin of diameter D: $P/A_c = 4/D$, so $\mathbf{m = \sqrt{4h/kD}}$. For a thin rectangular fin (thickness t, width w ≫ t): m ≈ √(2h/kt).
- Governing equation: $d^2\theta/dx^2 - m^2\theta = 0$, $\theta = T - T_\infty$.

Fin tip condition	Temperature distribution θ/θ_0	Heat transfer Q
Infinitely long fin	e^{-mx}	$\mathbf{\sqrt{hPkA_c}\,\theta_0}$
Insulated (adiabatic) tip	$\mathbf{\cosh m(L-x)/\cosh mL}$	$\mathbf{\sqrt{hPkA_c}\,\theta_0 \tanh(mL)}$
Convective tip	$[\cosh m(L-x) + (h/mk)\sinh m(L-x)] / [\cosh mL + (h/mk)\sinh mL]$	$\sqrt{hPkA_c}\,\theta_0\,[\sinh mL + (h/mk)\cosh mL] / [\cosh mL + (h/mk)\sinh mL]$
Specified tip temperature θ_L	$[(\theta_L/\theta_0)\sinh mx + \sinh m(L-x)]/\sinh mL$	$\sqrt{hPkA_c}\,\theta_0\,[\cosh mL - \theta_L/\theta_0]/\sinh mL$

- **Corrected length**: a convective-tip fin can be treated as an insulated-tip fin of length $L_c = L + A_c/P$ → **L + t/2** (rectangular), **L + D/4** (pin fin).
- tanh(mL) → 1 when mL ≈ 2.5–3; so a fin longer than mL ≈ 2.5 behaves practically as an **infinite fin** (tanh 2.5 = 0.987). Extra length adds little heat transfer.
- **Fin efficiency η_f** = actual heat transfer ÷ heat transfer if the whole fin were at base temperature. Insulated tip: $\mathbf{\eta_f = \tanh(mL)/(mL)}$. Infinite fin: $\eta_f = 1/(mL)$. η_f decreases as L increases.
- **Fin effectiveness ε_f** = heat transfer with fin ÷ heat transfer without fin (from base area A_c). Infinite fin: $\mathbf{\varepsilon_f = \sqrt{kP/hA_c}}$.
- $\varepsilon_f < 1$ → fin acts as insulation; $\varepsilon_f = 1$ → no effect. Use of fins is justified only if $\mathbf{\varepsilon_f \geq 2}$ (Cengel; some texts say $\varepsilon_f \geq 5$ is desirable).
- Effectiveness increases with **high k, high P/A_c (thin fins), low h**. Relation: $\varepsilon_f = \eta_f \times (A_{fin}/A_{base})$.
- Fin is worthwhile only if **Biot number $hA_c/(kP) < 1$** (in practice ≪ 1, i.e. Bi = ht/2k small for a thin plate fin).
- Overall surface efficiency: $\eta_o = 1 - (A_f/A)(1 - \eta_f)$.
- Error in a **thermometer well** (pocket) is treated as a fin – reduce error by using **longer, thinner, low-k well** and high h.

Biot number

- **$Bi = hL_c/k_{solid}$** = internal conduction resistance (L_c/k) ÷ external convection resistance (1/h).
- k in Bi is of the **solid**; k in Nusselt number is of the **fluid** – a classic trap.

Nandyal Banking Coaching Center

- Small Bi (< 0.1) → temperature nearly uniform inside the solid (lumped system valid). Large Bi → large internal gradients.

MEMORY TRICK

"**Cylinder K by H, Sphere doubles**" – r_c = k/h and 2k/h. For fins: "**Fins go where h is weak**" – air side, not water side.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Critical radius of insulation for a sphere? → **2k/h**.
- Heat loss from a bare pipe of radius smaller than r_c on adding insulation? → **Increases** (till r_c).
- Efficiency of an insulated-tip fin? → **tanh(mL)/mL**.
- Fins are provided on which side of a heat exchanger? → Side with **lower h** (gas side).
- Effectiveness of an infinitely long fin? → **$\sqrt{(kP/hA_c)}$**.
- In Biot number, k refers to? → **Solid**.

8.3 Transient (Unsteady) Conduction ★★☆

- Transient conduction: temperature varies with **time and position** (heat treatment, quenching, cooling of castings, thermocouple response, food freezing).
- **Lumped system analysis** (Newtonian heating/cooling): internal resistance negligible → temperature uniform throughout body at any instant. Valid if **Bi < 0.1** (error < ≈ 5%).
- Characteristic length **$L_c = V/A_s$**.

Body	$L_c = V/A_s$
Plane wall, thickness 2L (both sides exposed)	**L** (half-thickness)
Long cylinder, radius R	**R/2**
Sphere, radius R	**R/3** (= D/6)
Cube, side a	**a/6**

KEY FORMULAE – LUMPED SYSTEM

- $(T - T_\infty)/(T_i - T_\infty) = e^{-t/\tau} = e^{-(hA_s/\rho Vc)t} = e^{-Bi \cdot Fo}$
- Time constant **$\tau = \rho Vc/(hA_s)$** (seconds) = thermal capacitance (ρVc) × convective resistance ($1/hA_s$)
- Fourier number **$Fo = \alpha t/L_c^2$** (dimensionless time)
- Instantaneous heat rate $Q(t) = hA_s(T - T_\infty)$; total heat $Q_{total} = \rho Vc(T_i - T)$

- After **t = τ**: temperature difference falls to **e^{-1} = 36.8%** of initial, i.e. **63.2%** of total change completed. After 4τ ≈ 98% complete; after 5τ ≈ 99.3%.
- **Thermocouple**: small time constant = fast response → use **small bead diameter**, low ρc, high h. τ ∝ D for a spherical junction (V/A = D/6).
- **Fourier number** = rate of heat conduction ÷ rate of heat storage in the body. Large Fo → heat penetrates deeper, faster.
- **$Bi \times Fo = hA_s t/(\rho Vc)$** = t/τ.
- When Bi > 0.1 → use the **one-term series solution** or **Heisler charts** (plane wall, infinite cylinder, sphere): give centre temperature vs Fo with 1/Bi as parameter; valid for **Fo > 0.2**. Position-correction charts give off-centre temperature. **Gröber charts** give total heat transferred Q/Q_0.
- Multi-dimensional bodies (short cylinder, rectangular bar) solved by **product solution**: θ(short cylinder) = θ(infinite plane wall) × θ(infinite cylinder).
- **Semi-infinite solid**: extends to infinity in one direction (earth surface, thick walls for short times). For sudden change of surface temperature to T_s: **$(T - T_s)/(T_i - T_s) = erf[x/(2\sqrt{(\alpha t)})]$**. erf(0) = 0, erf(∞) = 1, erf(2) ≈ 0.995.
- Penetration depth ∝ **$\sqrt{(\alpha t)}$**; a finite body behaves as semi-infinite for short times (Fo small). Used for burial depth of water pipes (frost penetration).
- Surface heat flux in semi-infinite solid: $q_s = k(T_s - T_i)/\sqrt{(\pi\alpha t)}$. Two bodies brought into contact reach an interface temperature $T_s = (\sqrt{(k\rho c)_A}T_A + \sqrt{(k\rho c)_B}T_B)/(\sqrt{(k\rho c)_A} + \sqrt{(k\rho c)_B})$ – explains why metal feels colder than wood at the same temperature (higher $\sqrt{(k\rho c)}$, "thermal effusivity").

MEMORY TRICK

"**One tau = 63**" – after one time constant 63.2% of the change is over. Lumped L_c denominators: wall–cylinder–sphere = **1–2–3** (L, R/2, R/3).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Lumped capacitance analysis is valid when? → **Bi < 0.1**.
- Characteristic length of a sphere of diameter D? → **D/6**.
- Time constant of a thermocouple? → **ρVc/(hA)**.
- Fourier number is defined as? → **$\alpha t/L^2$**.
- Heisler charts are used for? → **Transient conduction with Bi > 0.1** (one-term solution).

8.4 Convection ★★★

- **Newton's law of cooling**: **$Q = hA(T_s - T_\infty)$**. h = convective heat transfer coefficient, W/m²K. h is **not a property** – depends on geometry, flow velocity, fluid properties, surface condition, type of flow.
- At the wall the fluid velocity is zero (**no-slip**), so heat is transferred by **pure conduction** in the fluid layer next to the wall: $h = -k_f(\partial T/\partial y)_{y=0}/(T_s - T_\infty)$.
- **Free (natural) convection**: motion due to **buoyancy** from density differences (needs gravity). **Forced convection**: motion by external means (fan, pump, wind).

Process	Typical h (W/m²K)
Free convection – gases (air)	**2–25**
Free convection – liquids	10–1000
Forced convection – gases	**25–250**
Forced convection – liquids (water)	50–20,000
Boiling and condensation	**2,500–100,000**
Dropwise condensation of steam	up to ≈ 10^5–5×10^5

- Order: **free gas < forced gas < free liquid < forced liquid < boiling/condensation**. Highest h: **dropwise condensation**.

Dimensionless numbers

Number	Definition	Physical meaning
Nusselt, Nu	**hL/k_f**	Convective ÷ conductive heat transfer across fluid layer; dimensionless temperature gradient at surface. Nu = 1 → pure conduction

Nandyal Banking Coaching Center

Reynolds, Re	$\rho VL/\mu = VL/\nu$	Inertia force ÷ viscous force; laminar/turbulent criterion (forced convection)
Prandtl, Pr	$\mu c_p/k = \nu/\alpha$	Momentum diffusivity ÷ thermal diffusivity; relative thickness of velocity and thermal boundary layers. A **fluid property**
Grashof, Gr	$g\beta\Delta T L^3/\nu^2$	Buoyancy force ÷ viscous force (plays role of Re in free convection)
Rayleigh, Ra	**Gr × Pr** = $g\beta\Delta T L^3/(\nu\alpha)$	Laminar/turbulent criterion in free convection (transition ≈ 10^9 on a vertical plate)
Stanton, St	$Nu/(Re{\cdot}Pr) = h/(\rho V c_p)$	Heat transferred ÷ heat capacity of fluid flow; used in analogies
Peclet, Pe	**Re × Pr = VL/α**	Advection ÷ conduction (important in liquid metals)
Graetz, Gz	Re·Pr·(D/L)	Thermal entry length problems (laminar tube flow)
Biot, Bi	hL/k_{solid}	Internal ÷ external resistance (conduction)
Fourier, Fo	$\alpha t/L^2$	Dimensionless time (transient conduction)
Eckert, Ec	$V^2/(c_p\Delta T)$	Kinetic energy ÷ enthalpy difference (high-speed flow)
Lewis, Le	α/D_{AB} = Sc/Pr	Simultaneous heat & mass transfer (Sc = ν/D_{AB}, Sh = h_mL/D_{AB})

- Forced convection: **Nu = f(Re, Pr)**. Free convection: **Nu = f(Gr, Pr)** or f(Ra).
- **Gr/Re^2**: ≫ 1 → natural convection dominates; ≪ 1 → forced dominates; ≈ 1 → **mixed convection**.
- β (volume expansion coefficient) for an **ideal gas = 1/T** (T in kelvin, film temperature).
- Properties are evaluated at the **film temperature $T_f = (T_s + T_\infty)/2$** for external flow; at bulk mean temperature for internal flow.

Fluid	Prandtl number	Boundary layers
Liquid metals (Hg, Na, NaK)	**≪ 1** (≈ 0.004–0.03)	$\delta_t \gg \delta$ (heat diffuses very fast)
Gases (air)	**≈ 0.7–1.0** (air ≈ 0.71)	$\delta_t \approx \delta$
Water	**≈ 1.7–13.7** (≈ 7 at room temperature)	$\delta_t < \delta$
Light organic liquids	5–50	$\delta_t < \delta$
Oils	**≫ 1** (50–100,000)	$\delta_t \ll \delta$
Glycerine	2,000–100,000	$\delta_t \ll \delta$

Boundary layers

- **Hydrodynamic (velocity) boundary layer** δ: region where velocity changes from 0 at the wall to 0.99 U_∞ (Prandtl, 1904).
- **Thermal boundary layer** δ_t: region where $(T_s - T)/(T_s - T_\infty)$ goes from 0 to 0.99.
- Laminar flow: **$\delta_t/\delta \approx Pr^{-1/3}$** (for Pr ≥ ≈ 0.6). Pr = 1 → $\delta_t = \delta$.
- Critical Reynolds number for flat plate = **5×10^5**; for pipe flow ≈ **2300** (fully turbulent above ≈ 4000–10,000).
- Laminar: δ = **$5x/\sqrt{Re_x}$** (Blasius; Cengel uses 4.91; von Kármán integral method gives 4.64). $\delta \propto x^{1/2}$. Turbulent: $\delta = 0.37x/Re_x^{1/5}$, $\delta \propto x^{4/5}$.
- Local h in laminar flow $\propto x^{-1/2}$ → h decreases along the plate; **average h over length L = 2 × local h at x = L**.
- Skin friction: laminar $C_{f,x} = 0.664/\sqrt{Re_x}$, average $C_f = 1.328/\sqrt{Re_L}$; turbulent $C_{f,x} = 0.0592\ Re_x^{-1/5}$.
- Turbulent flow gives much higher h than laminar because eddies mix fluid (eddy diffusivity).

KEY FORMULAE – FORCED CONVECTION CORRELATIONS

- Flat plate, laminar, local: **$Nu_x = 0.332\ Re_x^{1/2}\ Pr^{1/3}$** (Pr ≥ 0.6)
- Flat plate, laminar, average: **$Nu_L = 0.664\ Re_L^{1/2}\ Pr^{1/3}$**
- Flat plate, turbulent, local: $Nu_x = 0.0296\ Re_x^{0.8}\ Pr^{1/3}$; average (fully turbulent) $Nu_L = 0.037\ Re_L^{0.8}\ Pr^{1/3}$
- Tube, turbulent – **Dittus–Boelter**: **$Nu = 0.023\ Re^{0.8}\ Pr^n$**; **n = 0.4 for heating** of fluid, **n = 0.3 for cooling** (Re > 10,000, 0.7 ≤ Pr ≤ 160, L/D > 10)
- Tube, fully developed laminar: **Nu = 3.66** (constant wall temperature), **Nu = 4.36** (constant heat flux)
- Laminar pipe friction factor (Darcy) f = 64/Re
- Hydrodynamic entry length (laminar) ≈ 0.05 Re D; thermal entry length ≈ 0.05 Re Pr D; turbulent entry ≈ 10 D

- In fully developed laminar tube flow, h is **constant and independent of Re** (Nu = constant) – h ∝ k/D.
- In turbulent tube flow, $h \propto V^{0.8}/D^{0.2}$ (Dittus–Boelter): doubling velocity raises h by $2^{0.8} \approx$ **1.74 times**.
- Sieder–Tate correlation: $Nu = 0.027\ Re^{0.8}Pr^{1/3}(\mu/\mu_s)^{0.14}$ – for large property variation.
- Non-circular ducts use **hydraulic diameter $D_h = 4A_c/P$** (square duct side a → D_h = a; annulus → $D_o - D_i$).
- **Reynolds analogy** (Pr = 1): **$St = C_f/2$** – relates heat transfer to skin friction (momentum and heat transfer are analogous). For pipe: St = f/8 (Darcy f).
- **Chilton–Colburn analogy** (0.6 < Pr < 60): **$St{\cdot}Pr^{2/3} = C_f/2 = j_H$** (Colburn j-factor).
- Cross flow over cylinder: Hilpert/Churchill–Bernstein correlations; flow over sphere: Whitaker correlation; tube banks: Zukauskas correlation.

Free convection – basics

- General form: **$Nu = C\ (Gr{\cdot}Pr)^n = C\ Ra^n$**.
- **Laminar** (vertical plate, Ra ≈ 10^4–10^9): **n = 1/4** (C ≈ 0.59). **Turbulent** (Ra ≈ 10^9–10^{13}): **n = 1/3** (C ≈ 0.10).
- For n = 1/3, L cancels → **h independent of characteristic length** in turbulent free convection.
- Characteristic length: vertical plate/cylinder → height L; horizontal cylinder → diameter D; horizontal plate → A_s/P.
- Horizontal plate: hot surface facing **up** (or cold facing down) gives **higher** h than hot surface facing down.
- Enclosures (double-pane window): effective conductivity $k_{eff}/k = 0.18(Pr\ Ra/(0.2 + Pr))^{0.29}$ type relations; for Ra < ≈ 1708 (horizontal layer heated from below) heat flows by **pure conduction** (Nu = 1) – Bénard cells form above this.
- Simplified relations for air: laminar $h \approx 1.42(\Delta T/L)^{1/4}$; turbulent $h \approx 1.31(\Delta T)^{1/3}$ (vertical plate).

Nandyal Banking Coaching Center

MEMORY TRICK

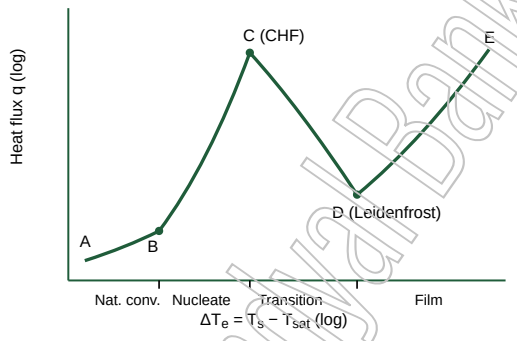
"**Heat up – 4, Cool down – 3**": Dittus–Boelter exponent n = 0.4 heating, 0.3 cooling. "**Temperature 3.66, Flux 4.36**" – flux (q) gets the bigger number. Flat plate laminar: **0.332 local, double it (0.664) for average**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Prandtl number is the ratio of? → **Momentum diffusivity to thermal diffusivity (ν/α)**.
- Nusselt number for fully developed laminar flow in a tube with constant heat flux? → **4.36**.
- Grashof number signifies the ratio of? → **Buoyancy to viscous force**.
- For liquid metals, thermal boundary layer is? → **Much thicker than velocity boundary layer** (Pr ≪ 1).
- Stanton number equals? → **Nu/(Re·Pr)**.
- Reynolds analogy is valid for Pr = ? → **1**.

8.5 Boiling & Condensation ★★☆

- **Boiling**: liquid → vapour at a solid–liquid interface when $T_s > T_{sat}$. Excess temperature $\Delta T_e = T_s - T_{sat}$.
- **Evaporation** occurs at a liquid–vapour interface when vapour pressure < saturation pressure – no bubbles.
- **Pool boiling** – stagnant liquid, motion only by natural convection and bubbles (kettle). **Flow (forced convection) boiling** – liquid pumped through heated tube (boiler tubes).
- **Subcooled (local) boiling**: bulk liquid below T_{sat}; bubbles collapse in the liquid. **Saturated (bulk) boiling**: bulk at T_{sat}; bubbles rise to the free surface.
- Pool boiling curve first obtained by **Nukiyama (1934)** using electrically heated nichrome/platinum wires in water.



Pool boiling curve for water at 1 atm (schematic). B ≈ 5 °C, C ≈ 30 °C (q_{max} ≈ 1 MW/m²), D ≈ 120 °C (q_{min}).

Regime (water, 1 atm)	ΔT_e range	Key features
Natural convection boiling (A–B)	**up to ≈ 5 °C**	No bubbles at surface; liquid slightly superheated; evaporation at free surface
Nucleate boiling (B–C)	**≈ 5–30 °C**	Bubbles form at nucleation sites (cavities); isolated bubbles → jets/columns; **very high h**; most desirable regime in practice (boilers, evaporators). $q \propto \Delta T_e^{\,3}$ (Rohsenow correlation)
Critical heat flux (point C)	≈ 30 °C	**Maximum heat flux** q_{max} ≈ 1 MW/m² for water (≈ 1.26 MW/m², Incropera); Zuber–Kutateladze correlation. Also called **burnout point / boiling crisis / DNB**
Transition boiling (C–D)	**≈ 30–120 °C**	Unstable, partial film; **q decreases as ΔT_e increases**; avoided in practice
Leidenfrost point (D)	≈ 120 °C (≈ 220–230 °C surface)	**Minimum heat flux** q_{min}; stable vapour blanket begins (water drops "dance" on a hot plate)
Film boiling (beyond D)	> ≈ 120 °C	Continuous vapour film covers surface; heat flows by conduction through vapour + **radiation** (significant at high ΔT); low h

- **Burnout**: in a heat-flux-controlled system (electric heater, nuclear fuel rod), increasing q beyond q_{max} makes the operating point jump from C to the film-boiling curve at a very high ΔT_e (≈ 1000 °C) – heater may **melt**. Hence boilers/reactors are designed to operate well below CHF (nucleate regime).
- Boiling curve **cannot** follow the transition region under heat-flux control – it needs temperature control. Cooling path shows **hysteresis** (jumps from D to the nucleate curve).
- Nucleate boiling h depends strongly on **surface roughness/condition and the surface–fluid combination** (C_{sf} in Rohsenow). CHF is almost independent of surface material but increases with pressure up to ≈ 1/3 critical pressure.
- Bubble growth requires liquid superheat; equilibrium bubble: $p_v - p_l = 2\sigma_s/r$ (σ_s = surface tension).
- **Flow boiling in tubes** regimes: single-phase liquid → bubbly → slug → annular → mist (dry-out) → single-phase vapour.

Condensation

- Occurs when vapour contacts a surface at temperature **below T_{sat}**.
- **Filmwise condensation**: condensate wets the surface and forms a continuous liquid film – the film offers **thermal resistance**. Occurs on clean, wettable surfaces; the usual mode in practice and the basis of design.
- **Dropwise condensation**: condensate forms drops that roll off, exposing bare surface – **h 5 to 10 times (up to ~10×) higher** than filmwise. Occurs on non-wetting surfaces; promoted by coatings like **oleic acid, stearic acid, Teflon (PTFE), silicones, gold/noble metal plating**. Difficult to sustain for long periods.
- **Non-condensable gases** (air) in a condenser drastically **reduce** h (gas blanket near surface) – hence air extraction pumps/ejectors in steam condensers.
- Film Reynolds number $Re_\delta = 4\dot{m}/(P\mu_l)$: laminar < 30 (wave-free), 30–1800 wavy laminar, > 1800 turbulent (Cengel).

KEY FORMULAE – NUSSELT'S FILM CONDENSATION THEORY (1916)

- Vertical plate (average): $\mathbf{h = 0.943\,[\rho_l(\rho_l - \rho_v)\,g\,h_{fg}\,k_l^3 / (\mu_l\,L\,(T_{sat} - T_s))]^{1/4}}$
- Horizontal tube (average): $h = \mathbf{0.725}\,[\rho_l(\rho_l - \rho_v)\,g\,h_{fg}\,k_l^3 / (\mu_l\,D\,(T_{sat} - T_s))]^{1/4}$
- Film thickness $\delta \propto x^{1/4}$; local $h \propto x^{-1/4}$; average h = **(4/3) $h_{x=L}$**
- Bank of N tubes in a vertical tier: $h_N = h_1\,N^{-1/4}$

- Nusselt's assumptions: laminar film, **linear temperature profile across film** (heat by conduction only), constant properties, pure

Nandyal Banking Coaching Center

vapour at T_{sat}, negligible vapour shear and film inertia.

- A horizontal tube gives higher h than a vertical tube of the same size; both equal when L ≈ **2.87 D**. Hence **horizontal tubes preferred in condensers**.
- h of condensation decreases with increasing ΔT ($\propto \Delta T^{-1/4}$) and with increasing height L.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Boiling regime with highest heat transfer coefficient used in practice? → **Nucleate boiling**.
- Point of minimum heat flux on the boiling curve? → **Leidenfrost point**.
- Dropwise vs filmwise condensation – which gives higher h? → **Dropwise**.
- Effect of non-condensable gases on condensation? → **Reduce h sharply**.
- In Nusselt's theory, local h varies with distance x as? → $\mathbf{x^{-1/4}}$.

8.6 Radiation ★★★

- Thermal radiation is electromagnetic energy emitted by matter due to its **temperature**; requires **no medium** (most efficient in vacuum); travels at $c = 3\times10^8$ m/s; $c = \lambda\nu$.
- Radiation is a **volumetric** phenomenon, but for opaque solids it is treated as a **surface** phenomenon.
- Radiation heat transfer $\propto$ **T^4 difference** (absolute temperatures), whereas conduction/convection $\propto \Delta T$.

Band	Wavelength
Gamma rays, X-rays	< 0.01 μm
Thermal radiation	**0.1–100 μm** (includes part of UV, all visible, part of IR)
Ultraviolet	0.01–0.4 μm
Visible light	**0.4–0.76 μm** (violet → red)
Infrared	0.76–100 μm (thermal part); radiation from bodies near room temperature lies in IR
Microwaves	10^2–10^5 μm
Solar radiation	0.3–3 μm (≈ 50% in visible band)

Surface properties

- Incident radiation (irradiation G) is absorbed, reflected or transmitted: **α + ρ + τ = 1**.
- **Opaque** body: τ = 0 → **α + ρ = 1** (most solids, liquids in thick layers). **Black body**: α = 1. **White body** (perfect reflector): ρ = 1. **Transparent (diathermanous)** body: τ = 1.
- **Gases**: ρ ≈ 0, so α + τ = 1. Symmetric molecules (**O_2, N_2, H_2**, dry air) are practically **transparent**; **CO_2, H_2O, CO, SO_2, NH_3, CH_4** absorb and emit in specific **bands** (gas radiation is selective and volumetric).
- **Specular** reflection: angle of incidence = angle of reflection (polished surfaces); **diffuse** reflection: reflected uniformly in all directions (rough surfaces).
- **Black body**: ideal surface that **absorbs all incident radiation** (all λ, all directions), is the **best emitter** at a given T and λ, and is a **diffuse emitter**. Practical approximation: **small hole in a large cavity** (hohlraum), lamp-black (α ≈ 0.95–0.97).
- **Emissivity ε** = E/E_b at the same temperature ($0 \le \varepsilon \le 1$). Depends on material, surface finish, temperature, wavelength, direction.
- **Grey body**: ε (and α) **independent of wavelength** (ε_λ = constant < 1). **Real body**: ε_λ varies with λ (selective emitter).
- **Kirchhoff's law**: at thermal equilibrium, **ε = α** (strictly $\varepsilon_\lambda = \alpha_\lambda$ at the same temperature; total ε = α holds for grey bodies). A good absorber is a good emitter.
- Total emissivity of metals **increases with temperature**; for most non-metals it decreases. Oxidation and roughness **increase** ε of metals.

Surface	Emissivity ε (approx.)
Polished silver	**≈ 0.02**
Polished aluminium / gold / copper	≈ 0.03–0.05
Oxidised / anodised aluminium	≈ 0.2–0.8 (anodised high)
Polished stainless steel	≈ 0.1–0.2
Oxidised iron / rusted steel	≈ 0.6–0.8
Red brick, concrete	≈ 0.9–0.93
White paint (in IR)	≈ 0.9
Water, ice, human skin	**≈ 0.95–0.97**
Snow	≈ 0.8–0.97 (high IR emissivity, though white in visible)
Lamp black / black paint	**≈ 0.95–0.98**

- White paint: low α for solar (short-wave) but high ε in IR – keeps roofs cool. **Selective surfaces** for solar collectors (black chrome, black nickel): **high α_{solar}, low ε_{IR}**.
- **Greenhouse effect**: glass is transparent to short-wave solar radiation (≈ 0.3–2.5 μm) but nearly opaque to long-wave IR (> ≈ 3 μm) emitted by objects inside.

Black-body laws

KEY FORMULAE – RADIATION LAWS

- **Stefan–Boltzmann law**: $E_b = \sigma T^4$, $\sigma = 5.67 \times 10^{-8}$ **W/m²K⁴**; real (grey) body $E = \varepsilon\sigma T^4$
- **Planck's law**: $E_{b\lambda} = C_1/\{\lambda^5[\exp(C_2/\lambda T) - 1]\}$; $C_1 = 3.742 \times 10^8$ W·μm⁴/m², $C_2 = 1.4388 \times 10^4$ μm·K
- **Wien's displacement law**: $\lambda_{max}T = 2898$ **μm·K** (≈ 2897.8 μm·K)
- Emissive power and intensity of a diffuse surface: **E = πI** (Lambert's cosine law: I(θ) direction-independent, emitted flux $\propto \cos\theta$)
- Radiation heat transfer coefficient: $h_r = \varepsilon\sigma(T_1 + T_2)(T_1^2 + T_2^2) \approx 4\varepsilon\sigma T_m^3$

- Planck's law reduces to **Wien's distribution law** for short λ (small λT) and to the **Rayleigh–Jeans law** for long λ (large λT; "ultraviolet catastrophe" if extended to short λ).
- Stefan–Boltzmann law = integration of Planck's law over all wavelengths (area under the curve). Stefan found it experimentally (1879), Boltzmann derived it thermodynamically (1884).
- As T rises, λ_{max} shifts to **shorter wavelengths** (colour changes dull red → white) and the curve rises at every λ (curves never cross).

Nandyal Banking Coaching Center

- Sun ≈ **5800 K** (effective ≈ 5762–5780 K) → λ_{max} ≈ **0.5 µm** (visible, green-yellow). Body at 300 K → λ_{max} ≈ **9.66 µm** (IR).
- Doubling absolute temperature increases E_b by 2^4 **= 16 times**.
- Fraction of radiation emitted in the band 0–λ, $F_{0-\lambda T}$, is a function of λT only (black-body radiation functions). ≈ 25% of energy lies below λ_{max}.

View factor (shape / configuration factor)

- F_{12} = fraction of radiation leaving surface 1 that **directly strikes** surface 2. Purely geometric (depends on size, shape, orientation, distance – not on temperature or emissivity). 0 ≤ F ≤ 1.
- **Reciprocity rule**: $A_1F_{12} = A_2F_{21}$.
- **Summation rule** (enclosure of N surfaces): $\Sigma_j F_{ij} = 1$.
- $F_{ii} = 0$ for flat or **convex** surfaces; $F_{ii} > 0$ for **concave** surfaces.
- Superposition rule: $F_{1\to(2+3)} = F_{12} + F_{13}$. Symmetry rule. Crossed-strings method (Hottel) for 2-D geometries.
- An N-surface enclosure needs N^2 view factors; only **N(N−1)/2** need be found directly.

Configuration	View factors
Small/inner convex body 1 fully enclosed by surface 2 (concentric spheres/long cylinders)	$F_{12} = 1$; $F_{21} = A_1/A_2$; $F_{22} = 1 - A_1/A_2$
Infinite parallel plates	$F_{12} = F_{21} = 1$
Flat base (1) covered by hemisphere (2)	$F_{12} = 1$; $F_{21} = 0.5$ ($A_1/A_2 = \pi r^2/2\pi r^2$); $F_{22} = 0.5$
Inside a cube – one face to any other face	**1/5 = 0.2**
Sphere to itself / plane to itself	0 (convex/flat); inner surface of a hollow sphere to itself = 1
Long duct of equilateral-triangle section – one side to another	0.5

Radiation exchange between grey surfaces

- **Radiosity J** = total radiation leaving a surface (emitted + reflected) = $\varepsilon E_b + \rho G$. Irradiation G = radiation incident per unit area.
- Network method (Oppenheim): **surface resistance (1 − ε)/(εA)** and **space resistance $1/(A_1F_{12})$**. For a black surface surface resistance = 0 (J = E_b).
- **Re-radiating (adiabatic / refractory) surface**: net radiation = 0, J = E_b; temperature independent of its emissivity (furnace walls).

KEY FORMULAE – GREY-BODY EXCHANGE

- Two-surface enclosure: $Q_{12} = \sigma(T_1^4 - T_2^4) / [(1-\varepsilon_1)/(\varepsilon_1 A_1) + 1/(A_1F_{12}) + (1-\varepsilon_2)/(\varepsilon_2 A_2)]$
- Infinite parallel plates: $\mathbf{q = \sigma(T_1^4 - T_2^4) / (1/\varepsilon_1 + 1/\varepsilon_2 - 1)}$
- Concentric cylinders / spheres (1 = inner): $\mathbf{Q = \sigma A_1(T_1^4 - T_2^4) / [1/\varepsilon_1 + (A_1/A_2)(1/\varepsilon_2 - 1)]}$; $A_1/A_2 = r_1/r_2$ (cylinders), $(r_1/r_2)^2$ (spheres)
- Small body in a large enclosure ($A_1/A_2 \to 0$): $\mathbf{Q = \varepsilon_1\sigma A_1(T_1^4 - T_2^4)}$ (independent of ε_2)
- Black surfaces: $Q_{12} = A_1F_{12}\sigma(T_1^4 - T_2^4)$

- Effective emissivity of two parallel plates with $\varepsilon_1 = \varepsilon_2 = \varepsilon$: $\varepsilon_{eff} = \varepsilon/(2 - \varepsilon)$. For ε = 0.5 → **1/3**.

- **Radiation shields**: thin, highly reflective (low ε) sheets placed between surfaces – add two surface resistances each, no conduction. With **n shields** and all emissivities equal: $\mathbf{Q_{with\ n\ shields} = Q_{without}/(n + 1)}$. One shield halves the heat flow.
- Shields of low ε (polished aluminium) are far more effective; used in cryogenic vessels (multilayer insulation), thermos flasks (silvered walls + vacuum), and around thermocouples to reduce radiation error in gas temperature measurement.
- Thermos flask: vacuum stops conduction & convection; silvering (low ε) reduces radiation.

Solar radiation basics

- **Solar constant** = solar flux on a surface normal to the sun's rays at the mean earth–sun distance, outside the atmosphere ≈ **1361–1367 W/m²** (older value 1353 W/m²).
- Reaching the earth's surface on a clear day ≈ **1000 W/m²** (attenuated by ozone (UV), H_2O and CO_2 (IR), scattering).
- Components: **beam (direct)** + **diffuse** = global radiation. Measured by **pyrheliometer** (beam) and **pyranometer** (global); sunshine duration by Campbell–Stokes recorder.
- Air mass m = $1/\cos\theta_z$ (θ_z = zenith angle); AM 1.5 is the standard for solar cell testing.
- Sky behaves as a black body at an effective sky temperature ≈ 230–285 K – allows night-time radiative cooling and frost formation even when air is above 0 °C.

MEMORY TRICK

"**Wien 2898 – hotter is shorter**". Shields: "**n shields, (n+1) share**" – heat flow becomes 1/(n+1). Kirchhoff: "**Good absorber = good emitter**" (ε = α).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Value of Stefan–Boltzmann constant? → **5.67×10^{-8} W/m²K⁴**.
- A grey body is one whose emissivity is? → **Independent of wavelength**.
- Wavelength of maximum emission from the sun (≈ 5800 K)? → **≈ 0.5 µm**.
- View factor from a flat base to its covering hemisphere? → **1** (and hemisphere to base = 0.5).
- Heat exchange between two plates reduces by how much with one shield of the same ε? → **To half**.
- For an opaque surface? → **α + ρ = 1**.

8.7 Heat Exchangers ★★★

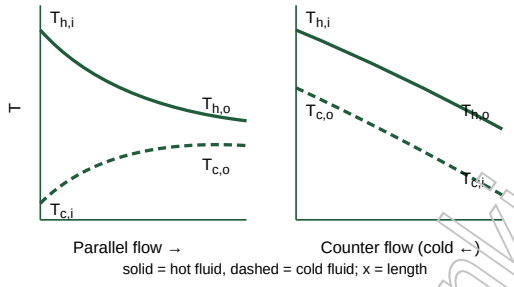
- **Heat exchanger**: device transferring heat between two fluids at different temperatures. Classified by nature of heat transfer, flow arrangement, construction, compactness.

Basis	Types / examples
Nature of heat exchange	**Recuperator** (direct transfer, fluids separated by a wall, steady – shell & tube, car radiator, economiser); **Regenerator** (same matrix alternately heated by hot and cooled by cold fluid – **Ljungström rotary air preheater**, Cowper stoves of blast furnace, glass-furnace checkers); **Direct contact** (fluids mix – **cooling tower**, jet condenser, desuperheater, deaerator)
Flow arrangement	**Parallel flow** (co-current), **counter flow**, **cross flow** (mixed / unmixed), multipass (1-2, 2-4 shell & tube)

Nandyal Banking Coaching Center

Construction	Double-pipe (tube-in-tube), **shell & tube** (most common in industry; TEMA standards; baffles increase shell-side h), **plate** (gasketed – high h, easy cleaning, dairy/food), plate-fin, tube-fin
Compactness	**Compact heat exchanger**: area density β ≥ **700 m²/m³** (gas side) – car radiator ≈ 1000, glass-ceramic gas turbine HX ≈ 6000, human lungs ≈ 20,000 m²/m³
Phase change	Condensers, evaporators, boilers

- Regenerators: **static** (fixed matrix, periodic valve switching) or **rotary/dynamic** (rotating matrix). Some mixing/carry-over of fluids possible.
- **Counter-flow** is the most efficient arrangement – for the same terminal temperatures it gives the **highest LMTD → smallest area**. Cold-fluid outlet can **exceed** hot-fluid outlet temperature (temperature cross possible). In parallel flow $T_{c,out}$ always < $T_{h,out}$.
- **Parallel-flow** advantage: more uniform wall temperature; used where the cold fluid must not overheat or wall temperature must be limited.
- **Capacity rate C = $\dot{m}c_p$** (W/K). The fluid with smaller C undergoes the larger temperature change. $Q = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i})$.
- Counter-flow with $C_h = C_c \rightarrow \Delta T$ **constant** along length; temperature profiles are **parallel straight lines**; LMTD = $\Delta T_1 = \Delta T_2$.



Temperature profiles: in parallel flow $T_{c,o} < T_{h,o}$; in counter flow $T_{c,o}$ may exceed $T_{h,o}$.

KEY FORMULAE – LMTD METHOD

- $Q = U A \Delta T_{lm}$ (× F for multipass/cross flow)
- **LMTD = $(\Delta T_1 - \Delta T_2)/\ln(\Delta T_1/\Delta T_2)$**
- Parallel flow: $\Delta T_1 = T_{h,i} - T_{c,i}$, $\Delta T_2 = T_{h,o} - T_{c,o}$
- Counter flow: $\Delta T_1 = T_{h,i} - T_{c,o}$, $\Delta T_2 = T_{h,o} - T_{c,i}$
- If $\Delta T_1 = \Delta T_2$, LMTD = ΔT_1 (limit of the formula)
- Overall U (tube, based on outer area): $1/U_o = 1/h_o + R_{fo} + r_o\ln(r_o/r_i)/k + (r_o/r_i)R_{fi} + (r_o/r_i)(1/h_i)$

- LMTD < arithmetic mean temperature difference (AMTD); they are nearly equal (error ≈ 1%) when $\Delta T_1/\Delta T_2 <$ ≈ 1.4–1.5 (≈ 4% error at a ratio of 2).
- Assumptions in LMTD derivation: steady flow, constant U and specific heats, no heat loss to surroundings, negligible axial conduction, negligible KE/PE changes.
- **Correction factor F** (for multipass and cross-flow): $Q = UA{\cdot}F{\cdot}LMTD_{counter\ flow}$. **F ≤ 1**; F = 1 for pure counter flow and whenever one fluid **changes phase** (R or P = 0). Design with **F < 0.75** is considered uneconomical. F is read from charts vs $P = (t_2 - t_1)/(T_1 - t_1)$ and $R = (T_1 - T_2)/(t_2 - t_1)$.
- **Condensers & evaporators**: one fluid stays at constant temperature (phase change) → **LMTD is the same for parallel and counter flow**; $C_{max} \rightarrow \infty$, so **$C = C_{min}/C_{max} = 0$**.
- LMTD method is convenient when **all inlet and outlet temperatures are known** (sizing problem). ε–NTU method is convenient when **only inlet temperatures** are known (rating problem) – avoids iteration.

Effectiveness–NTU method

KEY FORMULAE – E–NTU

- **$\varepsilon = Q_{actual}/Q_{max}$; $Q_{max} = C_{min}(T_{h,i} - T_{c,i})$**
- **NTU = UA/C_{min}** (number of transfer units – a measure of heat-exchanger size); capacity ratio $C = C_{min}/C_{max}$ ($0 \le C \le 1$)
- Parallel flow: $\varepsilon = [1 - \exp\{-NTU(1 + C)\}]/(1 + C)$
- Counter flow: $\varepsilon = [1 - \exp\{-NTU(1 - C)\}]/[1 - C\exp\{-NTU(1 - C)\}]$
- Counter flow, C = 1: **$\varepsilon = NTU/(1 + NTU)$**
- **Any type, C = 0 (condenser/evaporator/boiler): $\varepsilon = 1 - e^{-NTU}$**

- Maximum possible heat transfer would occur in an **infinitely long counter-flow** exchanger; the fluid with C_{min} undergoes the maximum temperature change ($T_{h,i} - T_{c,i}$).
- As NTU → ∞: counter flow ε → 1 (for C < 1); parallel flow ε → **1/(1 + C)** → for C = 1, max ε = **0.5**.
- ε increases with NTU but at a diminishing rate – beyond NTU ≈ 3 gains are small (uneconomical). For C = 0, NTU = 1 gives ε = 0.632; NTU = 2 → 0.865; NTU = 3 → 0.95.
- For a given NTU and C, **counter flow has the highest ε**, parallel flow the lowest; cross-flow lies between. When C = 0 all arrangements have the **same ε**.
- Effectiveness is dimensionless, $0 \le \varepsilon \le 1$.

Fouling

- **Fouling**: deposition of scale, rust, sludge, biological growth, soot on heat-transfer surfaces over time → adds thermal resistance and increases pressure drop. Accounted for by **fouling factor R_f** (m^2K/W): $1/U_{dirty} = 1/U_{clean} + R_f$.
- Fouling factor increases with time and temperature, decreases with fluid velocity. Leads to **over-design** (extra area).

Fluid	Typical R_f (m^2K/W)
Distilled water, seawater (< 50 °C)	≈ 0.0001
Treated boiler feed water	≈ 0.0002
River water	≈ 0.0002–0.001
Fuel oil	≈ 0.0009
Steam (oil-free)	≈ 0.0001
Refrigerant liquids	≈ 0.0002
Quenching oil	≈ 0.0007

- Typical overall U: water-to-water 850–1700; water-to-oil 100–350; steam condenser (water in tubes) 1000–6000; gas-to-gas 10–40; finned-tube (air side) 25–50 W/m²K.
- U is controlled by the **larger resistance (lower h)** – improve the poorer side first (e.g. add fins on gas side).
- In a shell-and-tube unit, the fluid that is more **corrosive, fouling, at higher pressure** or more expensive is usually put on the **tube side** (easier cleaning, cheaper high-pressure containment).

Nandyal Banking Coaching Center

MEMORY TRICK

"**Counter Conquers**" – counter flow gives highest LMTD and highest ε. "**Phase change → C = 0 → ε = 1 − e^{-NTU}**, and flow direction no longer matters."

PREVIOUSLY ASKED / LIKELY QUESTIONS

- For the same inlet/outlet temperatures, which arrangement needs least area? → **Counter flow**.
- Effectiveness of a heat exchanger when one fluid condenses (C = 0)? → **1 − e^{-NTU}**.
- NTU is defined as? → **UA/C_{min}**.
- Rotary air preheater of a boiler (Ljungström) is an example of? → **Regenerator**.
- Maximum effectiveness of a parallel-flow exchanger with equal capacity rates? → **0.5**.
- Cooling tower is which type of heat exchanger? → **Direct contact**.

Nandyal Banking Coaching Center

9. Engineering Materials & Heat Treatment

NOTE

Materials is a "sure-shot" scoring area: 4–6 MCQs are usually direct facts. Expect questions on **unit cells (APF, coordination number)**, **Fe–Fe_3C invariant reactions**, **TTT diagram and heat-treatment products**, **hardness tests**, **effect of alloying elements**, **stainless steels** and **alloy compositions (brass, bronze, duralumin, Babbitt)**. Learn the numbers in the tables exactly – they change little between textbooks, and where they do, both values are given.

9.1 Crystal Structure & Imperfections ★★★

- **Crystalline solid** = atoms in a regular, repeating 3-D arrangement (**long-range order**). **Amorphous** (glass, most polymers) = only short-range order; no sharp melting point.
- **Unit cell** = smallest repeating unit; described by **lattice parameters a, b, c and α, β, γ**.
- There are **7 crystal systems** and **14 Bravais lattices**. Cubic: a = b = c, α = β = γ = 90°. Tetragonal: a = b ≠ c, all 90°. Hexagonal: a = b ≠ c, α = β = 90°, γ = 120°. Triclinic: a ≠ b ≠ c, α ≠ β ≠ γ ≠ 90° (least symmetric).
- **Coordination number (CN)** = number of nearest neighbouring atoms touching an atom.
- **Atomic packing factor (APF)** = volume of atoms in the cell ÷ volume of the cell.
- **Polymorphism / allotropy** = the same element existing in more than one crystal structure. Iron: **α (BCC) → γ (FCC) at 912 °C → δ (BCC) at 1394 °C → melts at 1538 °C**. Others: Ti (HCP α ↔ BCC β at 882 °C), Co, Sn (white ↔ grey tin), carbon (graphite/diamond).

Structure	Atoms / cell	CN	APF	Relation a – r	Examples
Simple cubic (SC)	**1**	**6**	**0.52**	a = 2r	Polonium (only element)
Body-centred cubic (BCC)	**2**	**8**	**0.68**	a = 4r/√3	**α-Fe, δ-Fe, Cr, W, Mo, V, Na, K, β-Ti**
Face-centred cubic (FCC)	**4**	**12**	**0.74**	a = 2√2 r	**γ-Fe, Al, Cu, Ni, Au, Ag, Pb, Pt**
Hexagonal close-packed (HCP)	**6**	**12**	**0.74**	a = 2r; ideal c/a = **1.633**	**Zn, Mg, Ti (α), Cd, Co, Be, Zr**

- FCC and HCP are both **close-packed** (APF 0.74 – the maximum for equal spheres). Stacking: **FCC = ABCABC…**, **HCP = ABAB…**
- Close-packed directions: **BCC – body diagonal <111>**; **FCC – face diagonal <110>**. Close-packed plane in FCC = **{111}**; in HCP = **basal plane (0001)**.
- Voids in FCC: **4 octahedral + 8 tetrahedral** per unit cell. Carbon sits in octahedral voids of γ-Fe – the FCC octahedral void is larger than any BCC void, so **austenite dissolves far more carbon than ferrite**.
- Atom sharing: corner atom = 1/8, face atom = 1/2, edge atom = 1/4, body-centre atom = 1.
- Diamond cubic (C, Si, Ge): 8 atoms/cell, CN 4, APF **0.34**. NaCl structure: CN 6.

KEY FORMULAE

- Theoretical density: $\rho = n A / (V_c N_A)$; n = atoms per cell, A = atomic mass, $V_c = a^3$ (cubic), $N_A = 6.023 \times 10^{23}$
- Interplanar spacing (cubic): $d_{hkl} = a / \sqrt{(h^2 + k^2 + l^2)}$
- Bragg's law (X-ray diffraction): $n\lambda = 2d \sin\theta$
- Equilibrium vacancies: $n_v = N \exp(-Q_v/kT)$ – increase exponentially with temperature

Miller indices

- Procedure for a plane: take **intercepts** on x, y, z (in units of a, b, c) → take **reciprocals** → clear fractions to smallest integers → write as **(hkl)**. A plane parallel to an axis has intercept ∞ → index 0.
- Notation: **(hkl)** one plane, **{hki}** family of planes, **[uvw]** one direction, **<uvw>** family of directions. Negative index shown with a bar over the number.
- In cubic crystals the direction [hkl] is **perpendicular** to plane (hkl).
- Hexagonal crystals use 4-index **Miller–Bravais** indices (hkil) with **i = –(h + k)**.
- Example: intercepts 1, 2, ∞ → reciprocals 1, ½, 0 → **(210)**. Cube faces = {100}; face diagonals planes = {110}; octahedral planes = {111}.

Slip systems

Structure	Slip plane	Slip direction	No. of slip systems	Ductility
FCC	{111}	<110>	**12** (4 planes × 3 directions)	**Highly ductile** even at low temperature (Cu, Al, Au)
BCC	{110} (also {211}, {321})	<111>	12 on {110}; up to **48** in all	Moderately ductile; no truly close-packed plane → needs higher stress; **ductile–brittle transition**
HCP	Basal (0001)	<11$\bar{2}$0>	**3**	**Least ductile** (brittle at room temperature – Zn, Mg)

- **Slip system** = slip plane + slip direction; slip occurs on the **most densely packed plane** along the **most densely packed direction**.
- **Von Mises criterion**: a polycrystal needs at least **5 independent slip systems** for uniform plastic deformation.
- **Schmid's law**: resolved shear stress $\tau_R = \sigma \cos\varphi \cos\lambda$; maximum Schmid factor = **0.5** ($\varphi = \lambda = 45°$). Slip starts when τ_R = critical resolved shear stress (CRSS).
- **Twinning** = a part of the crystal becomes a mirror image across a twin plane; common in **HCP and BCC** at low temperature / high strain rate; small strains only.

Crystal imperfections

Class	Dimension	Examples
Point defects	0-D	Vacancy, self-interstitial, substitutional / interstitial impurity, **Frenkel**, **Schottky**
Line defects	1-D	**Edge**, **screw** and mixed dislocations
Surface (planar) defects	2-D	**Grain boundaries**, twin boundaries, tilt & twist boundaries, **stacking faults**, free surface, phase boundaries

Nandyal Banking Coaching Center

Volume defects	3-D	Voids, pores, cracks, inclusions, precipitates

- **Schottky defect** = a **pair of cation + anion vacancies** (maintains charge neutrality) – in ionic crystals with similar ion sizes (NaCl, KCl); **density decreases**.
- **Frenkel defect** = an ion (usually the smaller cation) **leaves its site and moves to an interstitial position** → vacancy–interstitial pair; **density unchanged** (AgCl, AgBr, ZnS).
- **Burgers vector (b)** = closure failure of a Burgers circuit drawn round a dislocation; gives the **magnitude and direction of lattice distortion**. |b| = **a/√2 in FCC** (a/2<110>), **a√3/2 in BCC** (a/2<111>).
- Strain energy of a dislocation ∝ **G b²** → dislocations prefer the shortest b (close-packed direction).
- Dislocations were proposed (1934) by **Taylor, Orowan and Polanyi** to explain why real metals yield at ~1/1000 of theoretical shear strength (≈ G/10–G/30).
- Dislocation density: annealed metal ~10^{10}–10^{12} m^{-2}; heavily cold-worked ~10^{15}–10^{16} m^{-2}.

Feature	Edge dislocation	Screw dislocation
Description	Extra half-plane of atoms inserted (symbol ⊥)	Spiral / helical ramp of atomic planes
Burgers vector vs dislocation line	**Perpendicular**	**Parallel**
Motion of line vs b	Parallel to b	Perpendicular to b
Climb (needs diffusion)	**Possible**	Not possible
Cross-slip	Not possible	**Possible**

Strengthening mechanisms (all work by impeding dislocation motion)

Mechanism	How it works	Example
Grain refinement	Grain boundaries block slip; finer grain → higher yield strength **and** better toughness (the only mechanism that raises both)	Normalising, controlled rolling, Al/Nb/V/Ti additions
Solid-solution strengthening	Solute atoms distort the lattice and pin dislocations	Zn in Cu (brass), Ni in Cu, C in Fe (interstitial)
Strain (work) hardening	Cold work multiplies dislocations which tangle	Cold-drawn wire, cold-rolled sheet
Precipitation (age) hardening	Fine coherent precipitates from a supersaturated solid solution	Duralumin, Be–Cu, maraging steel, 17-4 PH SS
Dispersion strengthening	Hard, insoluble, stable particles (oxides) – retain strength at high temperature	SAP (Al + Al_2O_3), TD-nickel (Ni + ThO_2), ODS alloys
Martensitic transformation	Supersaturated C distorts lattice to BCT	Hardened steel

KEY FORMULAE

- **Hall–Petch**: $\sigma_y = \sigma_0 + k_y d^{-1/2}$ (d = average grain diameter)
- Strain hardening (Hollomon): $\sigma_T = K \varepsilon_T^n$; n = strain-hardening exponent (≈ 0.1–0.5); necking begins when $\varepsilon_T = n$
- ASTM grain size: $N = 2^{n-1}$ (N = grains per in^2 at 100×); larger n → finer grain

- **Recovery → Recrystallisation → Grain growth** = the three stages of annealing a cold-worked metal. Recovery relieves internal stresses (little change in hardness); recrystallisation forms new strain-free grains (sharp fall in strength, rise in ductility); grain growth coarsens grains.
- **Recrystallisation temperature ≈ 0.3–0.5 T_m** (K), commonly taken as **0.4 T_m**. It is lower for heavier prior cold work, finer grain and purer metal. Pb, Sn, Zn recrystallise near room temperature → working them at room temperature is **hot working**.
- **Hot working** = working above recrystallisation temperature (no strain hardening); **cold working** = below it (strain hardening, better finish and tolerance).

MEMORY TRICK

"**1-2-4-6 / 6-8-12-12 / 52-68-74-74**" for SC-BCC-FCC-HCP (atoms, CN, APF). FCC metals are "**CAN-GAP**": **C**u, **A**l, **N**i, **G**old, **A**g, **P**b (+ γ-Fe) – all ductile. HCP metals "**Z-M-T-C**": Zinc, Magnesium, Titanium, Cadmium (+ Co, Be) – limited ductility.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- APF of BCC structure? → **0.68**; FCC/HCP → **0.74**.
- Coordination number of FCC? → **12**; BCC → **8**.
- Number of atoms per unit cell in HCP? → **6**.
- Burgers vector is parallel to the dislocation line in? → **Screw dislocation**.
- Defect in which a cation–anion vacancy pair forms? → **Schottky defect**.
- Yield strength ∝ $d^{-1/2}$ is called? → **Hall–Petch relation**.

9.2 Mechanical Properties & Testing ★★★

Property	Definition / measure
Strength	Ability to resist load without failure (yield, UTS)
Elasticity	Regains original shape on unloading
Plasticity	Permanent deformation without fracture (basis of forming)
Ductility	Can be drawn into wire under **tension**; measured by **% elongation** and **% reduction in area**. Most ductile: **gold** (then Ag, Pt, Fe, Cu…)
Malleability	Can be hammered/rolled into sheet under **compression**. Most malleable: **gold** (then Ag, Al, Cu, Sn, Pb)
Toughness	Energy absorbed up to **fracture** = total area under σ–ε curve; measured by impact test
Resilience	Energy absorbed up to **elastic limit**; modulus of resilience = $\sigma_y^2/2E$ (springs need high resilience)
Hardness	Resistance to indentation, scratching, abrasion
Brittleness	Fracture with little or no plastic deformation (cast iron, glass); elongation < 5 %
Stiffness	Resistance to elastic deformation; measured by **E**

Nandyal Banking Coaching Center

Fatigue	Failure under repeated/fluctuating stress below UTS
Creep	Slow time-dependent deformation under constant load at elevated temperature

Tensile test

- Points on mild-steel curve in order: **proportional limit → elastic limit → upper yield point → lower yield point → UTS (necking starts) → fracture**.
- **Upper and lower yield points** (yield-point phenomenon) are seen in **low-carbon (mild) steel**, caused by C/N atoms pinning dislocations (Cottrell atmospheres); accompanied by **Lüders bands** (stretcher strains).
- Materials without a clear yield point (Al, Cu, high-strength steel) use **0.2 % offset proof stress**.
- Necking starts at **UTS** (maximum load); beyond UTS the engineering stress falls but the true stress keeps rising.
- Standard gauge length (proportional specimen): $L_0 = 5.65\sqrt{A_0}$ (≈ 5d) as per IS 1608.
- Typical E: steel ≈ **200–210 GPa**, cast iron ≈ 100–130 GPa, copper ≈ 110–120 GPa, aluminium ≈ **70 GPa**, titanium ≈ 110 GPa, tungsten ≈ 400 GPa. Poisson's ratio: metals 0.25–0.35; rubber ≈ **0.5**; cork ≈ **0**.

KEY FORMULAE

- True stress $\sigma_T = \sigma(1 + \varepsilon)$; true strain $\varepsilon_T = \ln(1 + \varepsilon) = \ln(A_0/A)$ (valid up to necking)
- % elongation = $(L_f - L_0)/L_0 \times 100$; % reduction in area = $(A_0 - A_f)/A_0 \times 100$
- Approx. UTS of steel (MPa) ≈ **3.45 × BHN**

Hardness tests

Test	Indenter	Load	Measured	Remarks
Brinell (HB)	Hardened steel / tungsten-carbide ball, **10 mm** dia	**3000 kgf** (steel, CI); 1500 kgf; **500 kgf** (soft – Cu, Al); held 10–15 s	Diameter of impression (load ÷ curved area)	Large impression – good for **coarse/heterogeneous** materials (castings); not for very hard or thin parts. $P/D^2 = 30$ (steel), 10 (Cu), 5 (Al)
Rockwell (HR)	**B scale: 1/16 in (1.588 mm) steel ball; C scale: 120° diamond cone (Brale)**	Minor load **10 kgf** + major: **B = 100 kgf** total, **C = 150 kgf** total, A = 60 kgf (cone)	**Depth** of penetration (direct dial reading)	Fastest; used for production. HRC for **hardened steel**; HRB for soft steel, brass; HRA for carbides
Vickers (HV)	Square-based diamond pyramid, **136°** between opposite faces	1–120 kgf	Mean diagonal of square impression	Same scale for soft to very hard; HV ≈ HB up to ~300; geometrically similar impressions (load-independent)
Knoop (HK)	Elongated (rhombic) diamond pyramid; diagonals **7 : 1** (172.5° & 130°)	Very light (micro), < 1 kgf	Long diagonal	**Microhardness** of thin layers, coatings, case depth, brittle materials (glass, ceramics)
Shore scleroscope	Diamond-tipped hammer dropped from fixed height	Free fall	**Height of rebound**	**Dynamic** test; portable; large rolls. (Shore **durometer** – indentation test for rubber/plastics)
Mohs scale	Scratch with standard minerals	—	Which mineral scratches	**Scratch hardness**, 1 = talc … 10 = diamond (minerals, ceramics)

KEY FORMULAE

- BHN = $2P / [\pi D(D - \sqrt{(D^2 - d^2)})]$ (P in kgf, D and d in mm); impression d should be 0.24D–0.6D
- HV = $1.854\ P/d^2$ (d = mean diagonal, mm)
- HK = $14.2\ P/L^2$ (L = long diagonal)
- Meyer hardness = P / projected area = $4P/\pi d^2$

MEMORY TRICK

Mohs scale 1→10: "**T**he **G**irls **C**an **F**lirt **A**nd **O**ther **Q**uite **T**hings **C**an **D**o" = Talc, Gypsum, Calcite, Fluorite, Apatite, Orthoclase, Quartz, Topaz, Corundum, Diamond. Rockwell "**B**all-**B**rass, **C**one-**C**ase-hardened".

Impact test

Feature	Charpy	Izod
Specimen support	**Simply supported beam** (horizontal)	**Cantilever** (vertical, clamped)
Specimen size	10 × 10 × **55 mm**	10 × 10 × **75 mm**
Notch	V-notch 2 mm deep, 45°, root radius 0.25 mm (also U/keyhole)	V-notch 2 mm deep, 45°
Striker hits	Middle, on face **opposite** the notch	Free end, on the **same face** as the notch (22 mm above notch)
Result	Energy absorbed (J) = $W(h_1 - h_2) = WR(\cos\beta - \cos\alpha)$ – a measure of **notch toughness**	

- **Ductile–brittle transition temperature (DBTT)**: impact energy drops sharply below a temperature. Shown by **BCC metals (ferritic steels)** and HCP; **FCC metals (Al, Cu, austenitic SS) show no DBTT** → used for cryogenic service.

Nandyal Banking Coaching Center

- DBTT is raised by higher C, P, coarse grains, irradiation; lowered by **Ni, Mn, fine grain size**. Classic failures: **Liberty ships** (WW-II), Titanic hull plates.

Fatigue

- Fatigue failure = crack initiation (usually at surface / stress raiser) → slow propagation (**beach marks** visible to eye; **striations** under SEM) → sudden final fracture. Responsible for **~90 %** of service failures of machine parts.
- **S–N (Wöhler) curve**: stress vs log N. Ferrous metals show a flat **endurance limit** at about **10^6–10^7 cycles**; endurance limit of steel ≈ **0.5 σ_u**. **Non-ferrous (Al, Cu, Mg) have no true endurance limit** → fatigue strength quoted at 10^8 or 5×10^8 cycles.
- Fatigue life improved by: **shot peening** (compressive residual stress), surface rolling, polishing, carburising/nitriding, removing notches. Reduced by: surface roughness, decarburisation, corrosion, tensile residual stress, notches.
- Mean-stress lines: **Soderberg** ($\sigma_a/\sigma_e + \sigma_m/\sigma_y = 1/N$, most conservative), **Goodman** (σ_u instead of σ_y), **Gerber** (parabola – least conservative).

Creep

- Significant above about **0.4 T_m** (homologous temperature, K) – e.g. turbine blades, boiler tubes, steam pipes. Lead creeps at room temperature.
- **Primary (transient) creep** – decreasing creep rate (strain hardening dominates).
- **Secondary (steady-state) creep** – **constant, minimum creep rate**; balance between strain hardening and recovery; **longest stage**; used for **design**.
- **Tertiary creep** – accelerating rate; necking, grain-boundary voids, cracks → **rupture**.
- Creep resistance improved by: **coarse grains** (fewer grain boundaries – opposite of low-temperature strength), **single-crystal** turbine blades, high melting point, solid-solution and dispersed precipitates; Mo, W, Cr, V additions; Ni-base superalloys.
- **Stress relaxation** = fall in stress at constant strain with time (bolts, springs at high temperature).

KEY FORMULAE

- **Larson–Miller parameter**: $LMP = T\,(C + \log t_r)$; T in K, t_r = rupture time in hours, C ≈ **20**
- Steady-state creep: $\dot{\varepsilon}_s = K\,\sigma^n \exp(-Q_c/RT)$
- Griffith: $\sigma_f = \sqrt{(2E\gamma_s/\pi a)}$; fracture toughness $K_{IC} = Y\sigma\sqrt{(\pi a)}$, unit **MPa√m**

Fracture

Feature	Ductile fracture	Brittle fracture
Plastic deformation	Large, with necking	Little or none
Crack growth	Slow, stable, needs energy	Rapid, unstable
Appearance	**Cup-and-cone**, dull fibrous; **dimples** (microvoid coalescence) under SEM	Flat, bright granular; **chevron / river marks**; cleavage facets
Path	Shear at ~45° on outer lip	**Transgranular (cleavage)** or **intergranular** (along grain boundaries)
Example	Mild steel, Al, Cu in tension	Cast iron, glass, steel below DBTT; CI in torsion fails on a **45° helical** plane

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Angle between opposite faces of Vickers indenter? → **136°**.
- Rockwell C scale uses? → **120° diamond cone, 150 kgf total load**.
- Hardness test based on height of rebound? → **Shore scleroscope**.
- In Izod test the specimen is held as a? → **Cantilever**; Charpy → **simply supported beam**.
- Stage of creep used for design? → **Secondary (steady-state)**.
- Fracture surface showing beach marks indicates? → **Fatigue failure**.

9.3 Phase Diagrams & Iron–Carbon Diagram ★★★

- **Phase** = physically distinct, homogeneous portion of a system with uniform structure and composition. **Component** = chemically independent constituent (Fe and C, or Fe and Fe_3C).
- **Solid solution**: substitutional (similar atom size – Cu–Ni, Cu–Zn) or interstitial (small atoms C, N, H, B in Fe).
- **Hume-Rothery rules** for complete substitutional solubility: atomic size difference **< 15 %**, **same crystal structure**, similar electronegativity, same valency. Cu–Ni satisfies all → **isomorphous** (complete solubility in liquid and solid).
- **Liquidus** = above it all liquid; **solidus** = below it all solid; **solvus** = limit of solid solubility.
- Phase diagrams are drawn by **cooling curves (thermal analysis)**; a pure metal shows a horizontal arrest; an isomorphous alloy solidifies over a range.

KEY FORMULAE

- **Gibbs phase rule**: F = C − P + 2 ; at constant pressure (condensed systems) **F = C − P + 1**
- Pure substance triple point: F = 1 − 3 + 2 = **0** (invariant). Binary eutectic at constant p: F = 2 − 3 + 1 = **0**
- **Lever rule** (in a two-phase field, alloy C_0, tie-line ends C_α, C_β): $W_\alpha = (C_\beta - C_0)/(C_\beta - C_\alpha)$; $W_\beta = (C_0 - C_\alpha)/(C_\beta - C_\alpha)$ – "opposite arm ÷ total"

Invariant reaction	On cooling	Example
Eutectic	$L \rightarrow \alpha + \beta$	Fe–C (4.3 % C), Pb–Sn (61.9 % Sn, 183 °C), Al–Si (12.6 % Si)
Eutectoid	γ (solid) $\rightarrow \alpha + \beta$	Fe–C (0.76–0.8 % C, 727 °C) → pearlite
Peritectic	$L + \alpha \rightarrow \beta$	Fe–C ($\delta + L \rightarrow \gamma$ at 1495 °C)
Peritectoid	$\alpha + \beta \rightarrow \gamma$ (all solid)	Some Cu, Ni alloys
Monotectic	$L_1 \rightarrow L_2 + \alpha$	Cu–Pb
Syntectic	$L_1 + L_2 \rightarrow \alpha$	Na–Zn

Nandyal Banking Coaching Center

- "**-tectic**" involves a liquid; "**-tectoid**" is entirely in the solid state. "**Eu**" = one phase splits into two; "**Peri**" = two phases combine into one.
- Eutectic alloy has the **lowest melting point** in the system and solidifies at a **single temperature** → used for **solders** and casting alloys.

Iron–Iron carbide (Fe–Fe_3C) diagram

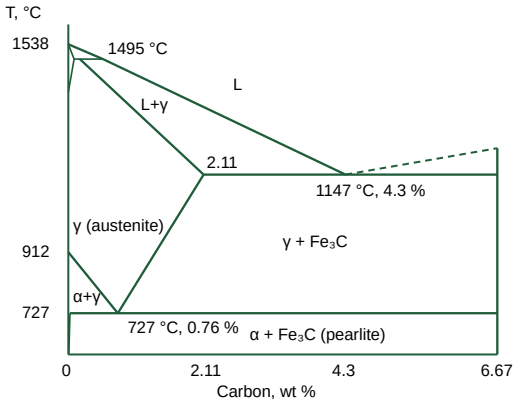


Fig 9.1 Simplified Fe–Fe_3C diagram (not to scale near the δ region). Lines: liquidus, solidus, A_3 (912 → 727 °C), A_{cm} (727 → 1147 °C), A_1 (727 °C).

Phase / constituent	Structure	Max. C solubility	Properties
Ferrite (α)	BCC, interstitial SS of C in α-Fe	**0.022 %** at 727 °C (≈ 0.008 % at room temp.)	Soft, ductile, **magnetic below 768 °C**
Austenite (γ)	FCC	**2.11 %** at 1147 °C (0.76–0.8 % at 727 °C)	Soft, ductile, tough, **non-magnetic**; not stable below 727 °C in plain C steel
δ-ferrite	BCC	**0.09 %** at 1495 °C	Exists 1394–1538 °C
Cementite (Fe_3C)	Orthorhombic intermetallic	**6.67 %** C (fixed)	**Hardest and most brittle** constituent; magnetic below 210 °C (A_0)
Pearlite	Lamellar α + Fe_3C (≈ **88 % ferrite + 12 % cementite**)	Eutectoid, 0.76–0.8 %	"Mother-of-pearl" look; strength + ductility
Ledeburite	γ + Fe_3C eutectic mixture	4.3 %	Below 727 °C its γ → pearlite ("transformed ledeburite")

Invariant reaction	Temperature	% C	Reaction (on cooling)
Peritectic	**1495 °C** (Callister: 1493 °C)	**0.17 %** (texts: 0.16–0.18)	δ (0.09 %) + L (0.53 %) → γ
Eutectic	**1147 °C** (Callister: 1148 °C)	**4.3 %**	L → γ (2.11 %) + Fe_3C = **ledeburite**
Eutectoid	**727 °C** (older: 723 °C)	**0.76 / 0.8 %** (0.77 % in many texts)	γ → α (0.022 %) + Fe_3C = **pearlite**

- **A_1** (lower critical) = **727 °C** eutectoid line. **A_3** (upper critical, hypo-eutectoid) = γ/(α+γ) boundary, falls from **912 °C to 727 °C**. **A_{cm}** = γ/(γ+Fe_3C) boundary for hyper-eutectoid, rises from 727 °C to 1147 °C. **A_2 = 768 °C** = **Curie point** of ferrite (magnetic change, no structural change). Suffix c (heating, "chauffage") and r (cooling, "refroidissement"): $Ac_1 > Ar_1$ due to thermal hysteresis.
- **Steel**: up to **2.11 % C** (commercially ≤ 1.5 %). **Cast iron**: **2.11–6.67 % C** (commercially 2–4.5 %). Wrought iron ≈ 0.02 % C with slag fibres.
- **Hypo-eutectoid steel** (< 0.8 % C) = **pro-eutectoid ferrite + pearlite**. % pearlite ≈ (C − 0.022)/(0.8 − 0.022) × 100 → 0.4 % C steel ≈ **50 % pearlite**.
- **Hyper-eutectoid steel** (0.8–2.11 % C) = **pro-eutectoid cementite (grain-boundary network) + pearlite**.
- **Hypo-eutectic CI** 2.11–4.3 % C; **hyper-eutectic CI** 4.3–6.67 % C.
- The Fe–Fe_3C diagram is a **metastable** diagram; Fe–graphite is the stable one (Si promotes graphite).

Cast irons

Type	Form of carbon	How produced	Properties / uses
White CI	All combined as **cementite**	Rapid cooling, low Si	Very hard, brittle, unmachinable, white fracture; wear parts, raw material for malleable CI
Grey CI	Free **graphite flakes**	Slow cooling, Si 1–3 %	Grey fracture; excellent **damping**, machinability, castability, compressive strength (≈ 3–4 × tensile); poor tensile, brittle. **Machine beds, frames, engine blocks**. IS designation **FG 200** (min UTS 200 MPa)
Malleable CI	**Temper-carbon rosettes** (irregular nodules)	Prolonged annealing of white CI (~850–950 °C, many hours)	Ductile, tough; pipe fittings, brackets, small castings (blackheart, whiteheart, pearlitic)
Nodular / SG / Ductile CI	**Spheroidal graphite** nodules	Add **Mg or Ce** to the melt (inoculation)	Steel-like strength + ductility; crankshafts, gears, pipes. IS designation **SG 400/15** (400 MPa, 15 % elongation)
Chilled CI	White surface, grey core	Chills in mould	Rolls, railway wheels
Mottled CI	Mixed white + grey	Intermediate cooling	—
Compacted graphite (CG) CI	Vermicular graphite	Controlled Mg/Ti	Between grey and SG; diesel engine blocks

- **Graphitisers** (promote graphite): **Si, Ni, Al, Cu**. **Carbide stabilisers** (promote white CI): **Cr, Mn, V, Mo, S**.
- **Carbon equivalent** CE = %C + (%Si + %P)/3; CE = 4.3 → eutectic CI.

MEMORY TRICK

"**Peri-14-95, Eu-11-47, Eu-oid 7-27**" and carbon "**0.17 – 4.3 – 0.8**". Solubility ladder "**0.022 – 0.8 – 2.11 – 4.3 – 6.67**" = ferrite – pearlite – max austenite – ledeburite – cementite.

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Maximum solubility of carbon in austenite? → **2.11 % at 1147 °C**.
- Eutectoid reaction in steel produces? → **Pearlite (at 727 °C, 0.8 % C)**.
- Ledeburite is a mixture of? → **Austenite and cementite (eutectic, 4.3 % C)**.
- Degrees of freedom at a binary eutectic point (constant pressure)? → **Zero**.
- Element added to get spheroidal graphite cast iron? → **Magnesium (or cerium)**.
- Cast iron with best damping capacity? → **Grey cast iron**.

9.4 Heat Treatment ★★★

- **Heat treatment** = controlled heating, soaking and cooling of a metal in the solid state to obtain desired properties. Cooling rate decides the product: **furnace (slowest) → air → oil → water → brine (fastest)**.

TTT and CCT diagrams

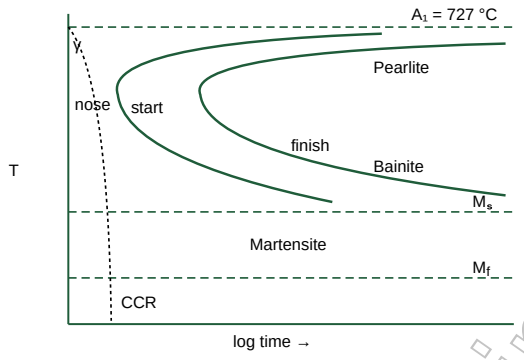


Fig 9.2 Schematic TTT diagram of eutectoid steel. Dotted curve = critical cooling rate (CCR), which just misses the nose.

- **TTT (Time–Temperature–Transformation) / isothermal transformation / "C-curve" / "S-curve"** diagram: austenite is quenched to a temperature and **held isothermally**; left curve = transformation start, right curve = finish. Developed by **Davenport and Bain** (1930).
- **Nose** of eutectoid steel ≈ **540–550 °C** at about **1 s** – austenite is least stable here.
- Above the nose (727 → ~550 °C): **pearlite** – coarse just below A_1, fine near the nose.
- Between nose and M_s (~550 → ~220 °C): **bainite** – **upper bainite** (feathery, ~350–550 °C) and **lower bainite** (acicular/needle-like, ~250–350 °C; tougher and harder).
- Below **M_s** (martensite start; ≈ **220 °C** for eutectoid steel): **martensite**, formed athermally (amount depends on temperature, **not time**). M_f (finish) of high-carbon steel lies **below room temperature** → **retained austenite**; removed by **sub-zero (cryogenic) treatment**.
- M_s and M_f are **lowered by increasing C** and most alloying elements.
- **Martensite** = supersaturated interstitial solid solution of C in α-Fe with **BCT (body-centred tetragonal)** structure; formed by **diffusionless shear** transformation at speed of sound; **hardest** and most brittle structure of steel. Its hardness depends mainly on **carbon content** (max ≈ 65 HRC at ~0.6–0.8 % C); c/a ratio increases with %C. Lath martensite (low C), plate/lenticular (high C).
- **Critical cooling rate (CCR)** = **minimum cooling rate** that just misses the nose → 100 % martensite. Plain C eutectoid steel needs ~ 200 °C/s or more (water quench).
- **All alloying elements except Co** (and Al) shift the TTT curves **to the right** → lower CCR → higher **hardenability** (oil or even air hardening).
- **CCT (Continuous Cooling Transformation)** diagram = for continuous cooling (real practice); curves shift **downward and to the right** of the TTT curves; for plain C eutectoid steel **no bainite** forms on continuous cooling.

Heat-treatment processes

Process	Heating	Cooling	Structure / purpose
Full annealing	Hypo: **30–50 °C above A_3**; Hyper: 30–50 °C above A_1	**Furnace cooling** (slowest)	**Coarse pearlite**; softest, max ductility, relieves stresses, improves machinability (medium C)
Process (sub-critical) annealing	Below A_1 (~550–650 °C)	Air / furnace	Recrystallises **cold-worked low-C steel** between drawing/forming passes
Spheroidising	Just below A_1 (~700 °C) for long time, or cycling about A_1	Slow	**Spheroidite** (globular Fe_3C in ferrite) – softest; **best machinability of high-C/tool steels**, cold formability
Stress-relief annealing	~550–650 °C (below A_1)	Slow	Removes residual stresses of welding, casting, machining; no structural change
Normalising	**40–50 °C above A_3** (hypo) / **above A_{cm}** (hyper)	**Still air**	**Fine pearlite**, refined grain; **stronger and harder** than annealed, slightly less ductile; breaks cementite network; homogenises castings/forgings
Hardening	Hypo: 30–50 °C above A_3; Hyper: 30–50 °C above **A_1** (not A_{cm})	**Quench** in water / oil / brine (faster than CCR)	**Martensite (BCT)**; max hardness, brittle, high residual stress
Tempering	Re-heat hardened steel **below A_1**	Air (or as required)	Relieves stress, restores toughness at some loss of hardness

Nandyal Banking Coaching Center

Austempering	Austenitise, quench into **salt bath above M_s** (~250–400 °C)	Hold isothermally till transformation complete, then air	**Bainite**; good toughness, less distortion, **no tempering needed**; limited to thin sections
Martempering (marquenching)	Austenitise, quench to **just above M_s**	Hold to **equalise** temperature (no transformation), then air-cool through M_s–M_f	**Martensite** with minimum distortion and cracking; **tempering still required**
Ausforming	Deform metastable austenite in the "bay" of TTT (~500 °C)	Then quench	Very high strength (thermo-mechanical treatment)

- Hyper-eutectoid steel is hardened from just above A_1 (not A_{cm}) because heating above A_{cm} gives coarse grains, more retained austenite and oxidation, while undissolved cementite adds **wear resistance**.
- Order of hardness of products: **martensite > lower bainite > upper bainite > fine pearlite (normalised) > coarse pearlite (annealed) > spheroidite**.

Tempering

Type	Temperature	Product	Typical use
Low	**150–250 °C**	Tempered martensite (ε-carbide)	Cutting tools, gauges, case-hardened parts, ball bearings – hardness retained
Medium	**350–500 °C**	**Troostite** (tempered martensite with very fine cementite)	**Springs**, chisels, hammers – high elastic limit
High	**500–650 °C**	**Sorbite** (coarser spheroidal cementite in ferrite)	Shafts, axles, gears, connecting rods – best **toughness**

- **Hardening + high tempering = "hardening and tempering" / "toughening"** (Q&T) – gives best combination of strength and toughness.
- Temper colours (oxide film): **light straw ≈ 220–230 °C**, brown ≈ 255 °C, purple ≈ 270 °C, **blue ≈ 300 °C**.
- **Temper embrittlement**: (i) **tempered-martensite (one-step / 350 °C) embrittlement** on tempering ~250–400 °C; (ii) **temper brittleness (two-step)** on slow cooling through / holding in ~375–575 °C – caused by P, Sb, Sn, As segregation at prior austenite grain boundaries; **prevented by Mo (0.2–0.5 %)** or fast cooling from tempering temperature. Detected by impact test (DBTT rises).
- **Secondary hardening**: in steels with strong carbide formers (Mo, W, V, Cr) hardness rises again at ~500–600 °C tempering due to fine alloy carbides – basis of **HSS** hot hardness.

Hardenability

- **Hardenability** = ability of steel to be hardened to a **depth** (i.e. to form martensite throughout a section). **Hardness** = maximum hardness attainable – depends on **carbon %**; hardenability depends on **alloying elements and grain size**.
- **Jominy end-quench test** (IS 3848 / ASTM A255): specimen **25 mm dia × 100 mm long**, austenitised, then **one end quenched by water jet**; hardness measured along the length at **1.6 mm (1/16 in)** intervals → Jominy curve. Flatter curve = higher hardenability.
- **Grossman's ideal critical diameter (D_I)** = bar diameter that gets 50 % martensite at centre in an ideal quench (H = ∞).
- Hardenability increased by: **Mn, Mo, Cr, Ni, Si, B (very small, 0.001–0.003 %)**, **coarse austenite grain**, more dissolved carbon. Decreased by: **Co**, fine grain, undissolved carbides/inclusions.
- Mass effect / **ruling section**: large sections cool slowly at centre → lower core hardness.

Quenching medium	Severity of quench H (still, approx.)
Brine (5–10 % NaCl) / caustic soda solution	**≈ 2** (most severe)
Water	**≈ 1** (reference)
Oil	≈ 0.25–0.3
Molten salt / lead	Mild (used for austempering, martempering)
Air	≈ 0.02 (least; for air-hardening tool steels)

- Three stages of liquid quenching: **vapour-blanket stage** (slow) → **boiling stage (fastest cooling)** → **convection stage** (slowest). Brine is faster because salt crystals break the vapour blanket. Agitation increases severity.
- Severe quench → risk of **quench cracks and distortion**; oil is preferred for alloy steels.

Surface (case) hardening

Process	Medium / element	Temperature	Key facts
Pack carburising	Charcoal + energiser **$BaCO_3$** (or Na_2CO_3) in sealed box; C diffuses	**900–950 °C** (austenite region)	Low-C steel (**0.1–0.2 % C**); case 0.5–2 mm with ~0.8–1 % C; slow; then quench + low temper. Case depth ∝ √t
Gas carburising	Methane, propane, natural gas in carrier (endothermic) gas	~900–950 °C	Most widely used in mass production; precise control
Liquid carburising	Molten **NaCN**-based salt bath	~850–950 °C	Fast, uniform; toxic cyanide salts

Nandyal Banking Coaching Center

Nitriding	**Ammonia (NH_3)** → nascent N; steels with **Al, Cr, Mo, V** (Nitralloy)	**~500–550 °C** (below A_1)	**No quenching** → minimum distortion; **hardest case** (~1000–1200 HV); long time (20–100 h); thin case (0.1–0.6 mm); good fatigue and corrosion resistance; done after hardening + tempering + finish machining
Cyaniding	Molten **NaCN** bath – adds **C + N**	~800–870 °C	Thin case (0.1–0.3 mm), quick; then water/oil quench; small parts
Carbonitriding ("dry/gas cyaniding")	Gas: hydrocarbon + **ammonia** – adds C + N	~700–900 °C	Lower temperature than carburising → less distortion; oil quench possible
Flame hardening	**Oxy-acetylene** flame + water spray	Surface above A_3	**No change in composition**; needs **medium/high C steel (0.4–0.6 % C)** or CI; large parts (lathe beds, gear teeth)
Induction hardening	High-frequency AC coil (eddy currents, **skin effect**) + water spray	Surface above A_3, in seconds	Case depth ∝ 1/√f (higher frequency → thinner case); crankshaft journals, gears, cam shafts; easy automation
Laser / electron-beam hardening	High-energy beam	Local	Very thin, precise; self-quenching by bulk

- **Case–core rule**: carburising, nitriding, cyaniding **change surface composition** (thermo-chemical); flame and induction hardening **do not** (only phase change).
- Boronising (boriding) gives very hard FeB/Fe_2B layer. Chromising, calorising, sherardising are diffusion coatings mainly for corrosion/oxidation resistance.

MEMORY TRICK

Tempering trio "**Low-Tool, Medium-Spring, High-Shaft**" and products "**T before S**": **T**roostite (medium) comes at lower temperature than **S**orbite (high). **Nitriding = 3 N's – No quench, NH_3, No distortion.**

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Crystal structure of martensite? → **Body-centred tetragonal (BCT)**.
- Cooling medium in normalising? → **Still air**; in full annealing → **furnace**.
- Austempering produces? → **Bainite**.
- Test used to determine hardenability? → **Jominy end-quench test**.
- Case-hardening process requiring no quenching? → **Nitriding**.
- Element that reduces hardenability of steel? → **Cobalt**.

9.5 Steels & Alloying Elements ★★★

Plain carbon steel	% C	Properties	Uses
Dead mild steel	< 0.15	Very ductile, weldable	Sheets, wires, rivets, pipes, deep-drawn parts
Low carbon (mild) steel	0.15–0.30	Ductile, tough, easily welded/machined; not hardenable by quenching	Structural sections, bolts, nuts, rods, boiler plates, case-carburised parts
Medium carbon steel	0.30–0.60	Balance of strength and toughness; hardenable	**Shafts, axles, gears, crankshafts, connecting rods, rails, forgings**
High carbon steel	0.60–1.5	Hard, wear-resistant, less ductile; poor weldability	**Springs** (0.6–0.9), hammers, chisels, **cutting tools, drills, files, razors, dies** (0.9–1.5)

- Increasing C → **↑ hardness, UTS (up to ~0.8–1 %), hardenability**; **↓ ductility, toughness, weldability, machinability**, and lowers melting point.
- **Weldability** judged by **carbon equivalent**: CE = C + Mn/6 + (Cr + Mo + V)/5 + (Ni + Cu)/15 (IIW); CE < 0.4 → good weldability.
- Impurity effects: **S → hot shortness** (FeS low-melting film at grain boundaries; cured by Mn → MnS); **P → cold shortness** (brittle at low temperature).
- **Austenite stabilisers** (γ-formers): **Ni, Mn, C, N, Co, Cu**. **Ferrite stabilisers** (α-formers): **Cr, Si, Mo, W, V, Ti, Al, Nb**. Strong **carbide formers**: Ti, Nb, V, W, Mo, Cr (decreasing order roughly).

Effect of alloying elements

Element	Main effects	Typical steels / uses
Nickel (Ni)	↑ **toughness** and strength, esp. low-temperature impact strength; γ-stabiliser; refines grain; corrosion resistance	Ni–Cr steels (gears, shafts); 18-8 SS; **Invar (36 % Ni)** – near-zero expansion; 9 % Ni cryogenic steel
Chromium (Cr)	↑ **hardenability, corrosion & oxidation resistance** (≥ 10.5–12 % → stainless), wear resistance; carbide former	Stainless steels, ball-bearing steel (1 % C, 1.5 % Cr – EN31), tool steels
Molybdenum (Mo)	↑ hardenability, **high-temperature (creep) strength, prevents temper brittleness**; secondary hardening; pitting resistance in SS	Cr–Mo boiler/pressure-vessel steels, 316 SS, Mo-HSS
Tungsten (W)	**Hot (red) hardness**, wear resistance, fine grain; hard carbides	**HSS**, hot-work dies, magnet steel
Vanadium (V)	**Grain refiner**, hot hardness, fatigue strength; very strong carbide former; secondary hardening	HSS, Cr–V spring and tool steels, HSLA steels

Nandyal Banking Coaching Center

Manganese (Mn)	↑ hardenability, strength; **combines with S → MnS (counteracts hot shortness)**; deoxidiser	All steels (0.3–1 %); **Hadfield steel 11–14 % Mn, 1–1.4 % C** – austenitic, **work-hardening**, wear-resistant (crusher jaws, rail crossings, excavator teeth)
Silicon (Si)	**Deoxidiser**; ↑ elastic limit; ↑ electrical resistivity, ↓ hysteresis loss; graphitiser in CI	**Electrical (transformer) steel ~3–4 % Si**; Si–Mn spring steel (55Si7)
Cobalt (Co)	↑ red hardness; ↑ magnetic properties; **only element that lowers hardenability**	**Super HSS** (5–12 % Co), permanent magnets (Alnico), cemented-carbide binder
Sulphur / Lead (S, Pb)	**Free machining** (chip breaking, lubrication)	Free-cutting steels for automats (e.g. 11C10S25)
Phosphorus (P)	↑ strength, machinability; **cold shortness**	Kept < 0.05 %
Aluminium (Al)	Strong **deoxidiser** (killed steel), grain refiner, nitride former	Nitriding steels
Titanium / Niobium (Ti, Nb)	Carbide formers; grain refiners; **stabilise SS against sensitisation**	321 (Ti), 347 (Nb) SS; micro-alloyed HSLA
Copper (Cu)	Atmospheric corrosion resistance	**Weathering steel** (Corten)
Boron (B)	Large ↑ in hardenability at **0.001–0.003 %**	Boron steels for fasteners

- **Killed steel** = fully deoxidised (Si, Al) – no gas evolution, sound; used for forgings/pressure parts; **rimmed steel** = little deoxidation, sound rim, blowholes in core; **semi-killed** = intermediate.
- **Maraging steel**: **~18 % Ni**, Co, Mo, Ti, very low C; low-carbon martensite + age hardening → UTS up to ~2000 MPa; rocket motor casings.
- **HSLA (micro-alloyed) steel**: low C + small Nb, V, Ti (< 0.1 %); high yield by grain refinement; pipelines, bridges.

Stainless steels (Cr ≥ 10.5–12 %; passive Cr_2O_3 film)

Type	Composition	Structure / magnetism	Hardenable?	Uses
Austenitic (300 series: 304, 316)	**18 % Cr – 8 % Ni** ("18-8"), C < 0.08 %; 316 adds 2–3 % Mo	FCC, **non-magnetic**	**No** (only by cold work)	Best corrosion resistance, weldable, cryogenic; kitchenware, chemical & food plant, 316 for marine
Ferritic (430, 409)	12–27 % Cr, low C, no Ni	BCC, magnetic	No	Auto exhaust, trim, appliances – cheaper
Martensitic (410, 420, 440)	12–18 % Cr, **0.1–1.2 % C**	BCT after hardening, magnetic	**Yes** (quench + temper)	**Cutlery surgica instrun** turbine blades, valves
Duplex (2205)	~22 % Cr, 5 % Ni, 3 % Mo, N	~50 % ferrite + 50 % austenite, magnetic	No	High strengt resists **chlorid stress-corrosi crackin** offshor desalin
Precipitation-hardening (17-4 PH)	17 % Cr, 4 % Ni, Cu, Nb	Martensitic/semi-austenitic	Yes (ageing)	Aerosp shafts

- **Sensitisation**: heating austenitic SS in **~425–815 °C** (e.g. HAZ during welding) precipitates **chromium carbide ($Cr_{23}C_6$)** at grain boundaries → Cr-depleted zone (< 12 % Cr) → **intergranular corrosion = "weld decay"** (occurs a little away from the weld bead).
- Remedies: **low-carbon grades (304L, 316L; C ≤ 0.03 %)**; **stabilised grades with Ti (321) or Nb (347)** which form TiC/NbC preferentially; **solution annealing** at ~1050–1100 °C followed by rapid quench.
- Stabilised grades may suffer **knife-line attack** (narrow band next to weld).

Tool steels

AISI group	Meaning	Example / use
W	Water-hardening (plain high-C)	Files, cheap tools
O / A	Oil-hardening / air-hardening (cold work)	Gauges, blanking dies
D	High-C, high-Cr (12 % Cr) cold work	D2 – long-run dies
H	Hot work	H13 – forging & die-casting dies
S	Shock-resisting	Chisels, punches
T	**Tungsten HSS**	**T1 = 18-4-1**
M	**Molybdenum HSS**	**M2 = 6 W – 5 Mo – 4 Cr – 2 V** (most used HSS today, cheaper)

- **HSS 18-4-1** = **18 % W, 4 % Cr, 1 % V**, ~0.7–0.8 % C; retains hardness (red/hot hardness) up to about **600 °C**. Cobalt HSS = "**super HSS**".
- HSS is hardened from ~1250–1300 °C (very close to melting) and tempered 2–3 times at ~550 °C (secondary hardening).

IS designation of steels (IS 1762 / IS 1570)

- **Plain carbon steel**: old **C45** = 0.45 % C (average C × 100). New style **45C8** = 0.45 % C, **Mn ≈ 0.8 %** (Mn × 10; i.e. 0.6–0.9 %).
- **Low-alloy steel**: C × 100 followed by element symbols with **% × multiplying factor**: **Cr, Co, Mn, Ni, Si, W → × 4**; **Al, Be, Cu, Mo, Nb, Pb, Ta, Ti, V, Zr → × 10**; **P, S, N → × 100**.
  - **40Cr4** = 0.40 % C, ~1 % Cr. **40Ni14** = 0.40 % C, 3.5 % Ni. **55Si7** = 0.55 % C, 1.75 % Si (spring steel). **40Cr4Mo2** = 0.40 % C, 1 %

Nandyal Banking Coaching Center

Cr, 0.2 % Mo. **11C10S25** = 0.11 % C, 1 % Mn, 0.25 % S (free cutting).

- **High-alloy steel** (any element ≥ 5 %): prefix **X**, C × 100, then actual % of elements: **X04Cr18Ni10** = 0.04 % C, 18 % Cr, 10 % Ni (≈ AISI 304).
- Cast irons: **FG 200** (grey, min UTS 200 MPa, IS 210), **SG 400/15** (IS 1865).
- **AISI/SAE**: 4-digit, first two = type, last two = C × 100. **10xx** plain carbon (1040 = 0.40 % C), 11xx free-cutting, 13xx Mn, 41xx Cr–Mo, 43xx Ni–Cr–Mo, 52xx Cr (52100 bearing steel).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Composition of 18-4-1 HSS? → **18 % W, 4 % Cr, 1 % V**.
- Element that prevents temper brittleness? → **Molybdenum**.
- 18-8 stainless steel is? → **Austenitic, non-magnetic, 18 % Cr + 8 % Ni**.
- Weld decay in austenitic SS is due to? → **Chromium carbide precipitation at grain boundaries (sensitisation)**.
- Designation 40Cr4 indicates? → **0.40 % C and 1 % Cr**.
- Steel used for transformer cores contains? → **~3–4 % silicon**.

9.6 Non-Ferrous Metals, Polymers, Ceramics & Composites ★★☆

Metal	Density (g/cm^3)	Melting point (°C)	Structure
Magnesium	**1.74** (lightest structural metal)	650	HCP
Aluminium	**2.70**	**660**	FCC
Titanium	**4.5**	1668	HCP (α) / BCC (β above 882 °C)
Iron	7.87	1538	BCC (α)
Copper	8.96	1085	FCC
Lead	11.34	327	FCC
Tungsten	19.3	**3422** (highest of metals)	BCC

Aluminium & its alloys

- Al: light, high conductivity (≈ 60 % of Cu), corrosion-resistant due to protective Al_2O_3 film; no DBTT. Ore: **bauxite** (Bayer process → alumina; Hall–Héroult electrolysis → Al).
- **Duralumin** ≈ **3.5–4.5 % Cu, 0.5 % Mg, 0.5 % Mn**, rest Al – **age (precipitation) hardenable**; aircraft structures, rivets. **Alclad** = duralumin clad with pure Al for corrosion resistance.
- **Y-alloy** ≈ 4 % Cu, 2 % Ni, 1.5 % Mg – retains strength at high temperature → **pistons, cylinder heads**.
- **Silumin / Alpax** = Al + ~12 % Si (near eutectic) – excellent castability (sodium modification refines Si).
- **Hindalium** (Al–Mg–Mn–Cr) – utensils; **Magnalium** (Al–Mg) – light, instrument parts.
- Wrought series: **1xxx** pure Al, **2xxx Cu**, 3xxx Mn, 4xxx Si, **5xxx Mg**, **6xxx Mg–Si** (6061, 6063 extrusions), **7xxx Zn** (7075 – highest strength, aircraft). Heat-treatable: **2xxx, 6xxx, 7xxx**.
- **Precipitation (age) hardening steps**: (1) **solution treatment** (~500 °C for duralumin) → single phase; (2) **quench** → supersaturated solid solution; (3) **ageing** – natural (room temperature, days) or artificial (100–200 °C) → GP zones/coherent precipitates. Over-ageing → coarse precipitates, strength falls. Condition: solubility must decrease with falling temperature.
- Duralumin rivets are stored in a **refrigerator** after solution treatment to delay natural ageing ("ice-box rivets").

Copper alloys

Alloy	Composition	Use
Cartridge brass	**70 Cu – 30 Zn** (α-brass, most ductile)	Cartridge cases, deep drawing, radiator cores
Muntz metal	**60 Cu – 40 Zn** (α + β, hot-workable)	Condenser tube plates, ship sheathing, bolts, hot forgings
Admiralty brass	70 Cu – 29 Zn – 1 Sn	Condenser tubes (sea water)
Naval brass	60 Cu – 39 Zn – 1 Sn	Marine hardware, propeller shafts
Gilding metal	95 Cu – 5 Zn	Coins, medals, jewellery
Bronze (tin bronze)	**Cu – Sn**	Bearings, bushes, statues
Phosphor bronze	Cu – Sn (~5–11 %) – P (up to 1 %)	**Springs, bearings, worm wheels**, gears
Gun metal	**88 Cu – 10 Sn – 2 Zn**	Valves, pump casings, bearings, steam fittings
Bell metal	~78–80 Cu – 20–22 Sn	Bells (sonorous)
Aluminium bronze	Cu + 5–11 % Al	Corrosion-resistant marine parts, gold-like coins
Beryllium copper	Cu + ~2 % Be (age-hardenable)	Springs, **non-sparking tools**
German silver (nickel silver)	Cu–Zn–Ni (**no silver**)	Cutlery, ornaments, resistances
Cupro-nickel	Cu–Ni (70/30, 90/10)	Coins, condenser tubes

- **Brass = Cu + Zn**; **Bronze = Cu + Sn** (and later Cu + Al, Si, Be). Gun metal and phosphor bronze are bronzes.
- Brasses suffer **dezincification** (selective leaching of Zn – prevented by As/Sn) and **season cracking** (stress-corrosion in ammonia).

Bearing metals, Ni and Ti alloys

- Bearing-metal requirements: low friction, **conformability**, **embeddability** (absorb dirt), good thermal conductivity, fatigue strength, corrosion resistance.
- **Babbitt (white metal)** – **tin-base: ~88–90 % Sn, 7–8 % Sb, 3–4 % Cu**; hard SbSn/Cu_6Sn_5 cuboids in soft matrix. Lead-base white metal: Pb–Sb–Sn (cheaper). Others: copper–lead, phosphor bronze, Al–Sn, sintered porous bronze (oil-impregnated, self-lubricating).
- **Monel** ≈ **67 % Ni, 30 % Cu** + Fe, Mn – excellent corrosion resistance (sea water, acids); pumps, propellers, chemical plant.
- **Inconel** ≈ Ni (~72–76 %) – Cr (~15 %) – Fe (~8 %) – high-temperature oxidation resistance; furnace parts, jet engine exhausts.
- **Nimonic** (Ni–Cr + Ti, Al) – **gas-turbine blades**; **Hastelloy** (Ni–Mo–Cr) – corrosive chemicals; **Nichrome** (80 Ni – 20 Cr) – **heating elements**; **Constantan** (55 Cu – 45 Ni) – thermocouples,

Nandyal Banking Coaching Center

resistors; **Invar** (Fe – 36 Ni) – minimum thermal expansion; **Permalloy** (Ni–Fe) – high magnetic permeability.

- **Titanium**: highest **strength-to-weight ratio** among common metals, excellent corrosion resistance and **biocompatibility**; **Ti-6Al-4V** is the workhorse alloy (aircraft, implants). Hard to machine (low thermal conductivity, reactive).
- **Superalloys** (Ni-, Co-, Fe-base) – turbine blades, often **directionally solidified / single crystal**.

MEMORY TRICK

"**B-Z, B-T**": **B**rass = **Z**inc, **B**ronze = **T**in. "Cartridge **70-30**, Muntz **60-40**, Gun **88-10-2**". "**Mo**nel = **Mo**stly Ni + Cu; **In**conel = **In**side furnace (Ni-Cr)".

Polymers

Feature	Thermoplastics	Thermosets
Molecular structure	**Linear / branched** chains, weak secondary (van der Waals) bonds between chains	**3-D cross-linked network** (strong covalent bonds)
Effect of heat	**Soften on heating, harden on cooling – repeatedly**	Set permanently on curing; do not soften, **char/decompose** on heating
Recycling	**Recyclable**	Not recyclable
Polymerisation	Mostly **addition**	Mostly **condensation**
Properties	Tougher, more ductile, lower temperature limit, solvent-soluble	Harder, stronger, more brittle, dimensionally stable, heat resistant
Processing	**Injection moulding**, extrusion, blow moulding, thermoforming	**Compression / transfer moulding**
Examples	**PE, PP, PVC, PS, PTFE (Teflon), nylon, PMMA (Perspex), PC, ABS, PET**	**Bakelite (phenol-formaldehyde), epoxy, polyester, urea-formaldehyde, melamine**, polyurethane (rigid)

- **Elastomers** = lightly cross-linked, amorphous polymers above T_g; large recoverable strains (> 200 %). Natural rubber = **cis-polyisoprene**; **vulcanisation** = cross-linking with **sulphur** (Charles Goodyear, 1839). Synthetic: **neoprene** (polychloroprene – oil resistant), SBR (tyres), butyl (inner tubes – low gas permeability), nitrile, silicone rubber (high/low temperature).
- **Glass-transition temperature (T_g)** = amorphous polymer changes from hard glassy to soft rubbery state.
- **Degree of polymerisation** = molecular weight of polymer ÷ molecular weight of mer (repeat unit).
- **PTFE** – lowest coefficient of friction (~0.04), chemically inert, non-stick; **PMMA** – transparent (aircraft windows, lenses); **polycarbonate** – high impact (bullet-resistant glazing, helmets); **nylon** – gears, bushes; **Bakelite** – first fully synthetic plastic (Baekeland, 1907), electrical switches.
- Additives: **plasticisers** (flexibility – soft PVC), fillers, stabilisers, colourants, flame retardants.

Ceramics

- Inorganic, non-metallic, **ionic/covalent** bonding → **high hardness, high melting point, high compressive strength**, good electrical & thermal insulation, chemical inertness; but **brittle**, low tensile strength, poor thermal-shock resistance.
- Engineering ceramics: **alumina (Al_2O_3)** – cutting tools, insulators; **silicon carbide (SiC, carborundum)** – abrasives, heating elements; **silicon nitride (Si_3N_4)** – bearings, engine parts; **zirconia (ZrO_2)** – **transformation toughening**; **cubic boron nitride (CBN)** – second hardest after diamond; SiAlON.
- Traditional: clay products, refractories, cement, glass (amorphous). **Cermet** = ceramic + metal (e.g. TiC–Ni).
- Ceramics are usually tested in **bending (flexural strength / modulus of rupture)**, not tension.

Composites

Type	Matrix	Examples / uses
PMC (polymer-matrix)	Epoxy, polyester	**GFRP** (boats, tanks), **CFRP** (aircraft, sports goods), Kevlar-epoxy
MMC (metal-matrix)	Al, Mg, Ti	Al–SiC (brake rotors, pistons), Al–B fibre
CMC (ceramic-matrix)	SiC, Al_2O_3, carbon	**Carbon–carbon** (re-entry nose cones, aircraft brakes), SiC–SiC (turbine parts) – improves toughness

- Composite = **matrix** (binds, transfers load, protects) + **reinforcement** (fibres, particles, flakes) – distinct phases with an interface.
- Fibres: **E-glass** (electrical grade, cheapest, most used), **S-glass** (higher strength); **carbon/graphite** (high modulus, low density, low CTE); **Kevlar (aramid)** – very high toughness/impact resistance – bullet-proof vests; boron; SiC.
- Types by reinforcement: **particle-reinforced** (concrete, cermets, carbide), **fibre-reinforced** (continuous/discontinuous), **structural** (laminates, **sandwich panels** – honeycomb core).
- Natural composites: **wood** (cellulose fibres in lignin), bone. **Reinforced concrete** = steel in concrete.

KEY FORMULAE

- Rule of mixtures (longitudinal, **iso-strain**): $E_c = E_f V_f + E_m V_m$ (upper bound)
- Transverse (**iso-stress**): $1/E_c = V_f/E_f + V_m/E_m$ (lower bound)
- Density: $\rho_c = \rho_f V_f + \rho_m V_m$

Smart materials

Material	Stimulus → response	Example
Shape-memory alloy	Heat → recovers original shape (martensite ↔ austenite transformation); superelasticity	**Nitinol (Ni–Ti)**, Cu–Zn–Al; stents, actuators, spectacle frames
Piezoelectric	Stress ↔ voltage	Quartz, **PZT**; sensors, actuators, igniters
Magnetostrictive	Magnetic field ↔ strain	Terfenol-D
ER / MR fluids	Electric / magnetic field → viscosity change	Dampers, clutches
Thermochromic / photochromic	Temperature / light → colour change	Sensors, lenses

Nandyal Banking Coaching Center

Corrosion & protection

- Corrosion = electrochemical deterioration; **anode corrodes** (oxidation), cathode is protected. Rust = hydrated Fe_2O_3 (needs **both O_2 and water**).
- **Galvanic series in sea water** (active/anodic → noble/cathodic): **Mg → Zn → Al → Cd → mild steel/CI → ... → brass → Cu → bronze → ... → Ni → passive SS → Ti → graphite → Au → Pt**.
- **Small anode + large cathode → rapid corrosion** of anode (avoid, e.g. steel rivets in copper plate).
- **Pilling–Bedworth ratio** (oxide vol./metal vol.): < 1 porous, non-protective (Mg, Na); **1–2 protective** (Al, Cr, Ni, Ti); > 2 oxide spalls off (Fe).

Form of corrosion	Key fact
Uniform	Most common, predictable (corrosion allowance)
Galvanic	Two dissimilar metals in electrolyte
Pitting	Localised holes; chlorides attack SS; resisted by Mo (316)
Crevice	Under gaskets, washers – differential aeration (O_2-starved region is anode)
Intergranular	Along grain boundaries – **weld decay** in SS
Stress-corrosion cracking	Tensile stress + specific environment: **brass – ammonia (season cracking)**; **austenitic SS – chlorides**; **mild steel – caustic (caustic embrittlement in boilers)**
Selective leaching	**Dezincification** of brass; graphitisation of grey CI
Erosion / fretting / cavitation	Corrosion aided by fluid velocity / small oscillating slip / bubble collapse (pump impellers)
Hydrogen embrittlement	Atomic H diffuses in (pickling, plating, welding) – high-strength steels

Protection method	Principle / details
Galvanising	**Zinc** coating by hot-dipping (~450 °C) – Zn is **anodic** to steel → protects even when scratched (**sacrificial**)
Tinning	Tin coating (food cans) – Sn is **cathodic** to steel → if scratched, steel corrodes faster
Sherardising / calorising / chromising	Diffusion coatings of **Zn powder / Al / Cr**
Electroplating	Ni, Cr, Cu, Cd, Zn deposited electrolytically
Anodising	Thickening the **Al_2O_3** film on Al electrolytically (Al is the **anode**)
Phosphating / chromating	Conversion coatings – base for paint (Parkerising, Bonderising)
Cathodic protection – sacrificial anode	More active metal (**Mg, Zn, Al**) connected to structure; anode corrodes and is replaced – ship hulls, buried pipelines, water heaters
Cathodic protection – impressed current	External **DC** source makes structure the cathode; inert anodes (graphite, high-Si iron, Pt-Ti) – long pipelines
Anodic protection	Keeps passivating metals (SS, Ti) in passive range
Inhibitors, paints, design	Chromates/phosphates in coolant; avoid crevices and dissimilar-metal contact; insulate joints

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Muntz metal composition? → **60 % Cu, 40 % Zn**.
- Duralumin is strengthened by? → **Age (precipitation) hardening**.
- Bakelite is an example of? → **Thermosetting plastic**.
- Metal commonly used as sacrificial anode? → **Magnesium / zinc**.
- Galvanising means coating steel with? → **Zinc**.
- Nitinol is a? → **Shape-memory alloy (Ni–Ti)**.

Nandyal Banking Coaching Center

10. Casting, Welding, Brazing & Fabrication

NOTE

Manufacturing processes give 8–12 questions in most PSU/SSC-JE/GATE-style papers, and casting + welding + forming are the core. Typical asks: pattern allowances, Chvorinov's rule, riser H = D, gating ratio, die-casting types, casting/weld defects, DCSP vs DCRP, flame types, carbon equivalent, brazing vs soldering temperature, μ^2R, LDR, punch/die sizing and blanking force. Learn the numbers in bold.

10.1 Casting – Patterns, Moulds & Cores ★★★

- **Casting** = pouring molten metal into a mould cavity and letting it solidify; oldest shaping process (≈ 4000 BC). Best for **complex shapes, large parts, internal cavities** and brittle metals such as **cast iron**.
- **Pattern** = replica of the casting used to make the mould cavity; it is made **larger** than the casting by the allowances.
- **Pattern-maker's (contraction) rule** = a scale already lengthened by the shrinkage allowance.
- **Flask** parts: **cope** (top), **drag** (bottom), **cheek** (middle, in 3-part moulds). The joint is the **parting line**.

Pattern material	Key points
Wood (teak, pine, mahogany, deodar)	Cheap, light, easy to shape; absorbs moisture and warps; **small runs**. Teak is the most used in India
Metal (Al, CI, brass, steel)	Long life, accurate; **mass production**, match plates. **Al** is the most common metal pattern (light, no rust)
Plastics (epoxy, polyurethane)	Light, strong, no moisture absorption, smooth finish
Wax	**Investment (lost-wax) casting** – pattern melted out
Expanded polystyrene (EPS)	**Full-mould / lost-foam** – pattern vaporised by metal
Plaster of Paris	Intricate small patterns, core boxes

Pattern type	Description / use
Single-piece (solid)	One piece, simple shapes, few castings (e.g. stuffing-box gland)
Split (two-piece)	Split along parting line; halves located by **dowel pins**; e.g. pipes, spindles
Match-plate	Cope & drag halves on opposite sides of one plate with gating; **machine moulding, mass production of small castings**
Cope & drag	Halves mounted on separate plates; cope and drag rammed separately – **large/heavy castings**
Loose-piece	Projections/bosses held by wooden pins; withdrawn separately after main pattern (**overhangs**)
Gated	Several patterns joined with gating; mass production of small parts
Sweep	Board rotated about a spindle – **large symmetrical (axisymmetric) shapes** such as bells, kettles
Skeleton	Wooden frame (ribs) filled with loam – **very large, few castings** (turbine casings, water pipes)
Follow-board	Board supports a thin/weak pattern during ramming
Segmental	Segment rotated and moved stepwise – large rings, gears

Pattern allowances

Allowance	Purpose / typical value	Effect on pattern
Shrinkage (contraction)	Compensates **solid-state contraction** only. Grey CI ≈ **10 mm/m (1%)**; steel ≈ **20 mm/m (2%)**; Al ≈ 13 mm/m; brass ≈ 15 mm/m	Larger (+)
Draft (taper)	Easy withdrawal; ≈ **1–3°**. **Internal surfaces need more draft** than external	Larger (+)
Machining (finish)	Extra metal for machining; more for **CI and ferrous**, large castings, cope side (top surface has more inclusions)	Larger (+)
Distortion (camber)	Pattern bent opposite to expected warpage (U, V, long flat shapes)	Shape changed
Shake (rapping)	Rapping enlarges the cavity, so pattern made smaller	**Smaller (–) → the only NEGATIVE allowance**

- Grey cast iron has the **lowest shrinkage** among common cast metals because **graphite precipitation causes expansion** at the end of solidification.
- **Colour code of patterns** (common Indian/American practice): **red** – surfaces to be machined; **black** – surfaces left unmachined (as-cast); **yellow** – core prints; **red stripes on yellow** – seats of loose pieces; **black stripes on yellow** – stop-offs.

Moulding sand

- Green sand = **silica sand + clay binder (bentonite/fire-clay) + water** (≈ 2–8 % moisture) + additives. "Green" means **moist**, not colour.
- Silica sand fusion point ≈ **1710 °C**. Special sands: **zircon** (high conductivity, low expansion), **chromite**, **olivine** – for steel castings.
- Bentonite (montmorillonite clay) – most common binder; sodium bentonite swells more than calcium bentonite.
- Additives: **sea coal (coal dust)** – better finish on CI, prevents metal penetration; **cereal/dextrin** – green strength & collapsibility; **wood flour/saw dust** – absorbs sand expansion, reduces scab; **silica flour** – hot strength, finish; **molasses** – core binder.
- **Facing sand** (fine, next to pattern), **backing (floor/black) sand** (re-used), **parting sand** (dry, clay-free – prevents sticking of cope and drag), **core sand**, **loam sand** (≈ 50% clay, large castings), **system sand** (machine moulding).

Property	Meaning / depends on
Permeability (porosity)	Allows gases/steam to escape; ↑ with **coarse, uniform, round grains**; ↓ with fines, clay, hard ramming. Low → **blow holes**
Refractoriness	Withstands high temperature without fusing; high for steel castings

Nandyal Banking Coaching Center

Green strength	Strength of moist sand – to hold shape after pattern withdrawal
Dry strength	Strength after moisture is driven off by hot metal (resists erosion)
Hot strength	Strength above 100 °C, prevents enlargement of cavity
Collapsibility	Mould/core breaks down as casting contracts; poor → **hot tears**, difficult knock-out
Flowability (plasticity)	Flows around pattern under ramming
Cohesiveness / adhesiveness	Sand-to-sand bonding / sand sticking to flask walls

- Grain shape: **angular** grains → higher strength, lower permeability; **rounded** grains → higher permeability, better flowability.
- Moisture: strength and permeability rise to an optimum (≈ 4–6%) and then fall.

Sand tests (AFS)

- **Standard AFS test specimen**: **50.8 mm dia × 50.8 mm high** (2 in × 2 in), rammed by **3 drops** of a **6.35 kg** weight from **50.8 mm**.
- **AFS Grain Fineness Number (GFN)** – by sieve analysis (shaking ≈ 15 min); GFN = Σ(% retained × multiplier) ÷ Σ % retained. It is the sieve number that would just pass the sand if all grains were of one size. **Higher GFN → finer sand** → better finish, lower permeability.
- **Permeability number** = volume of air (2000 cm^3) passing through the standard specimen per unit time under unit pressure.
- Other tests: moisture (by drying or **calcium carbide** moisture teller), clay content (washing with NaOH), **mould hardness** (spring-loaded ball tester), compression/shear strength (universal sand-testing machine), refractoriness.

KEY FORMULAE

- Permeability number $P_N = V \cdot H \div (p \cdot A \cdot t)$ (V = 2000 cm^3, H = 5.08 cm, A = 20.27 cm^2, p = g/cm^2, t = min)
- For standard specimen: $\mathbf{P_N = 501.2 \div (p \cdot t)}$ (t in minutes) ≈ 30 072 ÷ (p·t) with t in seconds
- Buoyancy (upward) force on core: $F_b = V_{core}(\rho_{metal} - \rho_{core})g$
- Metallostatic lifting force on cope: $F = \rho g h A_{projected}$ (h = metal head above the parting plane)

Cores, core prints & chaplets

- **Core** = sand body placed in mould to form **internal cavities/holes**. Needs high **collapsibility, permeability, dry strength, refractoriness**; made in a **core box**, baked (≈ 200–250 °C for oil-bonded).
- Core binders: **linseed oil** (core oil), resins, **sodium silicate + CO_2** (CO_2 process – hardens without baking by forming silica gel), dextrin, molasses.
- Core types: horizontal, vertical, balanced (supported one end), hanging (cover), drop, kiss core (no print – held by mould pressure).
- **Core prints** = projections on the pattern forming **seats** to locate and support the core. Painted **yellow**.
- **Chaplets** = metal supports (same/compatible metal as casting) used where core prints cannot support the core; they **fuse into** the casting. Must be clean (no rust/moisture), else blowholes.
- **Vent rods** – make vent holes in the cope for gas escape. **Core vents** – wax wires/straw inside the core.
- Moulding methods: **green sand, dry sand** (baked mould, larger castings), **skin-dried**, **loam**, **pit moulding** (very large), machine moulding – **jolt** (more density at the pattern face/bottom), **squeeze** (more density at the back/top), **jolt-squeeze** (uniform), **sand slinger** (large moulds).

MEMORY TRICK

"**S**hake **S**hrinks the pattern" – shake (rapping) is the only negative allowance; all others (shrinkage, draft, machining) make the pattern bigger.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Which pattern allowance is negative? → **Shake (rapping) allowance**.
- Pattern for large axisymmetric castings like bells? → **Sweep pattern**.
- Pattern best suited for mass production on moulding machines? → **Match-plate pattern**.
- Permeability number of standard specimen formula? → **501.2/(p·t)**, t in minutes.
- Higher AFS GFN means? → **Finer sand**, lower permeability.
- Metal supports for cores that fuse into the casting? → **Chaplets**.

10.2 Gating, Risering & Solidification ★★★

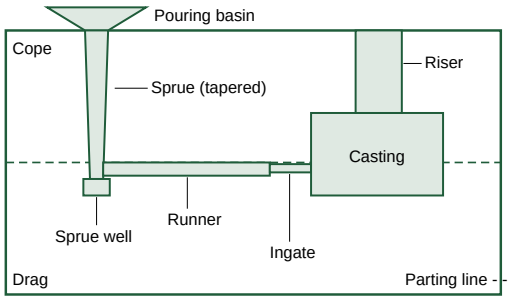


Fig. 10.1 Elements of a gating system (runner in drag, top riser on casting).

Element	Function
Pouring basin (cup)	Receives metal, reduces momentum/erosion, keeps sprue full, separates slag (with skimmer/dam)
Sprue (down-gate)	Vertical channel; **tapered (wider at top)** to avoid **aspiration**
Sprue well (base)	Enlarged base cushions the falling stream, reduces turbulence and erosion
Runner	Horizontal channel (usually in **drag**) distributing metal; **runner extension** traps first cold/dirty metal
Ingate (gate)	Entry into cavity; usually in drag/at parting
Skim bob / strainer / ceramic filter	Trap slag and sand inclusions
Riser (feeder)	Reservoir that feeds liquid to compensate **solidification shrinkage**

Nandyal Banking Coaching Center

Gate type	Features
Top gate	Metal falls directly; **favourable temperature gradient** (hottest metal on top); but turbulence, erosion, oxidation – for ferrous, simple shapes. **Not for Al/Mg**
Bottom gate	Metal enters at bottom, rises calmly – minimum turbulence; **unfavourable gradient** (hot metal at bottom); longer filling time
Parting-line gate	Most widely used compromise (side gate)
Step gate	Several ingates at different heights – tall castings
Horn gate	Curved bottom gate

Gating ratio (sprue : runner : ingate areas)

System	Pressurised	Unpressurised
Choke (min. area)	At **ingates**	At **sprue base**
Typical ratio	**1 : 0.75 : 0.5**, 1 : 2 : 1	**1 : 2 : 2**, 1 : 3 : 3, 1 : 4 : 4
Behaviour	System kept full, back-pressure, smaller & lighter gating (higher yield); **higher velocity, turbulence**	Low velocity, less turbulence; may not run full, lower yield
Used for	**Ferrous (grey CI, steel)**	**Al, Mg** (oxidisable, dross-forming)

KEY FORMULAE

- Continuity: $Q = A_1v_1 = A_2v_2$; Bernoulli: $p/\rho g + v^2/2g + h$ = constant
- Velocity at sprue base: $\mathbf{v = \sqrt{(2gh_t)}}$ (h_t = head above sprue base / gate)
- Sprue taper (no aspiration): $\mathbf{A_{top}/A_{bottom} = \sqrt{(h_{bottom}/h_{top})}}$ (heads measured from the free surface in the basin) → $A \propto 1/\sqrt{h}$
- Top gating filling time: $\mathbf{t_f = V \div (A_g\sqrt{(2gh_t)})}$
- Bottom gating filling time: $\mathbf{t_f = (A_m/A_g)\cdot(1/\sqrt{(2g)})\cdot 2(\sqrt{h_t} - \sqrt{(h_t - h_m)})}$ = 2× top-gated time when $h_m = h_t$
- Reynolds number in gating should be low; turbulent flow (Re > 2000) is common but Re > ≈20 000 causes severe oxide/air entrapment
- **Chvorinov's rule:** $\mathbf{t_s = B\,(V/A)^n}$, n ≈ **2** (1.5–2); B = mould constant (depends on mould & metal properties, pouring temperature)
- Casting modulus M = V/A; Riser condition: $\mathbf{t_{riser} \geq 1.25\ t_{casting}}$ (≈ $M_R \geq 1.1–1.2\ M_C$)
- Cylindrical side riser (all faces cooling): V/A = DH/(2D + 4H); maximum at **H = D** → **V/A = D/6**
- Top riser (bottom face in contact with casting not counted): optimum **H = D/2**
- Sphere: V/A = D/6; cube of side a: V/A = a/6; long plate thickness t: V/A ≈ t/2; long bar d: ≈ d/4
- **Caine's equation: X = a/(Y – b) + c**, X = freezing ratio = $(A/V)_{casting} \div (A/V)_{riser}$; Y = $V_{riser}/V_{casting}$; for steel a = 0.10, b = 0.03, c = 1.0. Sound casting if the point lies **above** the curve
- NRL (shape-factor) method: shape factor = (L + W)/T

- **Aspiration** = air sucked into metal stream where pressure falls below atmospheric – in a **straight (non-tapered) sprue** and sharp corners. Prevented by tapered sprue, sprue well, rounded corners, permeable mould.
- Three stages of shrinkage: (i) **liquid contraction** and (ii) **solidification shrinkage** → compensated by **riser**; (iii) **solid contraction** → compensated by **pattern shrinkage allowance**.
- Volumetric solidification shrinkage: **Al ≈ 6.6%**, Cu ≈ 4.9%, carbon steel ≈ 2.5–3%, **grey CI ≈ nil or slight expansion** (graphite).
- Pure metals and eutectics freeze at a **constant temperature** (plane/skin front, columnar grains); alloys freeze over a **freezing range** (mushy zone, dendrites) → more prone to dispersed porosity.
- Ingot/casting structure: **chill zone** (fine equiaxed at wall) → **columnar zone** → **central equiaxed zone**.
- **Directional (progressive) solidification**: solidification should proceed from the remotest point **towards the riser**; riser freezes last. Achieved by riser placement on the heaviest section, chills, padding, exothermic/insulating sleeves, top gating, gating through riser.
- **Chills** – metal inserts that increase local cooling rate. **External chills** (in mould wall; Cu, CI, steel, graphite) and **internal chills** (in cavity, fuse into casting, same metal).
- **Padding** – extra taper of metal added to a section to create a gradient towards the riser; removed later by machining.
- **Exothermic compounds** (on riser top) and **insulating sleeves** keep riser liquid longer (raise riser efficiency).
- **Open riser** (open to atmosphere at top) vs **blind riser** (enclosed; needs atmospheric vent – Williams core). **Top riser** vs **side riser**; side riser often connected to ingate (hot metal path).
- **Feeding distance** of a riser is limited (for steel plates ≈ 4.5 T including end effect) → long castings need several risers.
- **Fluidity** = ability to fill the mould; measured by **spiral fluidity test** (length of spiral filled). Increases with **superheat (pouring temperature)**; eutectics have high fluidity.
- **Casting yield** = casting weight ÷ (casting + gating + risers) weight.

MEMORY TRICK

"**P**ressurised = **P**inched at the gate" (ingate smallest, for ferrous). "**Un**pressurised = **Un**-reactive needs calm" (Al/Mg, choke at sprue). Riser shape: "side riser **H = D**, top riser **H = D/2**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Solidification time is proportional to? → **$(V/A)^2$** (Chvorinov).
- Optimum H/D for a cylindrical side riser? → **1** (V/A = D/6).
- Why is the sprue tapered? → To avoid **aspiration** of air.
- Gating ratio 1:2:2 is? → **Unpressurised** (used for Al).
- Riser compensates which shrinkage? → **Liquid + solidification shrinkage**.
- Bottom gating vs top gating filling time ($h_m = h_t$)? → **Twice**.

10.3 Casting Processes & Defects ★★★

Process	Mould / key features	Typical products
Sand casting	Expendable sand mould, permanent pattern; any metal, any size; poor finish/tolerance	Machine beds, engine blocks, pump bodies
Shell moulding (Croning / C-process)	Sand + **phenol-formaldehyde (thermosetting) resin** on **heated metal pattern** (≈ 175–370 °C) forming thin shell; good finish & accuracy, mass production	Crankshafts, camshafts, small gears

Nandyal Banking Coaching Center

Investment (lost-wax, precision)	Wax pattern + tree → ceramic slurry/stucco coats → dewax → fire → pour (hot mould). **Best finish/tolerance, very intricate, any alloy incl. high-melting**; costly	**Gas-turbine blades**, jewellery, dental, surgical
Hot-chamber die casting	Injection chamber (**gooseneck**) immersed in molten metal; fast (high cycle rate); pressures ≈ **7–35 MPa**	**Zn, Sn, Pb, Mg** (low-melting) – toys, carburettor bodies
Cold-chamber die casting	Metal ladled into separate shot chamber; higher pressure (≈ 20–70 MPa or more); avoids attack on plunger	**Al, Mg, Cu-alloys (brass)**
Permanent mould (gravity die)	Metal (CI/steel) mould, gravity pour; fine grain, good finish	Al pistons, gear blanks
Low-pressure die casting	Metal pushed up a riser tube by gas pressure (≈ 0.1 MPa)	Al wheels
Slush casting	Permanent mould; metal poured, thin skin solidifies, **remaining liquid poured out → hollow**	Ornaments, toys, lamp bases (Zn, Pb, Sn alloys)
True centrifugal	Rotation about own axis (usually **horizontal**); **no core**, outer shape by mould, bore by centrifugal force; dense outside, impurities at inner surface	**CI pipes**, liners, bushes
Semi-centrifugal	Vertical axis; axisymmetric solid shape; core may be used at centre	Wheels, pulleys, gear blanks
Centrifuging (pressure casting)	Several small cavities around central sprue; rotation gives feeding pressure; non-symmetric parts	Small intricate parts
Continuous casting	Water-cooled Cu mould, product withdrawn continuously; high yield	Billets, slabs, blooms
Full-mould / lost-foam (evaporative pattern)	**EPS pattern not removed** – vaporised by metal; no parting line, no cores, no draft; lost-foam uses unbonded sand	Engine blocks, manifolds
CO_2 moulding	Sodium silicate sand hardened by CO_2 gas	Moulds and cores
Plaster / ceramic mould	Plaster (Al, Zn, Cu – low mp only); ceramic for high-temperature alloys	Precision non-ferrous
Squeeze casting	Liquid metal solidified under high pressure in closed die	Al automotive parts

- Centrifugal casting speed is chosen to give ≈ **60–80 g** at the mould wall (horizontal machines); G-factor = $\omega^2 R/g$.
- Die-casting dies: hot-work tool steel (H13); die life highest for **Zn**, lowest for Cu-alloys.
- Die casting gives **thinnest sections** (≈ 0.5 mm Zn) and highest production rate; unsuitable for **ferrous** (too high mp).
- **Mg** can be die-cast in **both** hot- and cold-chamber machines; **Al** only in **cold chamber** (it attacks the immersed iron plunger/gooseneck).

Casting defects

Defect	Appearance / cause	Remedy
Blow holes	Smooth, rounded cavities near surface; **excess moisture, low permeability**, hard ramming, damp cores/chaplets	Control moisture, venting, permeable sand
Gas porosity / pin holes	Small dispersed holes; dissolved gas (H_2 **in Al**, N_2) released on cooling	Degassing (fluxing, vacuum), lower pouring temperature
Shrinkage cavity	Rough, dendritic cavity at last-freezing (hot-spot) section; **inadequate feeding**	Proper riser, chills, directional solidification
Misrun	Cavity **not completely filled**; low fluidity, low pouring temperature, thin sections, slow pouring	Raise pouring temperature, faster pouring, better gating
Cold shut	Two metal streams meet but **do not fuse** (visible line/discontinuity)	Same as misrun; more ingates
Hot tear (pulls)	Irregular cracks while metal is hot and weak; **restrained contraction**, poor collapsibility	Collapsible sand/cores, fillets, uniform sections
Scab	Rough metal patch – sand layer lifts/erodes and metal flows behind it; **sand expansion**, hard ramming, low permeability	Wood flour/cereal additions, uniform ramming
Rat tail / buckle	Line/depression on surface due to sand expansion compressed failure	As for scab
Swell	Local enlargement of casting; **soft ramming** – mould wall moves under metallostatic pressure	Hard, uniform ramming
Fins / flash	Thin metal projections at **parting line** or core seats; improper closing, worn patterns	Proper clamping, weights on cope
Inclusions (sand/slag)	Non-metallic particles; eroded sand, slag carried in	Skimming, strainers/filters, sprue well
Mould/core shift	Mismatch at parting line; misaligned flasks/core	Proper dowels, core prints
Drop	Cope sand falls into cavity; low green strength	Better sand strength, gaggers
Metal penetration	Metal enters sand voids – rough surface; coarse sand, high temperature	Finer sand, mould wash/coating
Run-out	Metal leaks out of mould	Proper mould closing
Warpage	Distortion from unequal cooling	Uniform sections, distortion allowance

- **Inspection of castings**: visual, dimensional; surface cracks – **dye penetrant** (any non-porous metal), **magnetic particle** (ferromagnetic only); internal – **radiography (X-ray/γ-ray)** best for porosity/shrinkage, **ultrasonic**; **pressure/leak test** for pressure-tight castings; eddy current.
- **Fettling/cleaning**: knockout (shake-out), removal of gates/risers (cutting, hammering), sand blasting/shot blasting, tumbling, grinding. **Impregnation** seals porosity.

MEMORY TRICK

"**Mis**run = **Miss**ing metal (not filled); **Cold** shut = **Cold** streams don't shake hands." "**S**well = **S**oft ramming; **S**cab = **S**and expansion."

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Die casting of Al is done in? → **Cold-chamber** machine.
- CI water pipes are made by? → **True centrifugal casting**.
- Gas-turbine blades of superalloys are cast by? → **Investment casting**.
- Hollow ornaments/toys of low-melting alloys? → **Slush casting**.
- Resin used in shell moulding? → **Phenol-formaldehyde**.
- Defect due to restrained contraction while hot? → **Hot tear**.

10.4 Welding Fundamentals & Arc Welding ★★★

Class	Principle	Examples
Fusion welding	Melting of faying surfaces (with/without filler), no pressure	Arc (SMAW, TIG, MIG, SAW, PAW), gas, thermit, EBW, LBW, electroslag
Pressure (plastic) welding	Heating to plastic/partly molten state + pressure	Resistance (spot, seam, projection, flash), forge
Solid-state welding	No melting; bonding by pressure/deformation/diffusion	Friction, FSW, ultrasonic, explosive, diffusion, cold, roll, forge

- **Welding** = permanent joint by coalescence; joints: butt, lap, tee, corner, edge. Welds: groove (butt) and fillet.
- **Arc temperature ≈ 5000–6000 °C** (SMAW). Arc initiated by touching and withdrawing (or by high-frequency spark in TIG).
- In a DC arc about **two-thirds of the heat is liberated at the anode (+)** and one-third at the cathode (–) (textbook statement for consumable/ordinary arcs).

Polarity	Connection	Heat distribution	Use
DCSP / DCEN (straight)	Electrode –, work +	**≈ 2/3 heat at work → deep penetration**, electrode cooler (TIG electrode life longer)	Thick sections; **TIG of steel, SS, Ti, Cu**; bare electrodes
DCRP / DCEP (reverse)	Electrode +, work –	≈ 2/3 heat at electrode → faster melt-off, **shallow penetration**	**Thin sheets**, non-ferrous; **cleaning action** (oxide removal by positive-ion bombardment of work) – Al, Mg; **MIG/GMAW standard polarity**
AC	Alternating	Heat equally shared; penetration intermediate; **practically no arc blow**	**TIG of Al & Mg** (cleaning + penetration); SMAW with transformer

- **Note**: in **GMAW (MIG)** DCEP actually gives stable spray transfer and **deeper** penetration than DCEN – an exception to the general SMAW/TIG rule; for exam MCQs use "DCSP → deeper penetration".
- **Arc blow** = deflection of DC arc by unbalanced magnetic fields (near ends/corners); cured by **using AC**, changing earth position, shorter arc, tack welds.
- Open-circuit voltage (OCV) ≈ **50–100 V** (AC usually 70–80 V max for safety); arc voltage ≈ **20–40 V**. Arc length ≈ electrode core diameter.

Power-source characteristics

Characteristic	Behaviour	Used for
Drooping / constant-current (CC)	Large change in voltage gives **small change in current** when arc length varies (manual hand wobble tolerated)	**SMAW, TIG**, (PAW)
Constant-voltage (CV, flat)	Small change of arc length → large change in current → **self-adjusting (self-regulating) arc** with constant wire-feed speed	**MIG/MAG, FCAW, SAW** (thin wire)
Rising characteristic	Voltage rises with current	Automatic GMAW (some)

- Power sources: **transformer** (AC only, cheapest, highest efficiency, least maintenance), **rectifier** (DC from AC), **motor/engine generator** (DC, site work), **inverter** (light, fast response). **Duty cycle** = % of a 10-min (5-min in some standards) period the source can deliver rated current; $I_2 = I_1\sqrt{(D_1/D_2)}$.

KEY FORMULAE

- Arc voltage (Ayrton): $V = A + B \cdot l$ (l = arc length)
- Linear power-source characteristic: $V/V_{OC} + I/I_{SC} = 1$; maximum arc power when $V = V_{OC}/2$, $I = I_{SC}/2$ → $P_{max} = V_{OC}I_{SC}/4$
- **Heat input per unit length** $H = \eta V I \div v$ (J/mm; v = travel speed)
- Melting efficiency η_m = (volume melted × heat to melt unit volume) ÷ (η V I t)
- Duty cycle relation: $I_2^2 D_2 = I_1^2 D_1$
- Heat-transfer (arc) efficiency η: **SAW ≈ 0.9–0.99 (highest)**, SMAW ≈ 0.7–0.85, GMAW ≈ 0.7–0.85, GTAW lowest (≈ 0.2–0.5 in Al to ≈ 0.6–0.7 in steel – texts vary)

Shielded Metal Arc Welding (SMAW / MMA / stick)

- Consumable **flux-coated electrode**; most versatile, cheap, all positions, site work; low deposition rate, slag removal needed, stub loss.
- **Functions of coating**: (1) **arc stabilisation** (K, Na silicates, TiO_2); (2) **gas shield** (cellulose, $CaCO_3 \rightarrow CO_2$); (3) **slag formation** – protects bead, slows cooling; (4) **deoxidation** (ferro-Mn, ferro-Si); (5) **alloying**; (6) increased deposition (**iron powder**); (7) binding (sodium silicate).
- Coating types: **cellulosic** (deep penetration, vertical-down, pipelines – E6010), **rutile (TiO_2)** (general purpose, easy arc, smooth bead – E6013), **basic / low-hydrogen ($CaCO_3$ + CaF_2)** (crack-free, high-strength & thick steels – E7018; must be **baked** ≈ 250–350 °C), **acid (iron oxide)**.
- **AWS A5.1 designation E XX Y Z**: E = electrode; first 2 digits = **min. UTS in ksi** (60 → 60 ksi ≈ 414 MPa; 70 → ≈ 483 MPa); 3rd digit = **position** (1 = all positions, 2 = flat & horizontal fillet, 4 = flat, horizontal, overhead & vertical-down); last two digits together = **coating type & current**.
- **E6013** = 60 ksi, all positions, **rutile (high-titania potassium)** coating, AC or DC (either polarity). E7018 = 70 ksi, all positions, low-H iron-powder, AC or DCEP.
- Indian code for covered electrodes: **IS 815**.

Process	Electrode / shielding	Key points & use

Nandyal Banking Coaching Center

GTAW / TIG	**Non-consumable tungsten** (pure W for AC; **2% thoriated**, ceriated or lanthanated for DC; zirconiated for AC); inert gas **Ar** or **He**; filler rod optional (autogenous possible)	Highest-quality, clean, no slag/spatter; **thin sections, Al, Mg, SS, Ti**, root passes of pipes; low deposition rate. Ar heavier than air → better shielding, smoother arc; He → hotter arc, higher voltage, deeper penetration. High-frequency arc start
GMAW – MIG	Continuous **consumable bare wire**, inert gas (Ar/He)	Non-ferrous (Al, Cu); semi-automatic, high deposition, no slag; **DCEP + CV source**
MAG	Wire + **active gas CO_2** or Ar + CO_2/O_2	Carbon steels; CO_2 cheap, deep penetration but spatter
FCAW	Tubular wire with flux core (self-shielded or gas-shielded)	Outdoor, structural steel
SAW	Bare wire under blanket of **granular flux**; arc **invisible** (submerged)	**High current (≈ 300–2000 A), highest deposition, deep penetration, no spatter/glare**; **flat & horizontal-fillet positions only**; **thick plates** – ship-building, pressure vessels, pipes, girders; unused flux recycled
PAW (plasma arc)	Tungsten electrode, arc constricted through water-cooled copper nozzle; orifice + shielding gas	Plasma ≈ **10 000–30 000 °C**; **keyhole** welding; **transferred arc** (work is part of circuit – welding/cutting metals) vs **non-transferred** (arc between electrode and nozzle – spraying, cutting non-conductors)
Electroslag (ESW)	Arc only to start; then heat by **electrical resistance of molten slag** (I^2R); water-cooled Cu shoes	**Vertical** position, **very thick plates (≈ 50–900 mm)**, single pass; coarse grain HAZ. Strictly **not an arc process**
Electrogas (EGW)	Continuous arc + shielding gas (or flux-cored wire), Cu shoes	Vertical, single pass, ≈ 12–75 mm thick – ship hulls, storage tanks
Stud welding	Stud acts as electrode; ceramic ferrule	Studs on plates, shear connectors
Carbon arc	Carbon/graphite electrode, **DCSP** (to avoid carbon pick-up)	Rarely used now
Atomic hydrogen	Arc between two W electrodes in H_2; H_2 dissociates and recombines on work releasing heat	Thin sheets, hard-to-weld alloys

- GMAW **metal-transfer modes**: **short-circuit (dip)** – low current, thin sheets, all positions; **globular** – medium current, CO_2; **spray** – above transition current, Ar-rich gas, flat position, high deposition; **pulsed spray** – spray at low mean current.
- Deposition rate order (typical): **SAW > GMAW/FCAW > SMAW > GTAW**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In DCSP, the electrode is connected to? → **Negative terminal** (deeper penetration).
- Polarity giving cleaning action for Al? → **DCEP (reverse)**; TIG of Al normally uses **AC**.
- Power-source characteristic for SMAW? → **Drooping (constant current)**; for MIG → **constant voltage**.
- Welding with invisible arc under granular flux? → **Submerged arc welding**.
- V_{OC} = 80 V, I_{SC} = 1000 A (linear); max arc power? → **20 kW** (40 V × 500 A).
- In E6013, "60" denotes? → Minimum tensile strength **60 ksi**.

10.5 Gas, Resistance & Solid-State Welding ★★★

Oxy-acetylene welding (OAW)

- **Primary combustion (inner cone)**: $C_2H_2 + O_2 \rightarrow 2CO + H_2$ (oxygen from cylinder, **1 : 1**); about one-third of total heat; hottest point just ahead of the inner cone tip.
- **Secondary combustion (outer envelope)**: $2CO + H_2 + 1.5\ O_2 \rightarrow 2CO_2 + H_2O$ (O_2 from **atmosphere**); about two-thirds of total heat; outer envelope also shields the pool.
- Total oxygen for complete combustion = **2.5 volumes per volume of C_2H_2** (1 from cylinder + 1.5 from air).
- **Maximum flame temperature ≈ 3200 °C** (neutral ≈ 3100–3300 °C) – highest of common fuel gases.
- Acetylene is stored **dissolved in acetone** in a cylinder filled with porous mass (free acetylene unsafe above ≈ **1 bar gauge (15 psig)**). Indian cylinder colours: **oxygen – black**, **acetylene – maroon**. Oxygen cylinder pressure ≈ 150 bar.
- Hoses: oxygen – green/blue (right-hand thread); acetylene – red (left-hand thread, grooved nut).

Flame	O_2 : C_2H_2	Appearance	Use
Neutral	**≈ 1 : 1**	Sharp, well-defined inner cone, no feather	Most metals – **mild steel, CI, SS, Cu, Al**
Oxidising	> 1 (≈ 1.15–1.5 : 1)	Short, pointed inner cone, hissing; **hottest**	**Brass, bronze** (oxide film reduces Zn fuming)
Carburising (reducing)	< 1	White **acetylene feather** around inner cone; lowest temperature	**High-carbon steel**, hard-facing (stellite), Monel, Ni alloys, non-ferrous

- **Leftward (forehand)** technique – thin plates (**≤ 5 mm**); **rightward (backhand)** – thicker plates (> 5 mm), faster, less gas.
- Other fuel gases: **oxy-hydrogen** (≈ 2800 °C, lower; for Al, Pb), oxy-LPG/propane (mainly cutting, preheating), MAPP.
- Flux in gas welding: borax-based for Cu alloys; chlorides/fluorides for Al; none for mild steel.

Oxy-fuel gas cutting

- Principle: preheat steel to **ignition (kindling) temperature ≈ 870 °C** (≈ 800–900 °C) then a jet of pure O_2 **oxidises** iron ($3Fe + 2O_2 \rightarrow Fe_3O_4$, exothermic) and blows away molten oxide. It is **chemical oxidation, not melting**.
- Conditions: ignition temperature < melting point of metal; oxide melting point < metal melting point; reaction exothermic; oxide

Nandyal Banking Coaching Center

fluid. → Works for **low/medium carbon steels** (ferrous).

- **Not** directly cut: **cast iron** (ignition temp > mp), **stainless steel** (Cr_2O_3 refractory), **Al, Cu** (oxide refractory / high conductivity) → use **powder (flux) cutting**, **plasma arc** or laser cutting.
- **Kerf** = width of cut; **drag** = lag of cut bottom behind top.

Resistance welding

KEY FORMULAE

- **Heat H = I^2 R t** (Joule's law); R = sum of bulk + contact resistances (**contact resistance between sheets** is the main heat source)
- Heat to melt nugget = volume × ρ × (c ΔT + latent heat); efficiency = useful heat ÷ I^2Rt

- **Very high current (≈ 3–100 kA; spot typically 5–20 kA) at low voltage (≈ 1–10 V)**, short time (fraction of a second to a few cycles), pressure applied. No filler, flux or shielding.
- **Electrodes**: **copper alloys** – high electrical & thermal conductivity with hardness: **Cu–Cr, Cu–Cr–Zr, Cu–Be**, Cu–W (RWMA classes); water cooled.
- Spot-weld cycle: **squeeze → weld → hold → off** time.
- Al is difficult (high conductivity → needs very high current); low-carbon steel is easiest.

Process	Features	Applications
Spot	Lap joint, two rod electrodes, discrete nuggets; thickness usually up to ≈ 3 mm each sheet	**Car bodies**, appliances, sheet assemblies
Seam	Rotating **roller (wheel) electrodes** – overlapping spots → continuous **leak-tight** seam	**Fuel tanks**, drums, mufflers, containers
Projection	Embossed **projections** localise current; several welds in one stroke; longer electrode life	Nuts/bolts to sheets, wire mesh
Upset butt	Ends pressed together, heated by resistance, then upset; no flashing	Wires, rods (small sections)
Flash butt	Light contact, **arcing/flashing** expels oxides, then rapid upset; better quality than upset butt	**Rails**, chain links, pipes, band-saw blades
Percussion	Very short discharge (capacitor) arc followed by impact	Electrical contacts, wires
High-frequency resistance	HF current along edges (skin/proximity effect)	ERW pipes and tubes

Other fusion processes

- **Thermit welding** – exothermic reaction of **Al powder + iron oxide** ($8Al + 3Fe_3O_4 \rightarrow 4Al_2O_3 + 9Fe$), ignited by Mg ribbon/barium peroxide; temperature ≈ **2500–3000 °C**; ratio ≈ 1 : 3 (Al : oxide) by weight. Uses: **rail joints**, repair of heavy sections (shafts, frames). Both fusion and (in pressure thermit) pressure.
- **Electron beam welding (EBW)** – focused high-velocity electrons (accelerating voltage ≈ 30–200 kV) in **vacuum**; very high power density; **deep, narrow welds, depth/width up to ≈ 25 : 1**, narrow HAZ, minimal distortion; generates **X-rays** (shielding needed); reactive & refractory metals (Ti, Zr, Mo), aerospace.
- **Laser beam welding (LBW)** – **CO_2 (10.6 µm)**, **Nd:YAG (1.06 µm)**, fibre lasers; **no vacuum needed**, no X-rays, keyhole mode, small HAZ, automated, micro-welding; difficulty with **highly reflective** metals (Al, Cu).

Solid-state welding

Process	Principle / features	Applications
Friction (rotary / inertia)	One part rotated against stationary part under axial pressure; frictional heat → plastic state → forging pressure; **flash** formed; no melting	**Dissimilar metals** (Al–steel), engine valves, axle shafts, drill bits
Friction stir welding (FSW)	Non-consumable rotating tool with **shoulder + pin** traversed along joint; invented by **TWI, UK (1991)**; no filler/shielding, low distortion	**Al alloys** (aircraft, ship, rail-coach panels)
Ultrasonic	High-frequency (≈ **20 kHz**, typically 10–75 kHz) **vibrations parallel** to interface under moderate pressure; low temperature	Foils, fine wires, **electronics**, battery tabs, **plastics**
Explosive	Controlled detonation drives flyer plate at high velocity; **wavy interface**	**Cladding** (dissimilar plates), tube-to-tubesheet
Diffusion	High temperature (≈ 0.5–0.7 T_m) + pressure for long time; atomic diffusion; vacuum/inert	Ti, superalloys, aerospace, dissimilar metals
Forge	Heating in forge + hammering; **oldest welding process** (blacksmith)	Low-carbon steel, wrought iron
Cold (pressure)	Pressure only at room temperature; ductile metals	Al, Cu wires
Roll bonding	Rolling pressure	Clad coins, bimetal strips

MEMORY TRICK

Flames: "**O**xidising for **B**rass (**OB**), **C**arburising for **C**arbon-rich steel (**CC**), **N**eutral for **N**early everything." Resistance: "**Spot** = **Spots** on car bodies, **Seam** = **Sealed** tanks, **Flash** = **Flat**-bottom rails."

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Flame used for welding brass? → **Oxidising flame**.
- Ratio of $O_2 : C_2H_2$ in neutral flame? → **1 : 1**.
- Why is cast iron not oxy-cut easily? → Its **ignition temperature exceeds its melting point**.
- Welding of rails in the field? → **Thermit** (also flash butt in depots).
- Process for leak-proof fuel tanks? → **Seam welding**.
- Welding process requiring vacuum and producing X-rays? → **Electron beam welding**.

10.6 Weldability, Defects & Inspection ★★☆

- **Weldability** = ease of producing a sound, crack-free joint with required properties. Low-carbon steel – **excellent**; high-carbon, alloy steels, CI – poor (martensite, cracking).
- **Heat-affected zone (HAZ)** = base metal not melted but whose microstructure/properties are altered. Zones across a steel weld: **fusion zone** (cast, columnar) → partially melted zone → **coarse-**

Nandyal Banking Coaching Center

grain HAZ (most brittle, near fusion line) → fine-grain (normalised) HAZ → intercritical → subcritical (tempered) → unaffected base metal.

- HAZ width ↑ with heat input; EBW/LBW give narrowest HAZ; gas welding and ESW give widest.
- **Cooling rate** ↑ with thickness and lower preheat → more martensite → hardness and cracking risk.

KEY FORMULAE

- Carbon equivalent (IIW): **CE = C + Mn/6 + (Cr + Mo + V)/5 + (Ni + Cu)/15** (wt %)
- **CE < 0.4** → good weldability, no preheat; **0.4–0.6** → **preheat** (≈ 100–250 °C) & low-H electrodes; **> 0.6** → difficult, preheat + post-weld heat treatment (some texts use 0.45 as the limit)
- Stress-relief annealing of C-steel welds: ≈ **550–650 °C (≈ 600 °C)**, hold ≈ 1 h per 25 mm thickness, slow cool

Weld defects

Defect	Cause	Remedy
Porosity / blow holes	Gas entrapment – moisture, rust, oil, damp electrodes, poor shielding, long arc	Clean joint, dry electrodes, correct gas flow
Slag inclusion	Slag not removed between passes, low current, poor manipulation	Inter-pass cleaning, proper current
Lack of fusion	Weld metal does not fuse with base/previous pass; low heat input, fast travel, wrong angle	Higher current, correct technique
Incomplete (lack of) penetration	Weld does not reach root; low current, small root gap, large root face, fast travel	Proper joint design, back-gouging
Undercut	Groove melted into base metal at toe, not filled; **excessive current, long arc**, wrong angle, high speed	Lower current, shorter arc
Overlap (cold lap)	Weld metal flows over base without fusing; **low current**, slow travel	Higher current, faster travel
Spatter	Metal droplets expelled; high current, long arc, CO_2 shielding, arc blow	Adjust parameters
Hot (solidification) cracks	At high temperature during solidification; **S, P** (low-mp films), high C, high restraint, concave/deep narrow beads	Low S/P, adequate **Mn : S** ratio, proper bead shape
Cold / hydrogen-induced (delayed, underbead) cracks	Below ≈ 200–300 °C, may appear hours/days later in **HAZ**; needs **hydrogen + martensite (hard HAZ) + tensile stress**	**Low-hydrogen (baked) electrodes, preheat**, post-heat, low-CE steel
Lamellar tearing	Step-like cracks in thick rolled plate under **through-thickness** stress; elongated MnS inclusions	Clean (low-S, Z-grade) steel, joint redesign, buttering
Crater crack	At end of bead when arc broken abruptly	Fill crater, run-off tabs
Distortion	Non-uniform heating/cooling; longitudinal & transverse shrinkage, **angular distortion**	Pre-setting, clamping, balanced/back-step welding, fewer passes, intermittent welds

Residual stresses, weld symbols, inspection

- **Residual stresses**: weld metal and adjacent zone in **longitudinal tension** (≈ up to yield strength), balanced by compression further away. Relief by **stress-relief annealing**, **peening** (cold work, compressive surface), vibratory stress relief, preheating.
- **Weld symbol (AWS A2.4 / IS 813)** elements: reference line, arrow, basic weld symbol, dimensions, supplementary symbols, tail (process/specification).
- Symbol **below** the reference line → weld on **arrow side**; **above** → **other side**; both sides → both. (ISO 2553 uses a solid + dashed dual reference line.)
- Basic symbols: fillet – right triangle; square butt – two parallel lines; V-groove – V; bevel – half-V (one vertical + one sloped line); U and J – curved symbols; plug/slot – rectangle; spot – circle; seam – circle with two lines. Supplementary: **circle at arrow-elbow = weld all around**; **flag = field (site) weld**; flush/convex/concave contour.
- **Inspection / NDT**: visual (most basic, first), **dye/liquid penetrant** (surface-open defects, any non-porous material), **magnetic particle** (surface & near-surface, **ferromagnetic only**), **radiography** (volumetric – porosity, slag; permanent film record), **ultrasonic** (planar defects – cracks, lack of fusion; thick sections, one-side access), eddy current (conductive, surface). Destructive: tensile, bend (face/root/side), nick-break, impact (Charpy), macro-etch, hardness.

MEMORY TRICK

Cold cracking needs "**H-M-S**" – **H**ydrogen + **M**artensite + **S**tress; remove any one (low-H electrode, preheat, stress relief) and it will not occur. "**U**ndercut = **U**p current; **O**verlap = **O**ff (low) current."

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Preheating is needed when carbon equivalent exceeds? → **≈ 0.4**.
- Main cause of undercut? → **Excessive current** / long arc.
- Hydrogen-induced cracking occurs in? → **HAZ**, delayed, at low temperature.
- Flag on the weld symbol indicates? → **Field (site) weld**.
- Best NDT for internal porosity in welds? → **Radiography**.

10.7 Brazing, Soldering & Adhesive Bonding ★★☆

Parameter	Welding	Brazing	Soldering
Base metal melted?	Yes (fusion)	**No**	**No**
Filler melting point	≈ base metal	**> 450 °C** (840 °F) but **below base-metal solidus**	**< 450 °C**

Nandyal Banking Coaching Center

Joining mechanism	Fusion / metallurgical	**Capillary action**, wetting, some diffusion	Capillary, wetting, intermetallic layer
Strength	Highest	Medium-high (can exceed filler strength)	Lowest
Dissimilar metals	Difficult	Easy	Easy
Distortion / HAZ	High	Low	Negligible

Brazing

- Joint clearance for capillary flow ≈ **0.025–0.125 mm** (too large → no capillarity; too small → no flow). Lap joints preferred (larger area).
- Fillers: **brass (Cu–Zn, "spelter")**, **silver alloys (Ag–Cu–Zn, "silver brazing/hard soldering")** ≈ 600–800 °C, **Cu–P** (self-fluxing on copper), **Al–Si** (for Al), Ni-based (high temperature), pure Cu (furnace brazing of steel).
- Fluxes: **borax**, boric acid, borates, fluorides (fluoride fluxes for Ag-brazing and Al). Flux removes oxides, prevents re-oxidation, promotes wetting.
- Methods: **torch**, **furnace** (controlled atmosphere/vacuum, no flux), **induction**, **dip** (salt or metal bath), resistance, infrared.
- Applications: carbide tool tips to shanks, bicycle frames, heat exchangers, radiators, pipe fittings, refrigeration tubing.
- **Braze welding** – filler (usually bronze/brass) deposited in a **groove/fillet like gas welding**; **no capillary action**; base metal not melted; used for **repair of cast iron** and joining galvanised steel.

Soldering

- Classic Sn–Pb solders: **63Sn–37Pb eutectic melts at 183 °C** (single temperature – electronics); **60Sn–40Pb** (≈ 183–190 °C); 50/50 (plumbing, wider pasty range ≈ 183–216 °C).
- **Lead-free** (RoHS, EU 2006): **SAC305 = Sn–3.0Ag–0.5Cu**, melting ≈ **217–220 °C**; Sn–0.7Cu (≈ 227 °C); Sn–3.5Ag (221 °C).
- **Fluxes**: **rosin (non-corrosive)** – electronics; **inorganic acid fluxes – $ZnCl_2$ + NH_4Cl ("killed spirit")** – corrosive, sheet metal/plumbing, residues must be washed; organic (water-soluble) fluxes.
- Methods: soldering iron (copper bit), torch, dip, **wave soldering** (through-hole PCBs), **reflow soldering** (SMD – solder paste), induction, ultrasonic (fluxless, Al).
- Solder joint clearance ≈ 0.075–0.125 mm; soldered joints are weak in tension → use mechanical interlock (seams, crimps).
- **Wetting**: contact angle $\theta < 90^\circ$ → wets (good); $\theta \to 0$ for ideal wetting.

Adhesive bonding

- Joins similar/dissimilar materials (metals, plastics, composites) with polymer adhesive; **uniform stress distribution**, no HAZ, sealing, vibration damping, light weight.
- Limitations: **low service temperature** (≈ ≤ 200 °C), long curing, surface preparation critical, **weak in peel and cleavage**, difficult inspection.
- Joints should load the adhesive in **shear** (lap joints) or compression – avoid peel.
- Types: **epoxy** (thermoset, structural, two-part), **cyanoacrylate** ("super-glue", instant, moisture-cured), **anaerobic** (cures without air – **thread-lockers**), polyurethane, acrylic, silicone, **hot-melt** (thermoplastic), phenolics; pressure-sensitive (tapes).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Temperature dividing brazing and soldering? → **450 °C** (840 °F).
- Melting point of 63/37 Sn–Pb solder? → **183 °C** (eutectic).
- Common flux for brazing? → **Borax**.
- Flux for electronic soldering? → **Rosin** (non-corrosive).
- Repair of cracked CI by bronze filler in a V-groove without capillary flow? → **Braze welding**.

10.8 Metal Forming & Sheet Metal Fabrication ★★★

Aspect	Hot working	Cold working
Temperature	**Above recrystallisation temperature** (≈ 0.6–0.8 T_m)	Below recrystallisation (usually room temperature, < 0.3 T_m)
Strain hardening	**None** (simultaneous recrystallisation)	**Yes** – strength, hardness ↑, ductility ↓
Force / power	Low	High
Surface finish & tolerance	Poor (scale, oxidation)	**Good**, close tolerances
Grain structure	Refined, porosity closed, inclusions broken; **fibre (grain flow)**	Elongated grains, anisotropy, residual stress
Deformation per step	Large	Limited – intermediate (process) annealing needed

- **Recrystallisation temperature ≈ 0.4 T_m** (absolute, K; range 0.3–0.5). Defined as temperature at which complete recrystallisation occurs in ≈ 1 h. Steel ≈ 450–550 °C (≈ 0.4 T_m).
- It **decreases** with more prior cold work, longer time, higher purity, finer initial grain.
- **Pb, Sn, Zn** recrystallise near/below room temperature → working them at room temperature is **hot working**.
- **Warm working** ≈ 0.3–0.5 T_m. **Homologous temperature** = T/T_m (in K).
- Flow stress (cold): $\sigma = K\varepsilon^n$ (n = strain-hardening exponent; n = true uniform strain at necking). Hot: $\sigma = C\,\dot{\varepsilon}^m$ (m = strain-rate sensitivity; superplastic m ≥ 0.3–0.5).

Rolling

KEY FORMULAE

- Draft $\Delta h = h_0 - h_f$; bite condition $\mu \geq \tan\alpha$ (α = angle of bite)
- **Maximum draft $\Delta h_{max} = \mu^2 R$** (R = roll radius)
- Angle of bite: $\cos\alpha = 1 - \Delta h/(2R)$; projected contact length **$L \approx \sqrt{(R\,\Delta h)}$**
- Roll separating force $F \approx L \cdot w \cdot \bar{\sigma}_f$ (w = width); torque per roll $T = F\cdot(L/2)$ (approx., $\lambda \approx 0.5$); power (2 rolls) $P = 2\pi N\cdot F\cdot L/60$ (N rpm, $\lambda = 0.5$)
- Volume constancy: $h_0 w_0 v_0 = h_f w_f v_f$ → exit velocity > entry velocity
- Forward slip = $(v_f - v_{roll})/v_{roll}$

Nandyal Banking Coaching Center

- **Neutral (no-slip) point**: strip velocity = roll surface velocity; **before it** strip moves **slower** than roll (friction acts towards exit); **after it** strip moves **faster**. Pressure (friction hill) is **maximum at the neutral point**.
- Neutral point moves towards **entry** as friction ↑ (towards exit as μ ↓; at limiting bite it reaches the exit). **Back tension** moves it towards exit, **front tension** towards entry; both reduce roll force.
- Roll force ↓ by: smaller rolls, lower friction, smaller reductions, front/back tension, higher temperature.
- Mills: **2-high** (reversing), **3-high**, **4-high** (small work rolls + large **backup rolls**), **cluster/Sendzimir (Z-mill, 20-high)** – very thin, hard strips (SS foils), **tandem** (continuous), planetary (large reduction in one pass).
- Hot rolling products: bloom (≥ 150 × 150 mm), billet (≈ 40–150 mm square), slab (rectangular, wide) → plates, sheets, sections. Ring rolling – bearing races, flanges. **Thread rolling** – cold, stronger threads (unbroken grain flow). **Pierce (Mannesmann) rolling** – seamless tubes (secondary tensile stresses open centre).
- Defects: **wavy edges** (roll bending – edges thinner), **zipper (centre) cracks** (centre elongated more), **edge cracks** (low ductility), **alligatoring** (split of slab ends – inhomogeneous deformation), folds, scale. Roll **camber/crowning** compensates roll deflection.

Forging

Type	Features
Open-die (smith/hand)	Flat dies, no lateral restriction; large parts, few pieces; operations: **upsetting, drawing out, fullering, edging, punching, bending**
Closed (impression) die	Shaped dies; **flash** through **flash land** into **gutter** – flash land restricts flow and builds pressure to fill the die; mass production
Flashless (precision)	Fully enclosed; exact volume of billet needed
Drop forging	Hammer (gravity/power) – impact, repeated blows
Press forging	Slow squeezing; deformation **penetrates deeper** (uniform), large parts
Upset forging (heading)	Increases diameter by reducing length – bolt heads, rivets
Roll forging, swaging, ring rolling	Tapered shafts, leaf springs; rotary swaging reduces tube/rod dia
Coining, heading	Closed-die cold forging; coins, medals (no flash)

- **Rules of upsetting**: (1) unsupported length ≤ **3 d** in one blow (some texts: 2.5–3 d); (2) beyond that, die cavity diameter ≤ 1.5 d; (3) unsupported length beyond the die face ≤ d.
- Forging gives **favourable grain flow (fibre)** → better fatigue and impact strength than casting/machining (crankshafts, connecting rods, spanners, gear blanks).
- Forging temperature for steel ≈ **1100–1250 °C** (start); finishing just above recrystallisation.
- **Forging defects**: **cold shut / lap / fold** (metal folds over – sharp corners, small fillet radius), **unfilled section** (insufficient metal/temperature), **die shift (mismatch)**, **scale pits**, **flakes** (internal cracks from hydrogen, rapid cooling), internal (centre) cracks, **burnt/overheated** metal, **barrelling** (friction at die faces), surface cracks.

KEY FORMULAE

- Upsetting (cylinder, sliding friction): $\mathbf{F = \sigma_f \pi r^2(1 + 2\mu r/3h)}$; average pressure $\bar{p} = \sigma_f(1 + 2\mu r/3h)$
- Upsetting of strip (plane strain): $\bar{p} \approx \sigma'(1 + \mu\, b/2h)$ ($\sigma' = 1.155\, \sigma_f$)
- Volume constancy: $A_0h_0 = A_fh_f$; true strain $\varepsilon = \ln(h_0/h_f)$

Extrusion

Type	Features	Products
Direct (forward)	Ram and product move in **same direction**; billet slides in container → **high friction, higher force**	Rods, sections, most common
Indirect (backward / inverted)	Die on hollow ram; product moves **opposite** to ram; **no billet–container friction → ≈ 25–30% lower force**; less scrap	Rods, tubes
Impact extrusion	Cold, fast punch blow on slug; usually **backward (reverse)** – thin-walled cups; **Hooker process** = forward impact extrusion of small tubes	**Collapsible tubes (toothpaste)**, cans, battery cases – **Al, Pb, Sn, Zn**
Hydrostatic	Billet surrounded by pressurised fluid; **no container friction**, high reductions; billet never touches container	**Brittle materials**, cladding
Tube extrusion	Mandrel/porthole (bridge) dies	Al window sections, tubes

- Extrusion ratio $R = A_0/A_f$ (typ. 10–100 for Al); **Al** is the most extruded metal (hot, ≈ 400–500 °C).
- Defects: **centre burst (chevron/cuppy cracks)** – high die angle with low reduction (low extrusion ratio) and high friction, hydrostatic tension at centre; **piping (tailpipe, fishtailing)** – oxides drawn into centre at end of stroke (leave discard/butt); **surface cracking (fir-tree, speed cracking)** – too high temperature/speed/friction.
- Glass (Séjournet process) used as lubricant for hot extrusion of steel.

KEY FORMULAE

- Ideal extrusion pressure $p = \sigma_f \ln R$; empirical (Johnson): $p = \sigma_f(a + b \ln R)$, $a \approx 0.8$, $b \approx 1.2–1.5$
- Extrusion force $F = A_0\, k \ln(A_0/A_f)$ (k = extrusion constant)

Wire, rod & tube drawing

- Material **pulled** through a die (tensile force) – a **cold** process; product is in tension at exit.
- Limit: drawing stress must not exceed flow stress of drawn wire. Ideal (frictionless, perfectly plastic): $\sigma_d = \sigma_f \ln(A_0/A_f) \le \sigma_f \rightarrow \ln(A_0/A_f) \le 1 \rightarrow \mathbf{A_0/A_f \le e = 2.718} \rightarrow$ **maximum reduction = 1 – 1/e ≈ 63%** per pass.
- Practical reduction per pass ≈ **20–45%** (friction, redundant work, strain hardening).
- Dies: **tungsten carbide**, **diamond** (very fine wire); die semi-angle ≈ 6–15°; regions: entry (bell), approach, **bearing (land)**, back relief.

Nandyal Banking Coaching Center

- Wire preparation: pickling, coating (lime, borax, phosphate), **pointing**. **Patenting** – heat treatment of high-carbon steel wire (fine pearlite) between draws.
- **Tube drawing**: **sinking** (no mandrel – only OD reduced), **fixed plug**, **floating plug** (longest lengths), **moving mandrel**.

KEY FORMULAE

- Drawing stress with friction (Sachs): $\sigma_d = \sigma_f \cdot [(1 + B)/B] \cdot [1 - (A_f/A_0)^B]$, $B = \mu \cot \alpha$
- Drawing force $F = \sigma_d A_f$; power = F × exit velocity

Sheet-metal working

- **Shearing** stages: elastic deformation → plastic (**rollover**) → **penetration (burnish, shiny)** ≈ 1/3 of t for soft steel → **fracture** (rough, with burr). Penetration ↓ for harder materials.
- **Clearance** per side ≈ **2.5–10% of t** (≈ 5–8% common for mild steel). Indian texts: **c = 0.0032 t √τ** (t in mm, τ in N/mm²). Too small → secondary shear, high force; too large → large burr, rollover.
- **Blanking** (piece cut out is the product): **die size = blank size**; **punch = blank − 2c**.
- **Piercing / punching** (hole is product): **punch size = hole size**; **die = hole + 2c**.
- **Shear** on punch (piercing) or die (blanking) reduces peak force; shear always on the tool that does **not** touch the product side, so the product stays flat.
- Stripping force ≈ 10–20% of cutting force. Operations: lancing, notching, nibbling, trimming, shaving, slitting, perforating, **fine blanking** (V-ring, clearance ≈ 1% t – square smooth edges).
- Press types: progressive die (stations in sequence, strip moves), **compound die** (two cutting ops – blanking + piercing – at one station in one stroke), **combination die** (cutting + non-cutting e.g. blanking + drawing in one stroke), transfer die.

KEY FORMULAE

- **Blanking/punching force $F = L \cdot t \cdot \tau_s$** (L = cut perimeter; circle: πd; $\tau_s \approx 0.7–0.8\ \sigma_u$)
- With shear on punch: **$F' \approx F \cdot (p \cdot t)/l$** (p = penetration fraction, l = shear on punch, $l \geq p \cdot t$) – work done remains the same
- Work done $W = F \cdot p \cdot t$
- Bending force (V-die): $F = k L \sigma_u t^2/W$ (k ≈ 1.33 V-die; ≈ 0.33 wiping/edge die; ≈ 0.67 U-die)
- Bend allowance $BA = \alpha(R + k t)$ (α in rad; k ≈ 0.33 for R < 2t, ≈ 0.5 for R ≥ 2t)
- Minimum bend radius: **$R_{min}/t = (50/r) - 1$** (r = % reduction of area in tensile test); $r \approx 50\% \rightarrow R_{min} = 0$
- Springback: $R_i/R_f = 4(R_iY/Et)^3 - 3(R_iY/Et) + 1$
- Deep drawing blank (cylindrical cup, sharp corner): **$D = \sqrt{(d^2 + 4dh)}$**
- Drawing ratio DR = D/d; reduction = (D − d)/D; **LDR ≈ 2** (≈ 1.8–2.2) → first-draw reduction ≈ 40–50%
- Drawing force (approx): $F = \pi d t \sigma_u (D/d - 0.7)$

- **Springback** ↑ with higher yield strength, **lower E**, larger R/t, thinner sheet. Compensation: **over-bending**, **bottoming/coining**, stretch bending, warm bending.
- Bend line should be **perpendicular to the rolling direction** (fibre) to avoid cracking; neutral axis shifts towards inside.
- **Deep drawing**: cup depth > ½ diameter (shallow drawing otherwise). Blank holder prevents **wrinkling** of flange. **Normal anisotropy $\bar{r}$** (Lankford) ↑ → LDR ↑ (better drawability). **Planar anisotropy Δr** → **earing**.
- Deep-drawing defects: **wrinkling** (flange – low blank-holder force; wall – large die clearance), **tearing/fracture** near punch nose (high blank-holder force, small radii), **earing** (anisotropy), **orange peel** (coarse grains), **stretcher strains / Lüders bands** (yield-point elongation in low-C steel – prevented by **temper (skin-pass) rolling**), surface scratches.
- Ironing – reduces wall thickness (beverage cans); redrawing (reverse); **spinning** (axisymmetric over mandrel); **stretch forming** (aircraft skins); **embossing** (matching male–female dies, raised on one side = depressed on other, thickness ≈ constant) vs **coining** (closed die, different relief on both faces, thickness varies – metal flows); **hydroforming**, rubber-pad (Guerin) forming.
- **HERF** (high-energy-rate forming): explosive forming, **electro-hydraulic** (spark under water), **electromagnetic (magnetic pulse)** forming – conductive tubes.

Powder metallurgy (P/M)

- Steps: **powder production → blending/mixing (with lubricant) → compaction (green compact) → sintering → secondary/finishing**.
- Powder production: **atomisation** (most common – gas/water), reduction of oxides, **electrolytic** deposition (high purity), **carbonyl** (Fe, Ni – fine, pure), mechanical comminution/milling.
- Compaction pressure ≈ **70–800 MPa** (≈ 100–1000 MPa); density gradient in single-action pressing → double-action pressing, **isostatic pressing** (CIP/HIP) for uniform density.
- **Sintering** at **≈ 0.7–0.9 T_m** (absolute) of main constituent in protective atmosphere; bonding by solid-state diffusion (liquid-phase sintering for WC–Co).
- Secondary ops: **sizing/coining**, **infiltration** (pores filled with lower-mp metal, e.g. Cu into Fe), **impregnation** (oil – **self-lubricating porous bronze bearings**), machining, heat treatment.
- Products: **cemented carbide tools (WC + Co binder)**, **tungsten lamp filaments**, porous bearings, filters, magnets, cermets, gears, refractory metals (W, Mo).
- Limitations: high powder & die cost, size/shape limits (no undercuts), lower strength/ductility (porosity), density variation.

MEMORY TRICK

"**B**lanking → **B**lank from **D**ie (die = blank, punch smaller); **P**iercing → **P**unch = hole (die bigger)." Rolling: "**Max draft = μ²R** – Mu-squared Radius." Drawing: "**e** limits drawing – 1 – 1/e ≈ 63%."

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Max possible draft in rolling with μ = 0.1, R = 300 mm? → **$\mu^2 R$ = 3 mm**.
- In blanking, clearance is provided on? → **Punch** (die = blank size).
- Force to punch a 20 mm hole in 2 mm sheet, τ = 300 MPa? → π×20×2×300 ≈ **37.7 kN**.
- Toothpaste collapsible tubes are made by? → **Impact extrusion**.
- Ideal maximum reduction in wire drawing? → **≈ 63%**.
- Earing in deep drawing is due to? → **Planar anisotropy**.

Nandyal Banking Coaching Center

11. Machining, Metal Cutting & Computer-Aided Manufacturing

NOTE

Production engineering gives 8–12 of the 60 questions in most PSU/AE papers. Favourite targets: chip thickness ratio and shear angle numericals, Merchant's equation, Taylor's tool-life numericals, tool-material order of hot hardness, grinding-wheel marking, the NTM "mechanism–medium–application" table, the 3-2-1 principle, G/M codes and the seven AM categories. Learn the tables. Most questions are direct recall.

11.1 Metal Cutting Mechanics ★★★

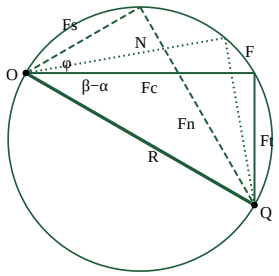
- **Orthogonal (2-D) cutting**: cutting edge is **perpendicular** to the cutting velocity; inclination angle λ = **0**; chip flows along the orthogonal plane; forces are only **Fc and Ft** (2 components). Examples: parting-off, turning a tube end, shaping with a straight edge. Used for **theoretical analysis**.
- **Oblique (3-D) cutting**: cutting edge is **inclined** (λ ≠ 0) to the velocity; chip flows sideways at the chip-flow angle; **3 force components** (Fc, Ff, Fr); most practical operations (turning, milling) are oblique. Stabler's rule: chip-flow angle ≈ inclination angle ($\eta_c \approx \lambda$).
- **Chip thickness ratio r = t_1/t_2** (uncut ÷ chip thickness) is always **< 1** in metal cutting; its reciprocal 1/r = **chip reduction coefficient** (ζ > 1). Volume constancy → r = $t_1/t_2 = l_2/l_1 = V_c/V$ (chip length ÷ uncut length).
- **Shear angle φ** = angle between the **shear plane** and the cutting-velocity direction. Larger φ → thinner chip, smaller shear-plane area, lower force and power, better finish. φ increases with **rake angle** and decreases with friction.
- **Uncut chip thickness** in turning t_1 = f sin κ (κ = approach/principal cutting-edge angle); width b = d/sin κ. For orthogonal turning (κ = 90°): t_1 = **f**, b = **d**.
- **Primary shear zone** = shear plane (chip formation). **Secondary deformation zone** = chip–tool interface (rake face; seizure + sliding zones; crater wear). **Tertiary zone** = tool–work interface (flank; flank wear, burnishing).

Chip type	Conditions favouring it	Remarks
Continuous	**Ductile** material, **high speed**, **large (positive) rake**, small feed/depth, sharp tool, good cutting fluid, low friction	Best finish, steady force, longest tool life; long chips need a **chip breaker**
Continuous with BUE (built-up edge)	Ductile material, **low speed**, high friction, small rake, no/poor coolant	Strain-hardened layers weld to the rake face, grow and break off. BUE gives a **poor finish**, changes effective rake (raises it) and protects the tool slightly. It disappears at higher speed (higher temperature).
Discontinuous (segmental)	**Brittle** material (grey CI, bronze, brass), very low speed, small/negative rake, large feed	Easy chip disposal; fluctuating force; good finish in brittle materials. In a ductile material it shows bad conditions.
Serrated (saw-tooth, non-homogeneous)	Materials of low thermal conductivity that soften with heat, e.g. **Ti alloys, Ni-based superalloys, austenitic SS**, high speed	Caused by **adiabatic (catastrophic) shear** bands. Semi-continuous.

KEY FORMULAE – CHIP GEOMETRY & KINEMATICS

- r = t_1/t_2 = **sin φ / cos(φ − α) ⇒ tan φ = r cos α / (1 − r sin α)**
- Shear strain γ = cot φ + tan(φ − α) = cos α / [sin φ cos(φ − α)]. For a given α, γ is least when φ = 45° + α/2.
- Chip velocity V_c = rV = V sin φ / cos(φ − α); shear velocity V_s = V cos α / cos(φ − α)
- Velocity relation (sine rule): V / cos(φ − α) = V_s / cos α = V_c / sin φ; vectorially $\vec{V}_s = \vec{V} + \vec{V}_c$
- Shear-plane length = t_1/sin φ; shear-plane area A_s = b t_1/sin φ; shear stress $\tau_s = F_s/A_s$
- Shear strain rate $\dot{\gamma} = V_s/\Delta y$ (Δy = thickness of shear zone): very high, about 10^3–10^6 s^{-1}

Merchant's circle diagram (orthogonal cutting). The resultant R is drawn as the diameter of a circle. Every pair of perpendicular components of R ends on the circle. The pairs are Fc–Ft (measured by the dynamometer), Fs–Fn (on the shear plane) and F–N (on the rake face).



Circle diameter = R; α = 10°, β = 40°, φ = 30°

Merchant's circle (to scale). R makes angle (β − α) with Fc. Fs lies along the shear plane at φ and N lies normal to the rake face. Here 2φ + β − α = 90°.

KEY FORMULAE – MERCHANT'S FORCE RELATIONS

- Friction force on rake face: **F = Fc sin α + Ft cos α**; normal: **N = Fc cos α − Ft sin α**
- **μ = tan β = F/N = (Ft + Fc tan α)/(Fc − Ft tan α)** (β = friction angle). For α = 0: μ = Ft/Fc.
- Shear force: **Fs = Fc cos φ − Ft sin φ**; normal to shear plane: **Fn = Fc sin φ + Ft cos φ**
- R = √(Fc² + Ft²); Fc = R cos(β − α); Ft = R sin(β − α); Fs = R cos(φ + β − α)
- Fc = τ_s b t_1 cos(β − α) / [sin φ cos(φ + β − α)]
- **Merchant (1st solution, minimum energy): 2φ + β − α = 90°**, i.e. φ = 45° + α/2 − β/2
- Merchant's 2nd solution (τ depends on normal stress, constant C = machining constant): 2φ + β − α = C
- **Lee & Shaffer (slip-line, rigid-plastic): φ + β − α = 45°**, i.e. φ = 45° + α − β
- Power: P = Fc·V (Ft does no work in orthogonal turning as there is no velocity along it, apart from feed). Power for shearing = Fs·V_s; friction power = F·V_c; P = Fs V_s + F V_c.
- **Specific cutting energy (pressure)** u = P/MRR = Fc V/(b t_1 V) = **Fc/(b t_1)** in N/mm² (MPa); note **1 J/mm³ = 1000 N/mm² = 1 GPa**

Nandyal Banking Coaching Center

- **Merchant's assumptions**: tool perfectly sharp (no flank contact); shear on a single **thin plane**; continuous chip without BUE; plane-strain (orthogonal) cutting; constant μ along the rake face; τ_s of the material constant (1st solution).
- **Specific cutting energy** (typical): aluminium alloys **0.4–1.1 J/mm³**, cast iron 1.1–5.4, carbon steels **2.7–9.3**, stainless steel 2–5, Ti alloys 2–5, high-temperature alloys 3.2–8.0 (Kalpakjian). Order: **grinding ≫ milling > turning**.
- **Size effect**: specific energy **increases as uncut chip thickness decreases**, because of ploughing, the edge radius and fewer defects in the small volume sheared. This is why grinding has very high specific energy.
- Of the total cutting energy, roughly **75% goes into shear** (primary zone) and about **25% into rake-face friction**. Almost all mechanical energy (≈ 98–99%) appears as **heat**.
- **Heat sources**: primary shear zone ≈ **60–80%** of heat (the largest), secondary/chip–tool zone ≈ **15–35%**, tertiary/flank ≈ **5%** (varies with author).
- **Heat distribution** (high speed): **chip ≈ 75–80%**, tool ≈ 10–20%, workpiece ≈ 5–10%. At higher speed a larger share leaves with the chip. Textbooks commonly quote "chip 80%, tool 10%, work 10%".
- **Maximum temperature** occurs on the **rake face a little away from the cutting edge** (where the crater forms), not at the tip itself. Temperature is measured by the tool–work **thermocouple**, embedded thermocouples, IR pyrometry and temperature-sensitive paints. It rises with V most strongly, then f, then d. Empirical: $\theta \propto V^{0.4-0.5} f^{0.2-0.3} d^{0.1}$.
- **Cutting-force measurement**: tool dynamometers. Strain-gauge type (octagonal ring, used for lathe/milling) or **piezoelectric** (quartz). A lathe dynamometer measures 2 or 3 components: Fc (largest), Ff, Fr.
- Increase in rake angle → φ increases → Fc decreases. Increase in feed → Fc increases (≈ linearly). Increase in speed → Fc decreases slightly (temperature softening, smaller BUE).

MEMORY TRICK

"**Merchant = 90, Lee-Shaffer = 45**": Merchant uses 2φ (two φ, sum 90); Lee & Shaffer uses one φ (sum 45). The terms are β − α in both. Chip ratio "**sin over cos**": r = sin φ / cos(φ − α).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Uncut chip thickness 0.25 mm, chip thickness 0.5 mm, α = 0°. Shear angle? → tan φ = r = 0.5, **φ ≈ 26.6°**.
- Continuous chip with BUE is formed at? → **Low cutting speed** with ductile material and high friction.
- Merchant's shear-angle relation? → **2φ + β − α = 90°**.
- In orthogonal cutting with α = 0°, Fc = 1000 N, Ft = 500 N. Coefficient of friction? → μ = Ft/Fc = **0.5**.
- Most of the heat generated in metal cutting is carried away by? → **The chip**.
- Saw-tooth (serrated) chips are typical of? → **Titanium alloys** (low thermal conductivity, adiabatic shear).

11.2 Tool Geometry, Tool Materials & Tool Wear ★★★

- **Tool signature** = the angles and nose radius of a single-point tool written in a fixed order.
- **ASA (American) system**, also called the **coordinate / machine reference system**. Order: **Back rake (α_b) – Side rake (α_s) – End relief (θ_e) – Side relief (θ_s) – End cutting-edge angle (C_e) – Side cutting-edge angle (C_s) – Nose radius (R)**.
- Example **8-14-6-6-6-15-1** means: back rake 8°, side rake 14°, end relief 6°, side relief 6°, ECEA 6°, SCEA 15°, nose radius 1 mm.
- **ORS (Orthogonal Rake System, ISO)**: **Inclination angle (λ) – Orthogonal rake (α_o) – Orthogonal clearance (γ_o) – Auxiliary clearance (γ_o') – Auxiliary cutting-edge angle (φ_1) – Principal cutting-edge (approach) angle (φ) – Nose radius (r)**. ORS is based on the cutting edge, so it is the **tool-in-hand** system preferred for analysis. NRS (normal rake system) is another ISO system.
- ASA SCEA C_s and ORS approach angle φ are related by **$\varphi = 90° - C_s$**.
- **Rake angle** = inclination of the rake face (along which the chip flows). Purpose: easy chip flow, lower force and power.
  - **Positive rake**: lower force, better finish, but a weaker edge. Used for soft/ductile materials (Al, Cu) and HSS tools.
  - **Negative rake**: stronger edge that takes the cutting load in **compression**. Used for **carbide/ceramic** tools, hard materials and interrupted cuts. Needs more power and rigid machines.
  - Brass is cut with **zero/slightly negative rake** to avoid digging in.
- **Relief (clearance) angle**: prevents the flank rubbing on the machined surface. It is **always positive** (typically 5–10°). Too large → weak edge; too small → rubbing and flank wear.
- **Side cutting-edge angle (lead angle)**: spreads the load over a longer edge, thins the chip, protects the tip on initial contact. Too large → chatter (higher radial force).
- **End cutting-edge angle**: gives clearance between the trailing edge and the machined surface.
- **Nose radius**: strengthens the tip and improves finish. Too large → chatter. Typical 0.4–1.6 mm.
- **Lip angle (wedge angle) β_w = 90° − (rake + clearance)** when the rake is positive.

Tool material (in order of increasing hot hardness)	Max working temp. (approx.)	Key facts
Plain carbon tool steel (0.8–1.3% C)	**≈ 200–250 °C**	Cheapest; hand tools, files, taps for low speed; V ≈ 5–10 m/min
High-speed steel (HSS)	**≈ 550–600 °C**	Taylor & White (1900). **18-4-1 = W 18%, Cr 4%, V 1%** (T-series). Mo-series M2 (cheaper; Mo replaces W). Co added (super HSS) for hot hardness. Drills, taps, form tools, broaches (toughest of the hard tool materials). V ≈ 30 m/min.
Cast cobalt alloys (**Stellite**: Co–Cr–W–C)	≈ 800–850 °C	Cast, not heat-treatable; brittle; little used now
Cemented carbides (WC + **Co binder**; TiC, TaC added)	**≈ 1000 °C**	Made by **powder metallurgy (sintering)**; as brazed tips or **throw-away inserts**. 3–5× HSS speed (V ≈ 100–200 m/min). Higher Co → tougher, less hard.

Nandyal Banking Coaching Center

Cermets (TiC/TiN + Ni/Mo binder)	≈ 1000–1100 °C	Finishing of steels at high speed
Ceramics (**Al_2O_3**, Al_2O_3–TiC "black", Si_3N_4, SiAlON)	**≈ 1200 °C+**	Very hard, chemically inert, **brittle**, poor shock resistance. High speed finishing; **negative rake**; no coolant (thermal shock). Whisker-reinforced grades are tougher. SiAlON is used for Ni superalloys.
CBN (cubic boron nitride; PCBN)	≈ 1400 °C (stable)	**Second hardest** material. Used for **hardened steels (> 45 HRC)**, chilled CI and superalloys ("hard turning"). Chemically inert to iron.
Diamond (PCD, single crystal)	Graphitises ≈ 700 °C in air	**Hardest** material, highest thermal conductivity. Used for Al–Si alloys, Cu, composites, plastics and ultra-precision. **Not for ferrous metals**: carbon diffuses into iron at high temperature.

- **Order of hardness / hot hardness**: carbon steel < HSS < cast alloy < carbide < cermet < ceramic < CBN < diamond. **Toughness runs the opposite way** (HSS is toughest; diamond/ceramic are most brittle).
- **ISO carbide grades (ISO 513)**:
  - **P (blue)**: steel, long-chipping. Contains TiC/TaC to resist crater wear.
  - **M (yellow)**: stainless steel.
  - **K (red)**: **cast iron**, short-chipping (straight WC–Co).
  - N (green): non-ferrous. S (brown): heat-resistant superalloys/Ti. H (grey): hardened steel.
  - A lower number within a group (e.g. P01) means harder and more wear-resistant, for finishing. A higher number (P50) means tougher, for roughing.
- **Coatings**, applied on carbide/HSS by **CVD** (≈ 1000 °C, thicker; mainly carbides) or **PVD** (≈ 500 °C; suits HSS and sharp edges), each 2–15 µm thick:
  - **TiN**: **gold colour**, low friction, most common on HSS drills.
  - TiC: grey, abrasion resistant; usually the first layer.
  - Al_2O_3: thermal barrier, crater-wear resistant.
  - TiCN, TiAlN: violet-grey; good for high-speed/dry cutting.
  - Diamond-like carbon.

Wear / failure mode	Location & measure	Cause & remedy
Flank wear (wear land)	Clearance face; land width **VB**	**Abrasion** by hard particles in the work. Governs **tool life** and finish/dimensions. Mainly a function of **cutting speed**.
Crater wear	Rake face, a little behind the edge; crater depth **KT**	**Diffusion** at high temperature (carbide cutting steel). Reduce with TiC/TaC (P grade), Al_2O_3 coating, lower speed.
Notch wear	At the depth-of-cut line (both flanks)	Work-hardened surface layer / scale, oxidation. Common in Ni alloys and SS.
Nose wear, chipping, thermal cracking (comb cracks), plastic deformation, catastrophic fracture	—	Interrupted cuts (milling) → thermal/mechanical shock. Use a tougher grade.

- **Wear mechanisms**: abrasion (dominant at low–moderate temperature), adhesion (BUE, at low speed), **diffusion** (at high temperature, the crater), oxidation (notch), chemical, and fatigue.
- **Flank-wear curve** has 3 stages: **initial (rapid) wear → uniform (steady, linear) wear → rapid/catastrophic wear**. The tool should be reground before the third stage.
- **Tool-life criteria (ISO 3685)** for HSS/carbide/ceramic tools:
  - Average flank wear **VB = 0.3 mm** (uniform), or maximum VB = 0.6 mm (irregular).
  - Crater depth KT = 0.06 + 0.3f (mm).
  - Catastrophic failure.

  Other practical criteria are loss of finish or size, a rise in force/power, chatter, and a bright band on the work.
- **Tool life** can be expressed as time (min), number of pieces, volume of metal removed, or equivalent cutting speed (V_{60} = speed giving 60 min life).

KEY FORMULAE – TOOL LIFE & FINISH

- **Taylor's tool-life equation: $V T^n = C$**. C = cutting speed for 1 min life; n = slope of the log V–log T line (straight on log–log plot).
- Exponent n: **HSS ≈ 0.1–0.15** (0.08–0.2); **carbide ≈ 0.2–0.25** (up to 0.4); **ceramic ≈ 0.5–0.7** (0.4–0.6 in some books); carbon steel ≈ 0.1. Higher n → life less sensitive to speed.
- Ratio form: $V_1T_1^{\,n} = V_2T_2^{\,n} \Rightarrow T_2/T_1 = (V_1/V_2)^{1/n}$. With n = 0.5, halving V gives **4×** life. With n = 0.25, halving V gives 16× life.
- **Extended Taylor: $V T^n f^a d^b = K$**. Usually a ≈ 0.5–0.8, b ≈ 0.2–0.4. Influence on tool life: **V > f > d**. To raise MRR with least loss of tool life, increase **d first, then f**, and V last.
- **Ideal surface roughness** with nose radius r and feed f: peak-to-valley $h_{max} = f^2/(8r)$; $R_a \approx f^2/(32r) = h/4$ (more exactly 0.0321 f^2/r).
- Sharp tool (no nose radius): $h_{max} = f/(\tan \psi + \cot \kappa')$, where ψ = SCEA (ASA) and κ′ = ECEA. In ORS: $h = f/(\cot \varphi + \cot \varphi_1)$. $R_a = h/4$.
- Machinability index (%) = (cutting speed of material for 20 min life ÷ cutting speed of standard for 20 min life) × 100. Standard = **free-cutting steel AISI B1112 = 100%**.

- **Machinability** = ease of machining. Judged by tool life (most common), forces/power, surface finish and chip form. It is **not an absolute property**.
- **Better machinability** comes from: lower hardness and strength (but very soft, gummy metals such as pure Al/Cu machine poorly), good thermal conductivity, and **free-machining additives**:
  - **Sulphur** (as MnS inclusions)
  - **Lead** (leaded steels, leaded brass)
  - Selenium, tellurium, bismuth, phosphorus (in low-C steel)
  - Calcium (deoxidised steels)

  These additives break chips and lubricate. **Grey CI** machines well because its graphite flakes act as chip breakers and lubricant.

Nandyal Banking Coaching Center

- Generally, **annealed/normalised** steel machines better than hardened steel. Coarse pearlite is better than fine pearlite for medium-C steel, and spheroidised structure is best for high-C steel. Austenitic SS, Ti and Ni alloys have poor machinability (work hardening, low conductivity).
- **Cutting fluids** – functions:
  - **Cooling** (at high speed, the main function)
  - **Lubrication** (at low speed, e.g. threading, broaching, tapping)
  - Flushing chips, preventing BUE and corrosion, improving finish and tool life

Cutting fluid type	Characteristics / use
Straight (neat) cutting oils – mineral, fatty, **EP additives (S, Cl, P)**	Best **lubricant**; low speed, heavy cuts – threading, broaching, gear cutting. Sulphurised oils are not for Cu alloys (staining).
Soluble oil (emulsion, "suds", oil 1:10–1:40 in water)	Most widely used; good cooling + some lubrication; milky white
Synthetic / semi-synthetic (chemical) fluids	Best **cooling**; grinding, high-speed machining; transparent
Gases / air, cryogenic (LN_2), MQL (minimum quantity lubrication, ~10–100 mL/h)	Near-dry, green machining
Dry machining	Grey CI (except threading), brass. Ceramics and often carbides are run dry to avoid thermal shock.

MEMORY TRICK

ISO carbide colours: "**P**olice **B**lue – **M**ango **Y**ellow – **K**etchup **R**ed". The P grade handles steel's long chips; K is for cast iron's short chips. Taylor n rises with hot hardness: "**HSS 0.1, Carbide 0.2, Ceramic 0.5+**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In ASA tool signature 10-20-7-7-10-15-2, what is "15"? → **Side cutting-edge angle**.
- Crater wear is mainly due to? → **Diffusion** (high temperature at chip–tool interface).
- Taylor's equation with n = 0.5, C = 400. Speed for 25 min tool life? → $V = 400/25^{0.5}$ = **80 m/min**.
- Diamond tools are unsuitable for machining steel because? → **Chemical affinity of carbon for iron** (diffusion/graphitisation).
- Feed 0.2 mm/rev, nose radius 0.8 mm. Peak-to-valley height? → $f^2/8r = 0.04/6.4$ = **6.25 µm** (Ra ≈ 1.56 µm).
- Element added to steel for free machining? → **Sulphur / lead**.

11.3 Machine Tools & Operations ★★★

- **Lathe** – the "mother of machine tools". The work rotates; the tool gets linear feed.
  - **Main parts**: bed (CI, with V and flat ways), headstock (spindle, cone pulley/all-geared), tailstock (dead centre; drilling/reaming), carriage, feed rod, **lead screw** (only for thread cutting; ACME thread), feed gearbox and **tumbler gears** (reversing feed direction).
  - **Carriage** = saddle + cross-slide + compound rest + tool post + **apron**. The apron holds the **half-nut (split nut)**, which is engaged only for threading.
  - **Size of lathe**: **swing over bed** (max diameter that can be turned) and **distance between centres** (max length), plus bed length.
  - Lathe centre angle = **60°**. Dead centre in the tailstock, live centre in the headstock (or a revolving centre in the tailstock).
- **Lathe operations**:
  - Turning (straight, step, taper), facing, chamfering
  - Knurling: a forming operation with no chip; diamond/straight patterns; low speed
  - Parting-off (low speed, rigid set-up)
  - Grooving, drilling, boring, reaming, threading, spinning, eccentric turning
- **Work-holding on lathe**:
  - **3-jaw self-centring chuck**: round/hexagonal bars; jaws move together.
  - **4-jaw independent chuck**: irregular shapes and eccentric work; each jaw set separately.
  - Collet chuck: bar stock; quick and accurate.
  - Face plate: irregular large work.
  - Between centres with lathe dog/carrier.
  - Mandrel: holds bored work for turning concentric to the bore.
  - **Steady rest** (fixed to bed) and **follower rest** (moves with carriage): support long slender jobs.
- **Capstan vs turret lathe**:
  - Capstan: the hexagonal turret is on a **ram** that slides on a saddle clamped to the bed. Suits light, short bar work.
  - Turret: the turret is mounted directly on a **saddle** that moves on the bed. Suits heavier, longer chucking work.
  - Both are semi-automatic and used for batch production.
- **Automats** are single-spindle/multi-spindle automatic lathes (mass production). The **Swiss-type** (sliding headstock) automat suits small, long precision parts.

Taper-turning method	Suitable for	Key point
Compound rest (swivelling)	**Short, steep** tapers, internal and external	Swivel angle α with tan α = (D − d)/(2l). Feed is by hand.
Tailstock set-over (offset)	**Long, slender, small-angle** tapers; external only; work between centres	Offset **s = L(D − d)/(2l)**, where L = total job length and l = taper length. The maximum offset is limited (a few mm), and the centres are misaligned. Power feed is possible.
Taper-turning attachment	Long tapers, internal and external; repeat jobs	Guide bar set at the half-angle; centres stay aligned; power feed
Form tool	Very short tapers/chamfers	Edge ground at the taper angle; length < tool edge
Combined feeds	Rough work	Longitudinal and cross feed together

- **Taper** = (D − d)/l; half-angle tan α = (D − d)/(2l). Standard tapers:

Nandyal Banking Coaching Center

- **Morse taper ≈ 1:20** (≈ 5/8 inch per ft): drill shanks, lathe centres; **self-holding**.
- **Metric taper 1:20**.
- Jarno 0.6 in/ft.
- Brown & Sharpe 1/2 in/ft (milling).
- **Self-releasing taper 7:24**: milling arbors, CNC spindles (BT/ISO/CAT).

- **Thread cutting**: the spindle and lead screw are linked through change gears.
  - Change-gear ratio **driver/driven = pitch of thread to be cut ÷ pitch of lead screw**, assuming a 1:1 spindle–stud ratio.
  - Example: cut 1.5 mm pitch with a 6 mm lead screw → 1/4 → 20T driver, 80T driven.
  - For pitches < lead-screw pitch the driver is smaller. A standard change-gear set runs 20 to 120T in steps of 5, plus **127T** for metric threads on an inch lead screw.
  - The **thread-chasing dial** shows when to re-engage the half-nut.
  - Metric V-thread angle = **60°**; BSW = **55°**; ACME 29°; square 0°; buttress 45° (7°/45°).
- **Cutting speed** $V = \pi D N /1000$ (m/min, D in mm). Feed f (mm/rev). Depth of cut $d = (D - d_f)/2$.

KEY FORMULAE – MACHINING TIME & MRR

- Turning time $T_m = L/(f N)$ per pass, where L = length + approach + over-travel. Number of passes = total depth ÷ d.
- **MRR (turning) = f d V** (×1000 for mm³/min when V in m/min). More exactly: $\pi D_{avg} d f N$.
- Facing time = (D/2)/(f N), with N constant (V varies).
- **Drilling**: MRR = $(\pi D^2/4)$ f N. Time = (L + 0.3D approach)/(f N). Cone approach = (D/2)·cot(point half-angle) ≈ **0.3D for 118°**. For enlarging a hole: MRR = $\pi(D^2 - d^2)f N/4$.
- **Milling**: table feed $f_m = f_z \times Z \times N$ (mm/min); MRR = $w \times d \times f_m$. Time = (L + approach)/f_m. For slab milling, approach = $\sqrt{(d(D - d))}$.
- **Shaping**: average cutting speed $V = N L (1 + m)/1000$ m/min, where m = return time/cutting time. Time = W/(f N), with W = width and N = strokes/min.
- Max undeformed chip thickness in slab milling ≈ $2 f_z \sqrt{(d/D)}$.

- **Twist drill** (HSS):
  - **Point angle 118°** (standard for steel and CI). Larger angles (135–140°) for hard materials; smaller (60–90°) for plastics and wood.
  - **Helix angle ≈ 30°** (the helix angle is the rake angle at the periphery).
  - Lip clearance 8–12°; chisel-edge angle 120–135°.
  - Two flutes. The **web thickness increases toward the shank**.
  - The **chisel edge** does not cut but extrudes/indents. It causes the high thrust and is reduced by web thinning or a pilot hole.
  - Rake angle varies along the lip: maximum at the periphery, negative near the chisel edge.
  - Shank: straight (small) or Morse taper (large). The **tang** is used to eject the drill with a drift.
- **Hole operations**:
  - **Drilling**: makes the hole.
  - **Boring**: enlarges a hole and **corrects its location/straightness** (single-point tool).
  - **Reaming**: sizes and finishes an existing hole. Small allowance (0.1–0.5 mm); low speed (≈ 2/3 of drilling speed), higher feed. **Cannot correct hole location.**
  - **Counterboring**: stepped flat-bottom recess for bolt heads.
  - **Countersinking**: conical recess (82°/90°) for flat-head screws.
  - **Spot facing**: a flat seat.
  - **Tapping**: internal threads. Set of 3 taps: taper (first), intermediate (second), bottoming (plug). Tap-drill size ≈ major dia − pitch.
  - **Trepanning**: large holes by cutting an annular groove (core removed).
- **Drilling machines**: sensitive (bench, hand feed, small holes), upright (pillar/column), **radial** (arm swings, for large heavy jobs), gang (several spindles in a row, one after another), multi-spindle (many holes at once), deep-hole (gun drill). Size of a radial drill = arm length and column diameter.
- **Milling**: the multi-point cutter rotates and the work feeds. It is an intermittent cut that produces chips of variable thickness.
  - **Horizontal mill**: plain/slab, side and face, straddle (two cutters on one arbor), gang (several cutters), form milling.
  - **Vertical mill**: face and end mills, slots, keyways (Woodruff key cutter).
  - **Universal milling machine**: the table swivels ±45°, for helical work with a dividing head.

Up (conventional) milling	Down (climb) milling
Cutter rotation **opposes** the feed	Cutter rotation is in the **same direction** as the feed
Chip thickness **zero → maximum**; rubbing/burnishing at entry	Chip thickness **maximum → zero**; no rubbing at entry
Force tends to **lift** the work → needs strong clamping	Force **presses** the work onto the table → simpler fixtures, good for thin work
Backlash in the feed screw is harmless	Needs a **backlash eliminator** (ball screw), otherwise the work is pulled in
Rougher finish; more tool wear; suited to castings/forgings with hard scale and to old machines	**Better finish, longer tool life**, lower power; preferred on CNC machines

- **Indexing (dividing head)**: the worm–worm-wheel ratio is **40:1**, so **40 turns of the crank = 1 revolution** of the spindle.
  - **Direct (rapid) indexing**: a plate on the spindle with 24 holes; divisions 2, 3, 4, 6, 8, 12, 24.
  - **Simple (plain) indexing**: crank turns = **40/N**. For N = 30: 1⅓ turns = 1 turn + 6 holes on an 18-hole circle.
  - **Angular indexing**: 1 crank turn = 360°/40 = **9°**; turns = θ/9°.
  - **Compound indexing**: two hole circles used in turn.
  - **Differential indexing**: for numbers not possible by simple indexing (e.g. 51, 53, 87, 127). The index plate is geared to the spindle through change gears and rotates as the crank moves. Gear ratio = (A − N)·40/A, where A is a near number that can be indexed.
- **Brown & Sharpe index plates**:

Nandyal Banking Coaching Center

- Plate 1: 15, 16, 17, 18, 19, 20
- Plate 2: 21, 23, 27, 29, 31, 33
- Plate 3: 37, 39, 41, 43, 47, 49

The Cincinnati plate has hole circles on both sides.

- **Shaper**: the **tool reciprocates**, the work is stationary and feeds intermittently (crosswise during the return stroke). Cuts flat surfaces, keyways and grooves on small jobs.
  - Drive: **crank and slotted-lever quick-return** mechanism (an inversion of the single slider-crank), Whitworth, or hydraulic.
  - Return is faster than cutting. **Cutting : return time ≈ 3:2** typically, i.e. ratio 1.5; the return stroke takes about 40% of the cycle.
  - Size = maximum **stroke length**.
  - Tool is held in a clapper box, which **lifts the tool on the return stroke** to avoid rubbing.
- **Planer**: the **work (table) reciprocates**, the tool is stationary and feeds at the stroke end. Used for large heavy jobs such as machine beds. Can take several tools at once. Size = table length × width × height. Drives: reversible DC motor, belt or hydraulic.
- **Slotter**: a **vertical shaper** (ram moves vertically). Internal keyways, splines, internal gears in small numbers. Quick return via Whitworth or a variable-speed reversible drive.
- **Broaching**: a multi-tooth tool with successive teeth higher by the **rise per tooth** (≈ 0.02–0.1 mm). Teeth run roughing → semi-finishing → finishing (finishing teeth have zero rise).
  - The whole operation is completed in a **single stroke**, giving a fine finish and close tolerance.
  - **Mass production** only: the broach is costly, made specially for each job, of HSS.
  - Products: internal keyways, splines, square/hexagonal holes, gun-barrel rifling, and external surfaces such as engine-block faces (surface broaching).
  - Types: **pull, push**, rotary. The feed is built into the tool, so the machine needs no feed mechanism.
  - Cannot be used on blind holes (the broach must pass through) or on very heavy stock.

Gear-cutting method	Principle	Key facts
Form milling (disc/end-mill form cutter)	Forming – the cutter profile equals the tooth space	One tooth space at a time with a dividing head. Slow and less accurate. A set of **8 cutters per module** (No. 1 = 135 teeth to rack; No. 8 = 12–13 teeth). Repair/small batches.
Hobbing	**Generating** – the hob (a worm with gashed teeth) and blank rotate like a worm–worm-wheel pair	Fast and accurate; **one hob for all tooth numbers of a module**. Cuts spur, helical gears, worm wheels and splines. **Cannot cut internal gears** or gears close to a shoulder.
Gear shaping (Fellows)	**Generating** – a pinion-shaped cutter reciprocates while cutter and blank rotate in mesh	Can cut **internal gears**, cluster gears and gears close to shoulders. Rack-type shaper = Sunderland/Maag process.
Bevel gear generators (Gleason)	Generating	Straight and spiral bevel gears
Gear finishing	Shaving (before hardening), **grinding** & lapping (after hardening), burnishing (rolling)	Shaving is the most common finishing method for soft gears

- Other gear methods: casting, sintering (P/M), stamping, extrusion, rolling, broaching (internal), template planing (large bevels).
- **Economics of machining (Gilbert)**:
  - Total cost per piece = machining cost + tool-change cost + tool cost + handling cost.
  - As V rises, machining cost falls while tool cost rises, so there is an optimum speed.
  - The **maximum-production-rate speed is higher** than the **minimum-cost speed**. The maximum-profit speed lies between them (this range is the "**high-efficiency (Hi-E) range**").

KEY FORMULAE – OPTIMUM CUTTING CONDITIONS

- Tool life for **minimum cost**: $T_o = (1/n - 1)(C_m t_c + C_t)/C_m$, where C_m = machine + labour cost per min, t_c = tool-change time, C_t = tool cost per cutting edge
- Tool life for **maximum production**: $T_p = (1/n - 1)\, t_c$
- Optimum speed $V_o = C/T_o^n$. Since $T_p < T_o \rightarrow V_p > V_o$.
- Quick-return ratio of shaper = cutting-stroke time ÷ return-stroke time = (360° − β)/β, where β = crank angle on the return stroke

MEMORY TRICK

"**Shaper – Tool moves; Planer – Plate (table) moves.**" Indexing: "**Forty over N**, nine degrees a turn." Gears: "**Hob can't do Inside; Shaper can.**"

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Bar Ø 50 mm turned at 300 rpm, f = 0.2 mm/rev, L = 150 mm. Machining time? → 150/(0.2 × 300) = **2.5 min**.
- Crank turns for indexing 40 divisions by simple indexing? → **1 turn**.
- Most suitable method for long, slender external tapers? → **Tailstock set-over**.
- Which milling gives better finish and needs a backlash eliminator? → **Down (climb) milling**.
- Standard point angle of a twist drill for steel? → **118°**.
- Internal gears are cut by? → **Gear shaping** (not hobbing).

11.4 Abrasive Machining & Finishing ★★★

- **Grinding**: material removal by many randomly oriented abrasive grains. Each grain acts as a micro cutting tool with a **large negative rake** (≈ −60° average).
  - Very small chips and very high speed (conventional wheel V ≈ **25–35 m/s**, ≈ 1500–2000 m/min; high-speed grinding up to 125 m/s with CBN).
  - Gives close tolerance (±0.0025 mm) and finish Ra 0.1–1.6 µm.
  - Can machine **hardened steel**.
- **Specific energy of grinding is very high** (≈ 10–60 J/mm³ for steel, about 10× turning). Causes: negative rake, **ploughing and**

Nandyal Banking Coaching Center

rubbing, the size effect (tiny chip thickness), and wear flats. The resulting heat can cause **grinding burn**, temper colours, residual tensile stresses and cracks.

- **Natural abrasives**: sandstone, emery, corundum, diamond. **Synthetic abrasives** are used most.

Abrasive	Symbol	Used for
Aluminium oxide (Al_2O_3) – brown/white	**A**	Tougher. For **high-tensile materials: steels**, alloy steel, HSS, malleable iron, wrought iron. White Al_2O_3 is friable (tool rooms).
Silicon carbide (SiC) – black/green	**C**	Harder but more brittle. For **low-tensile materials: grey cast iron, brass, bronze, Al, Cu**, non-metals (stone, glass, rubber). **Green SiC** for cemented carbide tools.
CBN (Borazon)	B (CB)	Hardened steels, HSS, superalloys; high-speed precision grinding
Diamond	D	Cemented carbides, ceramics, glass, stone. **Not for steel.**

- **Hardness order**: diamond (≈ 7000 HK) > CBN (≈ 4700) > SiC (≈ 2500) > Al_2O_3 (≈ 2100). Diamond and CBN are called **superabrasives**.
- **Grinding-wheel marking (IS 551 / ISO 525)**. Order: **[Prefix: abrasive maker's symbol] – Abrasive type – Grain size – Grade – Structure – Bond – [Manufacturer's record]**.
- Example **51 A 46 K 5 V 23**: 51 = maker's abrasive symbol; **A = Al_2O_3**; **46 = grain size (medium)**; **K = grade (medium; some texts: soft)**; **5 = structure (dense–medium)**; **V = vitrified bond**; 23 = maker's record.

Parameter	Range / symbols	Meaning
Grain (grit) size – mesh number	Coarse **10–24**; medium **30–60**; fine **70–180**; very fine **220–600**	Higher number = finer grain = better finish. Coarse for fast removal and soft, ductile materials; fine for hard, brittle materials and finishing.
Grade (hardness of the **bond**, i.e. how strongly it holds the grains)	**Soft A–H; medium I–P; hard Q–Z**	Not the hardness of the abrasive. **Hard work material → soft wheel** (worn grains shed easily, exposing fresh ones). **Soft material → hard wheel.**
Structure (grain spacing)	**Dense 0/1–8; open 9–15/16**	Open structure for soft, ductile materials and large contact areas (chip clearance). Dense for hard, brittle materials and fine finish.
Bond	**V** vitrified; **S** silicate; **B** resinoid; **R** rubber; **E** shellac; **O** oxychloride; **M** metallic	See next table

Bond	Features / application
Vitrified (V) – clay/feldspar, fired ≈ 1250 °C	**Most common (~75–80% of wheels)**. Porous, strong, rigid, unaffected by water, oil and acids. Brittle (no shock). Max speed ≈ 33 m/s.
Silicate (S)	Sodium silicate; mild action, cool cutting; large wheels, edge-tool grinding
Resinoid (B) – phenolic resin	Strong and elastic; **high speed (up to 50–80 m/s)**. Snagging, cutting-off, fettling castings; diamond/CBN wheels.
Rubber (R)	Flexible. **Thin cut-off wheels**, **regulating wheels of centreless grinders**, fine finish.
Shellac (E) – natural resin	Elastic. **Fine finish**: cams, rolls, cutlery, saws.
Metallic (M) – bronze/nickel	Diamond and CBN wheels; ECG wheels (conductive)

- **Glazing**: the wheel face becomes smooth and shiny because **dull grains are not shed**. Caused by a **wheel too hard**, excessive wheel speed, or grinding hard material. The wheel then rubs and burns the work. Cure: use a softer grade, reduce speed, and dress the wheel.
- **Loading**: the pores get **clogged with work-material chips**. Caused by grinding **soft, ductile materials** (Al, Cu), too dense a structure, or poor coolant. Cure: use an open structure and coarser grain, and dress the wheel.
- **Dressing**: removes glazed or loaded grains to **restore sharpness** (cutting ability). Tools: star-wheel (Huntington) dresser, diamond, abrasive stick, crush dressing.
- **Truing**: restores the **true geometry, form and concentricity** of the wheel. Usually done with a **single-point diamond**. Truing also dresses the wheel.
- **Wheel balancing** and the **ring test** before mounting: a cracked wheel gives a dull sound. Blotters (compressible washers) go between the flanges and the wheel. A maximum safe speed is marked on the wheel.
- **Grinding ratio G = volume of work material removed ÷ volume of wheel wear**. Typical conventional G ≈ 20–80; **CBN can reach several thousand**. A high G is desirable, but a G that is too high may mean glazing.
- **Surface grinding**: horizontal spindle with a reciprocating table (most common), vertical spindle with a rotary table. The work is held on a **magnetic chuck**.
- **Cylindrical grinding**: external (between centres, traverse or plunge), internal, and universal.
- **Centreless grinding**: the work rests on a **work-rest blade** between the **grinding wheel** and the **regulating wheel**. The regulating wheel is **rubber-bonded**, smaller, rotates in the same sense as the grinding wheel and at much lower speed, and controls the work speed.
  - No centres are needed and the work is self-centring.
  - The work centre is set slightly **above** the line of wheel centres to get roundness.
  - **Through-feed**: the regulating wheel is inclined about **0–8°** to give axial feed. Axial feed rate = π d N sin θ (d, N of the regulating wheel).
  - **In-feed (plunge)**: shouldered or formed parts.
  - **End-feed**: tapered parts.
  - Used for long slender pins and mass production. Cannot grind work with keyways or flats easily.
- Other grinding types:
  - **Creep-feed grinding**: very large depth with very slow feed, so it acts like milling.
  - Snagging: rough grinding of castings.

Nandyal Banking Coaching Center

- Tool and cutter grinding.
- Thread and form grinding.
- Jig grinding: precise holes.

Finishing process	Tool / medium	Typical R_a (µm)	Typical application / key point
Turning / milling (rough to fine)	Single/multi-point tool	0.8–6.3	General machining
Grinding	Bonded wheel	0.1–1.6	Hardened parts, precision
Honing	**Bonded abrasive sticks (stones)** in a rotating and reciprocating head, with plenty of fluid	0.1–0.8	**Internal bores** – engine cylinders, hydraulic cylinders. Leaves a **cross-hatch** pattern that holds oil. Corrects taper and out-of-roundness but **not hole location**. Low speed.
Lapping	**Loose fine abrasive in a vehicle (oil/grease)** between the work and a lap made of a **softer material** (CI, Cu, brass, wood). Abrasive grains embed in the lap (charging).	0.025–0.4	**Gauge blocks**, optical flats, valve seats, fuel-injector parts. Best flatness/size accuracy; removes very little stock (0.005–0.01 mm).
Superfinishing	Fine bonded stick with short-stroke, high-frequency oscillation and low pressure	0.01–0.2	Removes the amorphous "smear" layer left by grinding. Crankshaft journals, bearing races.
Polishing	Abrasive glued/fixed on flexible wheels or belts	0.1–0.8	Removes scratches; precedes buffing
Buffing	Very fine abrasive in a compound on a soft **cloth wheel**	0.05–0.2	**High lustre**/mirror finish, decorative; negligible removal; follows polishing
Burnishing	Hard rollers/balls – **plastic deformation**, no chips	0.05–0.4	Smooths and work-hardens the surface; compressive residual stress
Abrasive flow machining / magnetic abrasive finishing	Abrasive-laden putty / magnetic particles	~0.05	Internal passages, deburring

- Approximate order of finish (best last): turning/milling → grinding → honing ≈ polishing → lapping ≈ buffing → **superfinishing**. Textbooks differ on lapping vs superfinishing; both are the finest conventional processes.
- Surface-roughness symbols per IS 3073 / IS 10719: N1 = 0.025 µm … **N6 = 0.8 µm**, N7 = 1.6, N8 = 3.2, … N12 = 50 µm. Each N step doubles Ra. **Ra** (CLA, centre-line average) is the most used parameter in India.

MEMORY TRICK

"**Hard work – Soft wheel; Soft work – Hard wheel.**" Abrasive choice: "**A for Alloy steel, C for Cast iron.**" Grade letters: "**A**…**H** = **H**ardly holding (soft); **Q**…**Z** = **Z**ealously holding (hard)." **G**lazing = **G**rade too hard; **L**oading = **L**ight (soft) metal.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In wheel marking "A 60 K 5 V", the letter K denotes? → **Grade** (medium, I–P range).
- Most widely used bond for grinding wheels? → **Vitrified**.
- Abrasive best suited for grinding grey cast iron? → **Silicon carbide**.
- Regulating wheel of a centreless grinder is usually? → **Rubber bonded**, inclined a few degrees for through-feed.
- Process producing cross-hatch pattern inside engine cylinders? → **Honing**.
- Operation to restore the geometrical shape of a grinding wheel? → **Truing**.

11.5 Non-Traditional Machining ★★★

- NTM (unconventional / advanced machining) is needed when:
  - the work material is **very hard, brittle or tough** (carbides, ceramics, superalloys, Ti, hardened steel);
  - the shape is complex (3-D cavities, odd-shaped holes, micro-holes, high L/D holes);
  - a stress-free, burr-free surface is required.

  In NTM the tool need not be harder than the work.
- **Classification by energy / mechanism**:
  - **Mechanical** (erosion by abrasive/fluid impact): USM, AJM, WJM, AWJM.
  - **Electrochemical** (anodic dissolution): ECM, ECG, ECD.
  - **Thermal / electrothermal** (melting and vaporisation): EDM, WEDM, EBM, LBM, PAM, IBM.
  - **Chemical** (chemical dissolution): CHM, PCM.

Process	Mechanism / medium	Key parameters	Applications & points
USM (ultrasonic)	Brittle fracture by **abrasive grains in a slurry** (B_4C, SiC, Al_2O_3 in water) hammered by a tool vibrating at ultrasonic frequency	Frequency **≈ 20 kHz (19–25 kHz)**; amplitude ≈ 15–50 µm. **Magnetostrictive** or piezoelectric transducer, with a concentrator/horn. Tool made of tough, ductile material (**mild steel, brass**, SS).	**Hard brittle materials** – glass, ceramics carbides, precious stones, germanium; both conductors and non-conductors. Poor for ductile metals. Low MRR; tool wears.

Nandyal Banking Coaching Center

AJM (abrasive jet)	Erosion by a high-velocity **jet of dry gas carrying fine abrasive**	Al_2O_3/SiC grains 10–50 µm; gas = **air, N_2, CO_2** (never O_2). Pressure 2–10 bar; velocity 150–300 m/s; nozzle of **WC or sapphire**, 0.2–1 mm; stand-off distance ≈ 0.5–15 mm.	Brittle materials, thin sections, **deburring**, frosting/etching glass, cleaning, cutting fine slots. Abrasive is not reused. Stray cutting (taper).
WJM (water jet)	Erosion by a high-velocity pure **water jet**	Pressure **≈ 200–400 MPa** (up to 600); jet velocity ≈ 600–900 m/s; **sapphire/ruby orifice** 0.1–0.4 mm; intensifier pump	**Soft materials** – paper, rubber, leather, foam, plastics, wood, **food**, textiles. No heat-affected zone; no dust; environment-friendly.
AWJM	WJM + abrasive (**garnet** most common) mixed in a mixing tube (WC/boron carbide)	As WJM; focusing tube ≈ 0.8–1.2 mm	Metals, composites, stone, glass, thick plates. **No HAZ.**
ECM (electrochemical)	**Anodic dissolution** by electrolysis (reverse of electroplating), governed by **Faraday's laws**. Work = **anode (+)**, tool = **cathode (–)**.	Low DC voltage **5–30 V**, very high current (up to ~40,000 A; density ≈ 0.5–5 A/mm²). Gap ≈ 0.1–0.5 mm. Electrolyte **NaCl** (most common), $NaNO_3$, $NaClO_3$, pumped fast (≈ 30 m/s). Tool of Cu, brass or SS.	**No tool wear**; no residual stress or burr; no HAZ; good finish; **high MRR** for hard, electrically conducting materials (turbine blades, dies, forging die cavities, rifling). Work must be **conductive**. Corrosion and sludge disposal are issues. H_2 gas forms at the tool. Sharp corners are not possible.
ECG (electrochemical grinding)	ECM + abrasive action of a **metal-bonded** (conductive) wheel	**≈ 90% removal electrolytic, 10% abrasive**; voltage 4–10 V	**Sharpening carbide tools**, honeycomb, thin parts, burr-free and stress-free. Low wheel wear.
EDM (spark erosion)	**Melting and vaporisation by repeated sparks** between tool and work immersed in a dielectric. Usual polarity: **work +, tool –**.	Spark gap ≈ **0.01–0.05 mm** (0.025 typical); voltage ≈ 40–300 V; frequency kHz range. Spark temperature 8,000–12,000 °C. Dielectric: **kerosene / paraffin / hydrocarbon oil** (die-sinking), **deionised water** (wire EDM). Tool: **copper, graphite** (most used), brass, Cu–W.	Any **electrically conductive** material regardless of hardness – carbides, hardened die steels. Press-tool dies, **forging and moulding die cavities**, small/odd holes. Tool wears (wear ratio); **recast (white) layer** and HAZ with micro-cracks. Circuits: **RC relaxation (Lazarenko)**, rotary impulse, transistorised pulse.
Wire EDM (WEDM)	EDM with a travelling wire electrode	Brass/coated wire ≈ **0.05–0.3 mm** (0.25 typical), used once. **Deionised water.** CNC controlled.	Through-profiles: punch–die sets, extrusion dies, gears, intricate contours. Also taper cutting.
EBM (electron beam)	Focused high-velocity electron beam melts and vaporises the material	Must work in a **vacuum** (~10^{-5} torr); accelerating voltage **50–150 kV**; electron speed ≈ 0.4–0.65 c; tungsten filament cathode	**Micro-holes** (Ø 0.025–1 mm, high L/D), e.g. spinnerets, fuel filters, turbine cooling holes; any material. **X-rays** are generated (shielding needed). Costly; small job size.
LBM (laser beam)	Coherent monochromatic light focused to a spot – melting and vaporisation	**CO_2 (10.6 µm)** – metals and non-metals, cutting; **Nd:YAG (1.06 µm)** and fibre lasers – metals, drilling; ruby (0.694 µm); excimer (UV) – micromachining. **No vacuum** needed.	Cutting sheet, drilling small holes, marking, welding; any material including non-metals. Poor on highly **reflective** materials (Al, Cu); HAZ; low efficiency (~10–20% for CO_2).
PAM (plasma arc)	Melting by a constricted plasma arc; molten metal blown away by the gas	Plasma temperature **≈ 10,000–30,000 °C**; gas: Ar, N_2, H_2, air. Transferred arc for conductors.	Fast **cutting of thick plates** – stainless steel, Al, mild steel. **Highest MRR** of the NTM processes. Rough finish, wide HAZ, taper.

Nandyal Banking Coaching Center

CHM (chemical machining: chemical milling, chemical blanking, photochemical machining)	Controlled **chemical dissolution (etching)** of unmasked areas	**Maskants**: neoprene, butyl rubber, PVC, polyethylene; photoresist for PCM. **Etchants**: $FeCl_3$ (steel, Cu, PCBs), NaOH (Al), HNO_3, HF (Ti). **Etch factor = undercut ÷ depth** (some texts: depth ÷ undercut).	Aircraft panel weight reduction (chemical milling), **printed circuit boards**, thin metal parts, name plates. No burrs or stresses; slow; hazardous chemicals.

- **Typical MRR (mm³/min, Ghosh & Mallik)**: PAM ≈ **75,000** > ECM ≈ **15,000** > EDM ≈ 800 > USM ≈ 300 > CHM ≈ 15 > EBM ≈ 1.6 > AJM ≈ 0.8 > LBM ≈ 0.1. So ECM has the **highest MRR among the precision NTMs** and LBM/EBM the lowest. The **specific power** consumption is highest for LBM/EBM.
- **ECM MRR (Faraday)**: mass rate $\dot{m} = I \cdot E/F = I \cdot A/(Z \cdot F)$, where A = atomic weight, Z = valency, E = A/Z (chemical equivalent) and F = **96,500 C** (Faraday constant). Volumetric MRR = I A/(Z F ρ) × efficiency. The MRR depends on current, not on the hardness of the work.
- **Equilibrium gap** in ECM: the gap stabilises when the tool feed equals the dissolution rate. The gap is **inversely proportional to feed rate**.
- **EDM MRR** rises with discharge energy (current, pulse on-time) but the surface gets rougher. **Tool-wear ratio** = work removed ÷ tool wear. Graphite tools give the lowest wear. Flushing of debris is essential. The work is usually **positive** with short pulses; tool positive with long pulses (for low tool wear). Books vary on this.
- **EDM dielectric functions**: insulation until breakdown, constricting the spark, cooling, and flushing debris.
- **USM MRR** (Shaw's model) rises with **amplitude**, **frequency**, grain size, static feed force (up to an optimum, beyond which grains crush) and slurry concentration (up to ~30–40%). MRR falls as work-material hardness/toughness rises, so ductile metals machine poorly.
- **Hybrid processes**: ECG, ECDM, ultrasonic-assisted EDM, laser-assisted machining. **Micro-machining**: μ-EDM, LBM, EBM, FIB (focused ion beam).
- **Electrical conductivity of the work is required for**: ECM, ECG, EDM, WEDM, PAM (transferred arc). **Not required** for USM, AJM, WJM, LBM, EBM and CHM.

MEMORY TRICK

"**ECM – no tool wear; EDM – no contact but tool wears.**" Vacuum is needed only for **EBM** ("electrons hate air"). USM ≈ "**Ultra-20**" kHz. Mechanism: **M**echanical = **U**SM, **A**JM, **W**JM ("**MUAW**"); **T**hermal = **E**DM, **E**BM, **L**BM, **P**AM ("**The Lamp**").

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Material removal in ECM is based on? → **Faraday's laws of electrolysis**.
- Electrolyte commonly used in ECM? → **NaCl solution**.
- NTM process for drilling square holes in glass? → **USM**.
- Dielectric in wire EDM? → **Deionised water**.
- Process requiring vacuum? → **EBM**.
- Most suitable process for cutting food products, rubber and paper? → **Water jet machining**.

11.6 Jigs, Fixtures & Work Holding ★★☆

- **Jig**: holds and locates the work **and guides the cutting tool** (through hardened drill bushes). Used mainly for **drilling**, reaming, tapping and counterboring. Usually not clamped to the machine table.
- **Fixture**: holds and locates the work but **does not guide the tool**. The tool is set with **setting blocks / feeler gauges**. A fixture is **clamped to the machine table**. Used for **milling**, turning, grinding, welding, assembly and inspection.
- **Advantages**:
  - Interchangeability and accuracy
  - No marking-out or individual set-up
  - Semi-skilled labour can be used
  - Higher production and lower inspection cost

  Economical for batch and mass production.
- **Degrees of freedom**: a free body has **12 DOF** when counting each direction separately (±X, ±Y, ±Z translations and ± rotations about X, Y, Z). Conventionally it has 6 DOF (3 translations + 3 rotations).
- **3-2-1 principle** (six-point location principle) for a prismatic block:
  - **3 locators** on the primary (largest) base surface restrict **5 DOF** (−Z translation and ± rotations about X and Y).
  - **2 locators** on the secondary (longer side) surface restrict **3 DOF** (−Y translation and ± rotation about Z).
  - **1 locator** on the tertiary (end) surface restricts **1 DOF** (−X).
  - Total: **6 locating points restrict 9 DOF**. The remaining **3 DOF** (+X, +Y, +Z) are restrained by **clamps**.
- **Cylindrical work**: a long V-block or a pin in a hole fixes 4 DOF (2 translations + 2 rotations). A short V-block/pin fixes 2. Use **one round pin + one diamond (relieved) pin** to locate a plate with two holes. The diamond pin prevents redundant location while allowing for centre-distance variation.

Locator	Use / point
Rest pads / buttons, flat-headed pins	Machined flat surfaces (flat head); **spherical/serrated head** for rough castings
Adjustable locators (screw/jack pins)	**Rough/uneven** surfaces such as sand castings, varying stock
Cylindrical locating pins, **diamond pin**	Locating from holes; the diamond pin allows for tolerance in hole spacing
V-block / V-locator	**Cylindrical work**; self-centring along one axis. A 90° V is common.
Conical locators, centres	Locating from holes or centre holes; self-centring

Nandyal Banking Coaching Center

Nest (cavity) / profile locators	Locating irregular outlines (e.g. sheet blanks)
Sight locators	Rough location by eye (castings)

- **Locating principles**:
  - Locate from a **machined (datum) surface** whenever possible.
  - Place locators **as far apart as possible**.
  - **Avoid redundant locators** (over-constraint).
  - Provide **fool-proofing** (poka-yoke) so that the work can be loaded only the right way.
  - Provide chip relief: locators raised, with chip grooves.
  - Locators should be hardened and replaceable.
- **Clamping principles**:
  - The clamping force should be directed **towards the locators** (never against them, never pushing the work off them).
  - Clamp on **rigid, supported parts** of the work to avoid distortion.
  - Cutting forces should be taken by the **locators, not the clamps** (the cutting thrust should act against fixed stops).
  - Clamps should be quick-acting, simple and placed away from the tool path.
- **Clamp types**:
  - **Strap/lever clamp** (heel pin principle)
  - Screw clamp, swing clamp, hinged clamp, C-clamp
  - **Cam clamp** (quick action; not for vibration)
  - **Toggle clamp** (quick, high mechanical advantage at the dead centre)
  - Equalising clamp (two points)
  - Wedge clamp
  - Pneumatic, hydraulic and vacuum clamps
  - Magnetic chucks (grinding)

Drill bush	Feature / use
Press-fit (permanent / fixed) – headless or headed	Pressed directly into the jig plate; for **a single operation** in low/medium production. The headed type resists axial thrust.
Renewable (fixed-renewable)	Fits in a **liner bush**; locked by a screw; **replaced when worn** (long runs)
Slip (slip-renewable)	**Quick change** without tools for **successive operations** in the same hole (drill → ream → tap)
Liner (master) bush	Permanent hardened sleeve in the jig plate that receives renewable/slip bushes and protects the plate
Threaded, special (eccentric, extended) bushes	Special situations

- **Bush material**: hardened tool steel (≈ 60–64 HRC) or carbide. The bush length is usually 1–2 × the drill diameter. A gap of about **0.5–1 × drill diameter** is left between the bush end and the work for chip clearance (zero gap for very precise holes).
- **Types of jigs**:
  - **Template jig**: simplest; a plate with holes placed on the work.
  - Plate jig, channel jig, diameter jig.
  - **Leaf/latch jig**: hinged leaf.
  - **Box (tumble) jig**: holes on several faces; the jig is turned over.
  - **Indexing jig**: equally spaced holes on a circle.
  - Sandwich jig, angle-plate jig, pot jig, turnover jig.
- **Fixtures**:
  - **Milling fixture**: needs **tenons** (tongues) to align with the T-slots and a **setting block** for the cutter. It must be very rigid, since milling forces are heavy and intermittent.
  - Turning fixture: must be balanced.
  - Welding fixture: needs clearance for distortion and fumes.
  - Others: grinding, broaching, assembly and inspection fixtures.
- **Modular fixtures** (grid-hole plates with standard elements) suit small batches and FMS. Pallets with fixtures allow loading outside the machine on CNC/FMS.

MEMORY TRICK

"**J**ig = **J**ourney guide for the tool (drill); **F**ixture = **F**ixed to the table (mill)." 3-2-1 arithmetic: "**5 + 3 + 1 = 9 by locators; 3 by clamps; 12 total**."

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In the 3-2-1 principle, how many DOF are restricted by the six locators? → **9** (the remaining 3 by clamps).
- Device that holds the work and guides the tool? → **Jig**.
- Drill bush used when drilling and reaming are done in the same hole one after the other? → **Slip bush**.
- Locator suitable for rough cast surfaces? → **Adjustable (jack/screw) locator**.
- A milling fixture is aligned to the table T-slots by? → **Tenons**.

11.7 CNC, CAD/CAM & CIM ★★★

- **NC (numerical control)**: the machine is controlled by coded alphanumeric instructions on punched tape (8-channel, EIA RS-244 / ISO). Hard-wired control; the program is read block by block every time.
- **NC history**: **John T. Parsons** with **MIT**, for the US Air Force. The first NC milling machine was demonstrated in **1952**.
- **CNC**: an NC system with a **dedicated microcomputer** (soft-wired). The program is stored in memory, so editing, tool/cutter compensation, canned cycles, diagnostics and graphics simulation are possible.
- **DNC (direct numerical control)**: **one central computer controls several NC machines** directly in real time (the tape reader is bypassed).
- **Distributed NC**: a central computer **downloads programs** to several CNC machines and collects shop-floor data.
- **Adaptive control (AC)**: adjusts feed/speed during cutting from measured force, power or temperature. Two types: ACO (optimisation) and ACC (constraint).
- **Machining centre**: a CNC machine with an **automatic tool changer (ATC)** and a tool magazine (drum/chain), often with an automatic pallet changer (APC). It can mill, drill, bore and tap in one set-up.
- **Turning centre**: a CNC lathe with a turret, sometimes with live tooling.

Nandyal Banking Coaching Center

Feature	Description
Z-axis	Along (parallel to) the **main spindle** axis. **+Z points away from the work** (the tool retracts).
X-axis	Horizontal, perpendicular to Z, parallel to the work-holding surface; the **longest travel**. On a lathe, X is **radial** (cross-slide).
Y-axis	Completes the **right-hand Cartesian system**
Rotary axes	**A, B, C** about X, Y, Z respectively. Secondary linear axes U, V, W (parallel to X, Y, Z).
Standards	**ISO 841** / EIA RS-267 (axes); **ISO 6983 / EIA RS-274-D** (G-code program format)
Point-to-point (PTP)	Positions the tool at points; no cutting during movement; path between points not controlled. **Drilling, punching, spot welding**.
Straight-cut (paraxial)	Cutting along lines parallel to one axis at a time
Contouring (continuous path)	Two or more axes controlled together by an **interpolator** (linear, circular, parabolic, helical). **Milling profiles, turning contours**.
Open-loop control	**No feedback**; **stepper motors** (step angle e.g. 1.8° = 200 steps/rev). Cheap; light loads; accuracy depends on the motor not missing steps.
Closed-loop control	**Feedback** of position/velocity by **encoders** (optical rotary, linear), resolvers, **linear scales**, inductosyn and tachogenerators; driven by **DC/AC servo motors**. Accurate; most CNCs. "Semi-closed" = rotary encoder on the motor/screw.
Drive elements	**Recirculating ball screws** (low friction, preloaded to give near-zero backlash), linear guideways (LM guides), anti-friction slideways

- **BLU (basic length unit)** = the smallest movement the control can command (e.g. 0.001 mm). Resolution of a stepper drive = lead/(steps per rev × gear ratio).
- **Accuracy** = resolution/2 + mechanical errors. **Repeatability** is usually better than accuracy.
- **Part program structure**: program number (O____), then **blocks**. Word-address format, e.g. **N010 G01 X50.0 Y25.0 F100 S1200 T01 M03 ;**
  - N = sequence (block) number
  - G = preparatory function
  - X/Y/Z = coordinates
  - I, J, K = arc-centre offsets from the start point along X, Y, Z (R = radius alternative)
  - F = feed
  - S = spindle speed
  - T = tool number
  - M = miscellaneous function
  - ; or EOB = end of block
- **Modal G-codes** stay active until replaced by another code of the same group (e.g. G01, G90). **Non-modal** codes act only in their own block (e.g. G04 dwell, G28).

G-code (Fanuc/ISO)	Function	G-code	Function
G00	**Rapid traverse** (positioning, no cutting)	**G40**	**Cutter radius compensation cancel**
G01	**Linear interpolation** (cutting feed)	**G41**	Cutter compensation **left** (tool to the left of the path, looking along motion)
G02	**Circular interpolation clockwise**	**G42**	Cutter compensation **right**
G03	**Circular interpolation counter-clockwise**	G43 / G44 / G49	Tool length compensation + / – / cancel
G04	**Dwell** (non-modal)	G54–G59	Work-coordinate system (work offsets)
G17	**XY plane** selection	G80	Canned cycle cancel
G18	**ZX (XZ) plane** (default on lathes)	G81 / G83	Drilling cycle / **peck drilling** cycle
G19	**YZ plane**	**G90**	**Absolute** dimensioning (from the program zero)
G20	**Inch** input (ISO/older: G70)	**G91**	**Incremental** dimensioning (from the previous point)
G21	**Metric (mm)** input (ISO/older: G71)	G92	Set/preset work coordinate (mill). On a Fanuc lathe, G50 sets the max spindle speed and G92 is the thread-cutting cycle.
G28	**Return to reference point (machine home)**	**G94 / G95**	Feed **per minute** / feed **per revolution** (milling, ISO). Fanuc lathe uses G98/G99.
G33 / G32	Thread cutting (constant lead)	G96 / G97	**Constant surface speed** (m/min) / constant rpm

M-code	Function	M-code	Function
M00	Program stop (unconditional)	**M06**	**Tool change**
M01	Optional (planned) stop	M07	Coolant on (mist)
M02	**End of program**	**M08**	**Coolant on** (flood)
M03	**Spindle on clockwise**	**M09**	**Coolant off**
M04	**Spindle on counter-clockwise**	**M30**	**End of program with rewind/reset** to start
M05	**Spindle stop**	M98 / M99	Sub-program call / sub-program end (return)

- **Reference points**:
  - **Machine zero**: fixed by the manufacturer.
  - **Work/program zero**: chosen by the programmer.
  - Reference/home point.
  - Tool reference point.

  Tool offsets account for differences in tool length and radius.
- **Cutter radius compensation (G41/G42)** lets the programmer program the **part profile**. The control offsets the tool path by the cutter radius.
- **Canned (fixed) cycles**: one block for a repetitive sequence such as drilling, tapping, boring or rough turning (G71 stock removal in Fanuc lathe).
- **Programming methods**:

Nandyal Banking Coaching Center

- **Manual part programming** (G/M codes).
- **Computer-assisted**: **APT** (Automatically Programmed Tools), developed at **MIT (1956–59)**; the most widely used NC language. Others: ADAPT, EXAPT (German), COMPACT II, AUTOSPOT.
- CAD/CAM-based (graphics) programming.
- Conversational / shop-floor programming.

- **APT statements**:
  - **Geometry** (POINT, LINE, CIRCLE, PLANE)
  - **Motion** (GOTO, GODLTA, GOLFT, GORGT, GOFWD; TL/ON/PAST/TO)
  - **Post-processor** (MACHIN, FEDRAT, SPINDL, COOLNT)
  - **Auxiliary** (CUTTER, PARTNO, FINI)

  The APT processor produces **CL data** (cutter location file). The **post-processor** converts CL data into the G/M codes of a **specific machine**.

CAD concept	Key facts
Wireframe model	Edges and vertices only. Least memory; **ambiguous** (nonsense/multiple interpretations); no hidden-line removal or mass properties.
Surface model	Faces/surfaces (ruled, revolved, Bezier, B-spline, NURBS, Coons patch). Used for **NC tool-path generation**, car bodies and aircraft. No mass properties.
Solid model	Complete, unambiguous. Gives **mass properties** (volume, CG, MI), interference checks, FEA. Most memory.
CSG (constructive solid geometry)	Combines **primitives** (block, cylinder, sphere, cone, torus, wedge) by **Boolean operations** (union ∪, difference −, intersection ∩), stored as a binary **tree**. Compact, always valid; editing history kept.
B-rep (boundary representation)	Stores **faces, edges, vertices** and their topology. Validity checked by the **Euler–Poincaré formula V − E + F = 2** (simple polyhedra, e.g. cube 8 − 12 + 6 = 2). Fast display; more storage. Most modern modellers are hybrid/feature-based parametric.
Bezier curve	Passes through the **first and last** control points only. Degree = **n − 1** for n control points (Bernstein polynomials). **Global control**: moving one point changes the whole curve. Lies within the **convex hull**; end tangents lie along the first and last control-polygon legs.
B-spline	**Local control**; degree independent of the number of control points; C^2 continuity for cubics. **NURBS** (rational) can represent conics exactly. It is the industry standard (IGES/STEP).
Hermite cubic (Ferguson) curve	Defined by 2 end points + 2 end tangent vectors
Geometric transformations	Translation, scaling, rotation, reflection, shear. **Homogeneous coordinates**: 3×3 matrices for 2-D, **4×4 for 3-D**, so that translation also becomes a matrix multiplication. Concatenation is matrix multiplication (order matters).
Data exchange standards	**IGES** (first neutral format, 1980), **STEP (ISO 10303)** – product data incl. AP203/AP214/AP242; DXF (AutoCAD 2-D); **STL** (AM); Parasolid (x_t), ACIS (sat)

- **CAD hardware/graphics**: raster scan display (most common), refresh vector, storage tube. Input devices: digitiser, light pen, mouse, joystick. Line algorithms: DDA and **Bresenham's** (integer arithmetic). Hidden-surface removal: **Z-buffer**, painter's algorithm.
- **CAM**: use of computers to plan, manage and control manufacturing. Covers NC programming, CAPP, MRP, scheduling, shop-floor control and quality control (CMM, machine vision).
- **CAPP (computer-aided process planning)**:
  - **Variant (retrieval)**: based on **group technology**. A standard plan exists for each part family; a new part is coded, the family plan retrieved and edited. Simple; needs a human planner.
  - **Generative**: the plan is created **from scratch** for each part using decision logic, decision tables/trees, expert systems and AI, from part geometry and material data. No human editing. Harder to build.
- **Group technology (GT)**: groups parts with similar design or manufacturing attributes into **part families**. Machines are arranged in **cells** (cellular manufacturing). Methods:
  - Visual inspection.
  - **Production flow analysis (PFA, Burbidge)**: based on routing sheets. **Rank order clustering (King)** is used on the machine–part matrix.
  - Classification and coding.
- **GT coding systems**:
  - **Opitz** (Prof. H. Opitz, Aachen, Germany): **5-digit form code** (part class/shape, external shape, internal shape, plane-surface machining, auxiliary holes/gear teeth) + **4-digit supplementary code** (dimension, material, original shape of raw material, accuracy) = 9 digits, plus an optional secondary code. Hybrid structure.
  - **MICLASS** (TNO, Netherlands): 12 digits.
  - **DCLASS** (Brigham Young University): 8 digits, decision-tree.
  - CODE (MDSI): 8 digits, hexadecimal.
- **Code structures**: **monocode (hierarchical)** – the meaning of each digit depends on the previous one; compact. **Polycode (chain)** – each digit has a fixed meaning; easy. **Hybrid (mixed)** – e.g. Opitz.
- **FMS (flexible manufacturing system)**: a group of **CNC machine tools** (workstations) linked by an **automated material-handling and storage system** (AGVs, conveyors, robots, AS/RS) under **central computer control**.
  - Suited to **mid-volume, mid-variety** production (between transfer lines and stand-alone CNC).
  - Types of flexibility: machine, routing, process, product, volume, expansion.
  - Layouts: in-line, loop, ladder, open-field, robot-centred cell.
  - Flexible manufacturing cell (FMC) = 2–3 machines; an FMS usually has ≥ 4.
- **Transfer line (hard/fixed automation)**: high volume, low variety. **Programmable automation** (CNC, robots): batch production. **Flexible automation** (FMS): medium volume and variety.
- **CIM (computer-integrated manufacturing)**: integration of all functions – design (CAD), planning (CAPP, MRP/ERP), manufacturing (CAM, FMS), quality (CAQ), business – through a **common database** and networks. Term from J. Harrington (1973). The CASA/SME "CIM wheel" is the standard model. Protocols: MAP (GM, manufacturing) and TOP (Boeing, office).

Nandyal Banking Coaching Center

Robot configuration	Joints (arm, first 3)	Work envelope
Cartesian / rectangular (gantry)	P P P (3 linear)	**Rectangular box**
Cylindrical	R P P (base rotation + vertical + radial)	**Cylindrical (annular)**
Polar / spherical (e.g. Unimate)	R R P	**Partial sphere**
Jointed-arm / articulated / revolute (anthropomorphic)	R R R (waist, shoulder, elbow)	**Irregular (complex) sphere**; largest envelope for its size; most common industrial robot
SCARA (Selective Compliance Assembly Robot Arm)	R R P (two vertical-axis revolute joints + vertical linear)	Cylindrical-type. Rigid vertically, compliant horizontally, so it is ideal for **assembly / pick-and-place / insertion**.

- **Robot DOF**: **6 DOF** are needed to place an object at any position **and** orientation in space. These are 3 arm/body joints (position) + 3 **wrist** joints, namely **roll, pitch, yaw** (orientation). A robot with more than 6 DOF is **redundant**.
- **End effector** = **gripper** (mechanical, vacuum/suction cups, magnetic, adhesive) or **tool** (welding gun, spray gun, spindle).
- **Robot terms and history**:
  - **Drives**: electric (most common), hydraulic (heaviest loads), pneumatic (light pick-and-place).
  - **Programming**: lead-through (walk-through), teach pendant, offline.
  - "Robot" comes from **Karel Čapek**'s play R.U.R. (1920; Czech *robota* = forced labour). "Robotics" and the **Three Laws** come from **Isaac Asimov**.
  - The first industrial robot was **Unimate** (Devol & Engelberger), at GM, 1961. Engelberger is called the "father of robotics".
  - **Work envelope** = volume reachable by the wrist/end effector. **Payload**, **repeatability** and accuracy are the key specifications.
  - **Cobots** (collaborative robots) work alongside humans without guarding.
- **AGV (automated guided vehicle)**: driverless, battery-powered material-handling vehicle. Types: towing (driverless train), **unit-load carriers**, pallet trucks, fork-lift AGVs. Guidance: embedded **wire** (inductive), magnetic/optical **tape**, **laser** navigation, vision/SLAM (free-ranging AMRs, autonomous mobile robots). Widely used in **warehouses** (relevant to CWC) together with **AS/RS** (automated storage and retrieval systems: stacker cranes in high-bay racks, unit-load and mini-load).
- **Automation types**: **fixed (hard)** – high volume, inflexible (transfer lines); **programmable** – batch (CNC, robots); **flexible** – FMS. Levels: device → machine → cell → plant → enterprise.
- **Industry 4.0** (term from Germany, **Hannover Fair 2011**): the "fourth industrial revolution".
  - Earlier revolutions: 1.0 = steam/mechanisation; 2.0 = electricity/mass production; 3.0 = electronics, IT, PLC automation.
  - Pillars: **cyber-physical systems (CPS)**, **Industrial IoT**, cloud computing, **big data analytics**, AI/ML, **digital twin**, additive manufacturing, augmented reality, autonomous robots, cyber-security and horizontal/vertical integration.
  - Leads to "smart factories", predictive maintenance and mass customisation.
  - **Industry 5.0** adds human-centric, sustainable and resilient production (EU, 2021).
  - India's initiatives: SAMARTH Udyog Bharat 4.0.
- **PLC** (programmable logic controller): invented for GM (Dick Morley, Modicon, 1968–69). Ladder logic is the most common language (IEC 61131-3). Scan cycle: **input scan → program execution → output update**. **SCADA**: supervisory control and data acquisition.

MEMORY TRICK

"**0-Zoom, 1-Line, 2-Clock, 3-Counter**" = G00 rapid, G01 linear, G02 CW, G03 CCW. "**17-18-19 = XY, ZX, YZ**" (XY first, then the plane with Z, alphabetical cycle). **G90 = Absolute** (90° = "right" reference); **G91 = Incremental**. M-codes: "**3 = Clockwise, 4 = Counter, 5 = Stop, 6 = Swap tool, 8 = Wet, 9 = Dry, 30 = Rewind**". GT coding: **Opitz = 5 + 4**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- In CNC machines the Z-axis is? → **Parallel to the main spindle axis**.
- G02 stands for? → **Circular interpolation clockwise**.
- M06 denotes? → **Tool change**.
- Feedback device used in closed-loop CNC? → **Encoder / resolver / linear scale**.
- Process planning that creates a new plan from scratch using decision logic? → **Generative CAPP**.
- Robot configuration best suited for assembly operations? → **SCARA**.

11.8 Additive Manufacturing ★★★

- **Additive manufacturing (AM)** / 3-D printing / rapid prototyping = "the process of **joining materials to make parts from 3-D model data, usually layer upon layer**". It is the opposite of subtractive (machining) and formative (casting, forging) methods. Definition per **ISO/ASTM 52900** (first 2015; revised 2021).
- **Process chain**:
  - **3-D CAD model**
  - Conversion to **STL** (tessellation)
  - File repair/verification
  - Orientation and **support generation**
  - **Slicing** into layers and tool-path generation (G-code for FDM)
  - Building layer by layer
  - Part removal and **post-processing**: support removal, curing, sintering/infiltration, heat treatment/HIP, machining, finishing
- **STL file format** (from "**stereolithography**", developed by **3D Systems, 1987**):
  - Approximates the surface by a mesh of **triangles (facets)**. Each facet is defined by its **3 vertices + a unit outward normal vector**.
  - Available as **ASCII or binary** (binary is more compact).

Nandyal Banking Coaching Center

- Stores **no colour, material, texture or units**, and is redundant (shared vertices are repeated).
- Errors include gaps/cracks, inverted normals, degenerate facets and non-manifold edges. The **vertex-to-vertex rule** applies.
- Newer formats: **AMF** (ISO/ASTM 52915; colour, materials, curved triangles), **3MF** (3MF Consortium), OBJ, PLY. A finer tessellation gives better accuracy but a larger file.

- **Staircase (stair-step) effect**: inclined and curved surfaces show steps because of the finite **layer thickness** (typically 0.02–0.3 mm). Thinner layers give a better finish but longer build time. **Build orientation** affects finish, supports, strength (parts are anisotropic and weakest in Z) and build time.

ISO/ASTM 52900 category (7)	Principle	Representative processes	Materials / key facts
1. Vat photopolymerisation	Liquid **photopolymer resin in a vat** is selectively cured by light (UV)	**SLA (stereolithography)** – UV laser scanning; **DLP** (digital light projector, whole layer at once); CLIP/CDLP	Acrylic/epoxy resins. **Best surface finish and accuracy** among polymer processes. Needs supports and **post-curing** (UV oven). **First commercial AM process**: Charles (Chuck) **Hull**, patent 1986, **3D Systems** (SLA-1, 1987).
2. Material extrusion	Material **selectively dispensed through a heated nozzle**	**FDM** (Fused Deposition Modelling; S. Scott Crump, **Stratasys**, 1989; trademarked) = **FFF** (fused filament fabrication; open-source name, RepRap)	Thermoplastic filament: **PLA** (biodegradable, easy), **ABS**, PETG, nylon, PC, TPU, PEEK/ULTEM. **Cheapest and most common** (desktop printers). Needs supports (often a soluble support material). Rough finish; anisotropic. Also concrete printing and bio-printing.
3. Powder bed fusion (PBF)	Thermal energy (laser/electron beam) **selectively fuses regions of a powder bed**; a recoater spreads each layer	**SLS** (selective laser sintering; Carl **Deckard**, Univ. of Texas Austin, 1986–89) – **polymers** (nylon PA12); **SLM / DMLS / LPBF** – **metals** (full melting); **EBM (electron beam melting, Arcam)** – metals in **vacuum**, preheated bed; MJF (HP Multi Jet Fusion)	In SLS the **surrounding unfused powder supports the part**, so no supports are needed. Metal PBF needs supports/anchors and an inert gas (Ar/N_2). Ti-6Al-4V, Inconel 718/625, SS 316L, AlSi10Mg, Co–Cr. Residual stresses; porosity; **HIP** post-treatment.
4. Material jetting	Droplets of build material (photopolymer/wax) **selectively deposited** (ink-jet heads) and UV-cured	**PolyJet** (Objet, now Stratasys), MultiJet (3D Systems), NanoParticle Jetting	**Mult and f** high and s finish weak Reali proto medi
5. Binder jetting	A liquid **bonding agent (binder) is jetted** onto a powder bed	"3DP" (MIT, Emanuel **Sachs**, 1993 – origin of the term "3-D printing"); ExOne, Desktop Metal, HP Metal Jet	Powd gypsu (full- mode (**sand and c casti** ceran heat build supp **Gree** weak curin and/ infiltr bronz
6. Directed energy deposition (DED)	Focused thermal energy (laser, electron beam, plasma/arc) **melts material as it is deposited** (powder or wire fed through a nozzle)	**LENS** (Laser Engineered Net Shaping; Sandia Labs/Optomec); laser metal deposition (LMD); DMD; **WAAM** (wire arc AM); EBAM (Sciaky)	Meta turbi moul cladd featu existi large depo poor mach need mach
7. Sheet lamination	**Sheets of material bonded** layer by layer and cut to shape (laser/knife)	**LOM** (Laminated Object Manufacturing; Helisys, 1991) – paper with adhesive; **UAM** (ultrasonic AM / ultrasonic consolidation) – metal foils; SDL (Mcor)	Pape meta comp cost, large wood mode mate into ("dec poor much

- **Liquid-based**: SLA, PolyJet. **Solid-based**: FDM, LOM. **Powder-based**: SLS, SLM, EBM, binder jetting, DED (powder-fed).
- **Laser use**: SLA – UV laser (He–Cd/solid-state); SLS – **CO_2 laser** (polymers); SLM/DMLS – **fibre (Yb) laser** (metals); LOM – CO_2 laser cutter.
- **Supports**: needed in **SLA, FDM, material jetting and metal PBF**. Not needed in **SLS (polymer), binder jetting and LOM**, where the unused powder or sheet supports the part.
- **Advantages**:
- **Complex geometry at no extra cost** ("complexity is free"): internal channels, **lattices**, **topology-optimised** lightweight parts, conformal cooling channels in moulds.
- **No tooling**; short lead time; economical for small batches and **mass customisation**.

Nandyal Banking Coaching Center

- Part consolidation (many parts printed as one; e.g. the GE LEAP fuel nozzle, 20 parts → 1).
- Less material waste (near-net shape); digital inventory and spare parts on demand.

- **Limitations**:
  - **Slow** build; unsuitable for mass production of simple parts.
  - Limited build volume; limited materials; high machine and material (powder) cost.
  - **Staircase effect** and poorer finish and accuracy than machining; post-processing needed.
  - Anisotropic properties, porosity, residual stress and distortion; qualification and certification issues.
- **Applications**:
  - **Rapid prototyping** (form/fit/function models; the original application).
  - **Rapid tooling**: moulds and patterns, sand cores (binder jet), conformal-cooled injection-mould inserts, investment-casting patterns (wax/SLA QuickCast).
  - **Direct/rapid manufacturing**: end-use parts.
  - **Aerospace** (fuel nozzles, brackets, Ti parts – high "buy-to-fly" savings).
  - **Medical**: patient-specific **implants** (Ti-6Al-4V hip cups, cranial plates, porous lattices for bone in-growth – EBM/SLM), surgical guides, dental crowns/aligners, prosthetics, hearing aids, anatomical models from CT/MRI scans, **bio-printing**.
  - Automotive, jewellery (castable resin), architecture models, consumer goods, defence spares, construction.
- **Related terms**:
  - **Reverse engineering**: a 3-D scanner or CMM produces a point cloud, then a CAD/STL model, then AM.
  - **4-D printing**: printed parts that change shape over time in response to a stimulus (shape-memory materials). Term from MIT's Skylar Tibbits (2013).
  - **Hybrid manufacturing**: DED + CNC milling in one machine.
  - **Rapid prototyping** is the older umbrella name. **Additive manufacturing** is the standard ISO/ASTM term for industrial use; "3-D printing" is the popular term.

Feature	SLA	FDM	SLS	SLM/DMLS
Raw material form	Liquid resin	Filament (solid)	Polymer powder	Metal powder
Energy source	UV laser	Heated nozzle (resistance heating)	CO_2 laser	Fibre laser
Supports	Yes	Yes	**No**	Yes (anchors, heat sink)
Finish / accuracy	**Best**	Poorest (visible layers)	Grainy, porous	Rough (needs machining)
Post-processing	Washing (IPA), UV post-cure	Support removal	De-powdering, bead blasting	Stress relief, HIP, support removal by wire EDM/sawing

CODE UPDATE

The AM terminology standard **ISO/ASTM 52900:2015** (which replaced ASTM F2792) was revised as **ISO/ASTM 52900:2021**. The **seven process categories** are retained in the revision. Data formats: AMF is **ISO/ASTM 52915**; 3MF is maintained by the 3MF Consortium.

MEMORY TRICK

The seven AM categories: "**V**ery **M**any **P**eople **M**ake **B**ig **D**esign **S**hapes":

- **V**at photopolymerisation
- **M**aterial extrusion
- **P**owder bed fusion
- **M**aterial jetting
- **B**inder jetting
- **D**irected energy deposition
- **S**heet lamination

SLA = **L**iquid + **L**ight; SLS = **S**and-like powder, **S**upport-free.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Standard file format used for AM / rapid prototyping? → **STL** (triangulated surface).
- FDM belongs to which ISO/ASTM 52900 category? → **Material extrusion**.
- AM process using liquid photopolymer cured by UV laser? → **Stereolithography (SLA)**.
- AM process in which no support structures are needed as unfused powder supports the part? → **SLS**.
- LENS is an example of? → **Directed energy deposition**.
- Number of AM process categories in ISO/ASTM 52900? → **Seven**.

Nandyal Banking Coaching Center

12. Metrology, Inspection & Maintenance Engineering

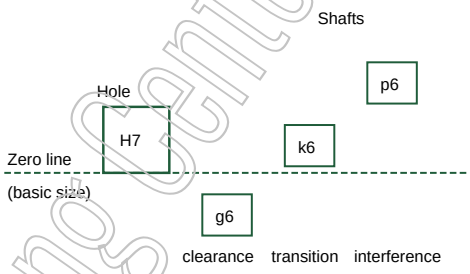
NOTE

CWC runs warehouses, cold storages and a large fleet of material handling equipment (forklifts, stackers, EOT cranes, conveyors, DG sets, refrigeration plants), so an Assistant Manager (Mechanical) must know **inspection, NDT, condition monitoring and maintenance planning**. Exams ask direct facts: least counts, IT-grade formula, fit types, Taylor's principle, sine bar limits, N-grades, control-chart limits, Cp/Cpk, NDT selection, MTBF/availability, bathtub curve, OEE, RPN, NLGI grades and Factories Act lifting-machine rules. Expect **3–6 MCQs** from this chapter.

12.1 Limits, Fits & Tolerances ★★★

- **Basic (nominal) size** = size from which limits are fixed by applying deviations; same for hole and shaft of a fit.
- **Actual size** = size obtained by measurement. **Limits of size** = maximum and minimum permissible sizes.
- **Deviation** = algebraic difference between a size and the basic size. **Upper deviation** (ES for hole, es for shaft) = max limit − basic size. **Lower deviation** (EI for hole, ei for shaft) = min limit − basic size.
- **Zero line** = straight line representing the basic size (deviation = 0); positive deviations above, negative below.
- **Fundamental deviation** = the deviation (upper or lower) **nearest to the zero line**; it fixes the **position** of the tolerance zone and is indicated by a **letter**.
- **Tolerance** = max limit − min limit = upper deviation − lower deviation; always **positive**, has no sign; fixes the **size (width)** of the zone and is indicated by the **IT grade number**.
- **Unilateral tolerance** = variation on one side of basic size (e.g. $25^{+0.021}_{+0.000}$) – preferred in the limit system. **Bilateral** = on both sides (e.g. 25 ± 0.01) – used for location dimensions.
- **Allowance** = **intentional difference between the maximum material limits** of mating parts = **minimum clearance** = LL(hole) − UL(shaft). Positive allowance → clearance; negative allowance → interference.
- **Maximum Material Condition (MMC)** = part contains maximum material: **hole at its lower limit, shaft at its upper limit**. **Least Material Condition (LMC)**: hole at upper limit, shaft at lower limit.
- **Fit** = relationship between mating parts arising from the difference in their sizes before assembly.
- **Max clearance** = UL(hole) − LL(shaft); **min clearance** = LL(hole) − UL(shaft). **Max interference** = UL(shaft) − LL(hole).
- **Fit tolerance (variation)** = hole tolerance + shaft tolerance = max clearance − min clearance.

Fit	Tolerance zones	Examples (hole basis)	Typical use
Clearance	Hole zone entirely above shaft zone; always clearance	H11/c11 loose running; H9/d9 free running; H8/f7 close running; **H7/g6 sliding**; **H7/h6 locational clearance** (min clearance = 0)	Bearings, sliding gears, pistons, spigots
Transition	Zones overlap; may give small clearance **or** small interference	**H7/k6** (locational transition, push fit); H7/n6 (locational, tighter); H7/js6	Gears and pulleys on shafts with key, coupling hubs, spigot location
Interference	Shaft zone entirely above hole zone; always interference	**H7/p6** locational interference / press fit; **H7/s6** medium drive fit; H7/u6 force (shrink) fit	Bushes, bearing outer race in housing, wheel on axle, rim on hub



Hole-basis system (Ø25, to scale): H7 hole 0/+21 µm; g6 −7/−20, k6 +15/+2, p6 +35/+22 µm.

- **Hole-basis system**: hole is the constant member, **H hole – lower deviation = 0** (hole minimum size = basic size); different fits obtained by varying the shaft. **Preferred** (IS 919/ISO) because holes are made with **fixed-size tools (drills, reamers, broaches)** and checked with fixed plug gauges, while shaft sizes are easily varied by turning/grinding.
- **Shaft-basis system**: **h shaft – upper deviation = 0** (shaft maximum size = basic size); fits obtained by varying the hole. Used when standard **bright/drawn bar stock** is used without machining (textile, agricultural machinery, line shafts carrying several parts with different fits), and for **rolling-bearing outer race** in housing.
- **Rolling bearings**: inner race on shaft follows **hole basis** (shaft tolerance e.g. k5/m5/j6); outer race in housing follows **shaft basis** (housing e.g. H7/J7).

IS 919 / ISO 286 item	Value / fact
Number of standard tolerance grades	**20** – IT01, IT0, IT1 … IT18
Number of fundamental deviations	**28** for holes (A … ZC) and 28 for shafts (a … zc)
Letters not used	**I, L, O, Q, W** (to avoid confusion); two-letter symbols CD, EF, FG, JS, ZA, ZB, ZC added
Holes A–G	Zone **above** zero line (EI positive)
Shafts a–g	Zone **below** zero line (es negative) → clearance with H hole
H / h	EI = 0 / es = 0
JS / js	Tolerance zone **symmetrical** about zero line (± IT/2)
Shafts k–zc	Zone above zero line (ei positive) → transition / interference
Size range covered (standard formula)	Up to **500 mm** (extended range 500–3150 mm uses I = 0.004D + 2.1 µm)

Nandyal Banking Coaching Center

Diameter D used in formula	**Geometric mean** of the diameter step, D = $\sqrt{(D_1 D_2)}$
IT01–IT4	Gauges, slip gauges, precision instruments
IT5–IT11 (approx.)	Fits / machined mating parts (IT5–IT7 grinding, lapping, honing; IT7–IT9 fine turning, reaming)
IT12–IT18	Non-mating parts, casting, forging, rolling (large tolerances)

KEY FORMULAE

- Standard tolerance unit: **$i = 0.45\sqrt[3]{D} + 0.001\,D$** (i in µm, D in mm) – first term covers manufacturing error, second term covers measurement error and temperature effects.
- IT5 = 7i, **IT6 = 10i**, **IT7 = 16i**, **IT8 = 25i**, IT9 = 40i, IT10 = 64i, IT11 = 100i, IT12 = 160i, IT13 = 250i, IT14 = 400i, IT15 = 640i, IT16 = 1000i.
- From IT6 onward each grade ≈ **1.6 × previous** (R5 preferred series); tolerance **multiplies by 10 every 5 grades**.
- IT01 = 0.3 + 0.008D; IT0 = 0.5 + 0.012D; IT1 = 0.8 + 0.020D (µm); IT2–IT4 in geometric progression between IT1 and IT5.
- Example Ø25 (step 18–30): D = √(18×30) = 23.24 mm → i = 0.45(2.854) + 0.023 = **1.31 µm** → IT6 ≈ 13 µm, IT7 ≈ 21 µm, IT8 ≈ 33 µm.

Limit gauges & Taylor's principle

- **Limit gauges** check whether a part lies within limits (not actual size). **GO** gauge made to the **MMC limit**; **NO-GO** (NOT-GO) gauge made to the **LMC limit**.
- **Plug gauge** (for holes): GO end = **lower limit of hole**; NO-GO end = upper limit of hole. NO-GO end is **shorter**, often marked by a red band/groove. **Progressive plug gauge**: GO and NO-GO on same end, one after the other.
- **Snap (gap) / ring gauge** (for shafts): GO = **upper limit of shaft**; NO-GO = lower limit of shaft. Ring gauge used as GO (checks full form); snap gauge often for NO-GO.
- **Taylor's principle** (William Taylor, 1905): the **GO gauge should check all related dimensions (form + size) simultaneously** at MMC – i.e. full-form, length equal to the length of engagement; the **NO-GO gauge should check only one dimension at a time** at LMC – i.e. point contact / short gauge.
- **Gauge maker's (gauge) tolerance** ≈ **10% of work tolerance**. **Wear allowance** ≈ 10% of gauge tolerance (some books: ~5% of work tolerance), applied **only to the GO gauge** (GO gauge wears as it enters the work; NO-GO normally does not enter) – the GO gauge size is shifted **into** the work tolerance zone.
- Gauge tolerances are kept **inside** the work tolerance zone (IS 3455 system) so that no bad part is accepted (some good parts at the limits may be rejected).
- **Interchangeability** = any part from a lot fits any mating part without fitting/selection – basis of mass production; needs standard limits & gauging. Introduced by **Eli Whitney** (muskets).
- **Selective assembly** = parts are measured and sorted into groups; group-1 holes matched with group-1 shafts etc. Gives **close fits from wider (cheaper) tolerances**; used for piston–cylinder, ball bearing races and balls. Loses full interchangeability.
- Indian standards: **IS 919 (Parts 1 & 2)** = ISO system of limits and fits (based on **ISO 286**); IS 2709 = guide for selection of fits; IS 3455 = gauging; IS 2102 = general tolerances.

MEMORY TRICK

"**GO = MMC, NO-GO = LMC**"; plug for hole – GO = small (lower) limit; snap for shaft – GO = big (upper) limit. Grade multipliers **7–10–16–25–40–64–100** (each ≈ ×1.6). Fits with H7: **g**o sliding, **h**and push, **k**ey transition, **p**ress, **s**hrink/drive.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Number of IT grades in IS 919/ISO 286? → **20 (IT01 to IT18)**.
- Standard tolerance unit i (µm) = ? → **$0.45\sqrt[3]{D} + 0.001D$**.
- In hole-basis system, lower deviation of the hole is? → **Zero**.
- H7/s6 is which fit? → **Interference (drive) fit**; H7/k6 → **Transition**.
- According to Taylor's principle, GO gauge checks? → **Maximum material condition, all dimensions simultaneously**.
- Wear allowance is provided on? → **GO gauge only**.

12.2 Linear & Angular Measurement ★★★

- **Standards of length**: **line standard** – length between two engraved lines (steel rule, International Prototype Metre of **platinum–iridium** (90:10), Tresca/X-section, 1889); **end standard** – length between two flat end faces (**slip gauges**, end bars, micrometer anvils); **wavelength standard** – 1960: 1 650 763.73 wavelengths of **Kr-86** orange-red line; since **1983**: metre = distance travelled by light in vacuum in **1/299 792 458 s**.
- **Standard reference temperature** for dimensional measurement = **20 °C** (ISO 1).
- **Abbe's principle**: the axis of measurement should be **in line with** the axis of the scale. Micrometer **obeys**; vernier caliper **violates** it (jaws offset from scale → Abbe error).

Instrument	Principle / LC	Key facts
Vernier caliper	LC = **1 MSD − 1 VSD** = (value of 1 MSD)/(no. of VSD); 49 MSD (1 mm) = 50 VSD → **0.02 mm**; 20 VSD → 0.05 mm; 10 VSD → 0.1 mm	Measures OD, ID, depth, step. Positive zero error is **subtracted**. Pierre Vernier (1631).
Outside micrometer	Screw & nut; LC = pitch / no. of thimble divisions = **0.5/50 = 0.01 mm**	**Ratchet stop** gives uniform measuring pressure; vernier micrometer LC **0.001 mm**; range usually 25 mm (0–25, 25–50 …).
Depth / inside / bore micrometer	Same screw principle	Three-point bore gauge (Intrimik) for holes; depth micrometer scale reads in reverse.
Dial indicator (dial gauge)	Rack & pinion + gear train; LC **0.01 mm** (fine: 0.001 mm)	Comparative measurement, run-out, alignment, flatness on surface plate. Lever-type (dial test indicator) for inaccessible spots.

Slip gauges (gauge blocks)

- Invented by **C.E. Johansson** (Sweden) – also called **Johansson gauges**; rectangular blocks of hardened, stabilised alloy steel (also tungsten carbide, ceramic); two faces lapped flat and parallel.
- They are **end standards** and the practical working standard of length in workshops.

Nandyal Banking Coaching Center

- **Wringing** = sliding and twisting two clean gauges together; they adhere by **molecular attraction + atmospheric pressure + a very thin liquid (oil) film** (~0.006–0.025 µm). Wrung blocks can bear a substantial pull.
- Build a size with the **minimum number of blocks**; select from the **last decimal place** first.
- **Grades** (BS 4311 / IS 2984; ISO 3650 equivalents K, 0, 1, 2): **Calibration grade (K)** – for calibrating other gauges; **Grade 00** – reference, standards room; **Grade 0** – inspection; **Grade I** – precise tool-room work; **Grade II** – workshop use. Accuracy decreases from K/00 to II.

Set	Composition (mm)	Pieces
M-87	1.0005 (1); 1.001–1.009 step 0.001 (9); 1.01–1.49 step 0.01 (49); 0.5–9.5 step 0.5 (19); 10–90 step 10 (9)	**87**
M-112	1.0005 (1); 1.001–1.009 (9); 1.01–1.49 (49); 0.5–24.5 step 0.5 (49); 25, 50, 75, 100 (4)	**112**
Example (M-87)	41.125 = **1.005 + 1.12 + 9.0 + 30**	4 slips

Comparators

- **Comparator** = instrument that compares a part with a **standard (slip gauge/master)** and magnifies the **difference**; does not measure absolute size. Gives quick, accurate inspection in mass production.

Type	Examples / principle	Notes
Mechanical	Dial indicator; **Johansson Mikrokator** (twisted thin metal strip – "**twisted strip**"); Reed comparator; **Sigma** comparator	Simple, robust; friction and inertia limit magnification
Optical	Lever + mirror (optical lever); Zeiss optimeter; **profile projector**	High magnification, few moving parts; needs light source and darkened view
Electrical / electronic	**LVDT** (linear variable differential transformer), strain gauge, capacitive	Very high magnification, remote reading, easily digitised
Pneumatic	**Solex** (back-pressure) comparator; flow-type (rotameter)	**Non-contact**, self-cleaning jet, very high magnification; best for **bores, internal diameters, ovality and taper**

Angular measurement

- **Sine bar**: hardened steel bar with two equal rollers; **L = centre distance of rollers** (100, 200, 250, 300 mm; commonly 200 mm). Angle set by slip gauges of height h under one roller: **sin θ = h/L**.
- Sine bar is **not recommended above 45°** – the error in angle for a given error in h ($d\theta = dh/(L \cos\theta)$) increases rapidly with θ; it is impractical beyond about 60°. Best accuracy at small angles.
- **Sine centre** – sine bar with centres for conical work held between centres; **sine table** – for heavy work; compound sine table for compound angles.
- **Angle gauges** (**Dr. G.A. Tomlinson**, NPL UK, 1939): hardened steel blocks; set of **13** (12 gauges + 1 square block): 1°, 3°, 9°, 27°, 41°; 1′, 3′, 9′, 27′; 6″, 18″, 30″. Gauges can be added or subtracted; any angle from 0° to 360° in steps of **3″**.
- **Universal bevel protractor**: least count **5 minutes** (vernier: 12 divisions on vernier = 23° on main scale; LC = 1/12° = 5′); acute angle attachment.
- **Autocollimator** = optical instrument (telescope + collimator) measuring **small angular deviations** to fractions of a second; mirror tilted by θ deflects reflected beam by **2θ**; image displacement $x = 2f\theta$. Used for **straightness and flatness** of machine guideways and surface plates, squareness, angle-gauge calibration.
- **Spirit level**: bubble moves along a curved vial of radius R; sensitivity = movement of bubble per unit tilt; typical engineer's level ≈ **10″ per division** (≈ 0.05 mm/m). **Clinometer** = spirit level on a rotary member with circular scale – measures inclination.

Screw thread measurement

- Elements: major diameter, minor diameter, **effective (pitch) diameter**, pitch, thread angle, flank form. Effective diameter is the **most important** (governs fit of threads).
- Major diameter – bench micrometer / floating carriage micrometer; minor diameter – using **V-pieces/prisms**; pitch – screw pitch measuring machine, pitch gauge; angle & form – **toolmaker's microscope**, **profile projector**.
- **Effective diameter** – **two-wire** or **three-wire method** (wires of known size placed in the grooves; measured over wires). Three-wire method is more accurate (micrometer anvils seat squarely, no tilting).
- **Best wire size** = wire that touches the flanks exactly at the **pitch line (effective diameter)** – eliminates the effect of thread-angle errors.

KEY FORMULAE

- Best wire size: $d_b = (p/2)\sec(\theta/2) = p/(2\cos(\theta/2))$; metric/ISO (θ = 60°): $d_b = 0.577\,p$; Whitworth (55°): 0.564 p.
- Two-wire: E = T + P, where T = dimension under wires and P = $(p/2)\cot(\theta/2) - d[\operatorname{cosec}(\theta/2) - 1]$; for 60° thread: **P = 0.866p – d**.
- Three-wire (60° thread): M (over wires) = E + 3d – 0.866p → **E = M – 3d + 0.866p**.
- Sine bar: $\sin\theta = h/L$. Autocollimator: $x = 2f\theta$.

Gear measurement

- **Gear tooth vernier caliper**: two verniers – vertical scale set to **chordal addendum**, horizontal scale measures **chordal thickness** at the pitch circle.
- Chordal thickness $w = N\,m\sin(90°/N)$; chordal addendum $h = m[1 + (N/2)(1 - \cos(90°/N))]$.
- **Constant chord method** – constant chord (chord joining the points where the tooth flanks touch a basic rack) is **independent of number of teeth**: $c = (\pi\,m/2)\cos^2\varphi$; depth from tip $h_c = m[1 - (\pi/4)\sin\varphi\cos\varphi]$. For φ = 20°: **c = 1.387 m**, $h_c = 0.748$ m.
- **Base tangent (span) method** – **David Brown tangent comparator**; span measured over several teeth with a disc micrometer.
- **Coordinate Measuring Machine (CMM)**: measures X, Y, Z coordinates of points with a **probe** (touch-trigger – e.g. Renishaw; scanning; non-contact laser/optical) and computes size, form and position by software. Configurations: **bridge (most common)**, cantilever, column, horizontal-arm, **gantry** (large parts). Air bearings and granite base; used in a temperature-controlled room (20 °C). Advantages: flexibility, complex geometry, reduced set-up, direct CAD comparison.

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- 49 MSD of 1 mm divided into 50 VSD – least count? → **0.02 mm**.
- Micrometer pitch 0.5 mm, 50 thimble divisions – LC? → **0.01 mm**.
- Sine bar of 200 mm, angle 30° – slip gauge height? → **100 mm**.
- Best wire size for metric thread of pitch p? → **0.577 p**.
- Comparator best suited for internal bores, non-contact? → **Pneumatic (Solex)**.
- Instrument for measuring small angular deviations / straightness of guideways? → **Autocollimator**.

12.3 Surface Finish & Form Measurement ★★☆

- **Surface texture** = **roughness** (primary texture – fine, closely spaced irregularities from the cutting action/feed marks) + **waviness** (secondary texture – wider spaced, due to vibration, chatter, deflection, heat treatment) + **lay** (direction of the predominant surface pattern) + flaws.
- **Sampling length (cut-off length)** = length over which roughness is evaluated; filters out waviness. Standard values 0.08, 0.25, **0.8**, 2.5, 8, 25 mm; **0.8 mm** is the default for most machined surfaces. **Evaluation length** normally = **5 × sampling length**.

Parameter	Definition	Remark
Ra (CLA / AA)	Arithmetic mean of absolute deviations from mean line: Ra = (1/L)∫|y|dx	**Most widely used** (India, UK, ISO); CLA = centre line average; AA = arithmetic average (USA)
Rq (RMS)	Root-mean-square of deviations: √[(1/L)∫y² dx]	For a **sinusoidal** profile Rq ≈ **1.11 Ra** (Rq ≥ Ra always); triangular ≈ 1.155 Ra
Rz	Ten-point height (old ISO): mean of 5 highest peaks + mean of 5 deepest valleys; ISO 4287 (1997): max height within each sampling length averaged	Sensitive to scratches/peaks; typically Rz ≈ 4–7 Ra
Rt / Rmax / Ry	**Maximum peak-to-valley height** within sampling (Rt: within evaluation) length	Critical for sealing and fatigue surfaces

- **Stylus instruments**: diamond stylus (tip radius ≈ 2–10 µm) traced across the surface. **Tomlinson surface meter** – mechanical-optical (lever and smoked glass trace). **Taylor-Hobson Talysurf** – **electronic** (carrier-modulating/inductive transducer), most common; gives Ra directly and a profile trace. Skid provides datum (skidded) or a straight datum is used (skidless).
- Stylus methods are **contact** methods; very soft surfaces may get scratched. Non-contact: optical (light section, interference microscope, laser scattering), white-light interferometry.

N-grade	Ra (µm)	N-grade	Ra (µm)
N1	**0.025**	N7	1.6
N2	0.05	N8	3.2
N3	0.1	N9	6.3
N4	0.2	N10	12.5
N5	0.4	N11	25
N6	0.8	**N12**	**50**

- Each N-grade step **doubles** Ra. 12 grades in all. Ra values follow the R10/3 series (0.025, 0.05, 0.1 …).
- **Surface texture symbol** (IS 10719/ISO 1302): basic symbol = two unequal legs at 60° (√ shape); **horizontal bar** on the symbol → **material removal required** (machining); **circle** in the symbol → **material removal not permitted**. Ra value written above the symbol.
- **Lay symbols**: **=** parallel to the projection plane, ⊥ perpendicular, **X** crossed in two oblique directions, **M** multidirectional, **C** approximately circular, **R** approximately radial, **P** particulate (non-directional).

Process	Typical Ra (µm)
Sand casting	12.5–25 (rough, up to 50)
Hot rolling / forging	3.2–25
Die casting, investment casting	0.8–3.2
Drilling	1.6–6.3
Turning, milling	0.8–6.3
Grinding	0.1–1.6
Honing	0.1–0.8
Lapping, superfinishing, polishing	**0.025–0.4** (finest)

(Ranges are typical textbook values and vary slightly between books.)

Form measurement

- **Straightness**: straight edge + feeler/light gap, **spirit level** or **autocollimator** (step-by-step along guideways), laser alignment, taut wire and microscope.
- **Flatness**: surface plate + dial indicator; autocollimator/level on a grid ("Union Jack" pattern for surface plates); **optical flat** for small lapped surfaces (slip gauges, micrometer anvils, seal faces).
- **Optical flat**: disc of fused **quartz** or glass with one/both faces optically flat; placed on the work at a slight angle and lit by **monochromatic light** (sodium, helium, mercury-green) → air wedge gives **interference fringes**; each fringe = change in air-gap of **λ/2** (for sodium λ = 0.589 µm → ≈ **0.29 µm per fringe**).
- Fringe reading: **straight, parallel, equally spaced** fringes → surface flat; curved fringes → convex or concave (finger-press test tells which); concentric rings → spherical (convex/concave).
- **Roundness (circularity)**: measured on a **rotating-spindle/rotating-table instrument (Talyrond)** – reference circles: **LSC** (least squares circle, most common), **MZC** (minimum zone), MIC (maximum inscribed), MCC (minimum circumscribed). Roundness error = radial separation of two concentric circles enclosing the profile.
- **Lobing**: a part with an **odd number of lobes** (e.g. 3-lobed, from centreless grinding) shows **constant diameter** with a micrometer – detected only with a **V-block** or rotating-spindle method. Between-centres checking gives **run-out**, not true roundness.
- **Geometric tolerances (GD&T)**: form – straightness (—), flatness, circularity (○), cylindricity (form tolerances need **no datum**); orientation – parallelism (∥), perpendicularity (⊥), angularity (∠); location – position, concentricity (◎), symmetry; run-out – circular and total run-out; profile – of a line, of a surface. 14 characteristics in all.

Nandyal Banking Coaching Center

CODE UPDATE

ASME Y14.5-2018 removed **concentricity and symmetry** as separate symbols (use position, profile or run-out instead); ISO GPS (ISO 1101) still retains them. ISO 21920 (2021) has replaced ISO 4287 for profile surface texture parameters.

MEMORY TRICK

"**N1 = 0.025, double each step → N12 = 50**" (N6 = 0.8 is the middle). Optical flat: "**one fringe = half a wave**" (λ/2 ≈ 0.3 µm). Rq/Ra for sine = **π/(2√2) = 1.11**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- CLA value is the same as? → **Ra**.
- Roughness grade N7 corresponds to Ra of? → **1.6 µm**.
- Standard cut-off length for most machined surfaces? → **0.8 mm**.
- Each interference fringe with an optical flat represents a height of? → **λ/2**.
- Talysurf works on which principle? → **Electronic (stylus with inductive / carrier-modulating transducer)**.
- Circle in the surface texture symbol means? → **Material removal not permitted**.

12.4 Errors, Calibration & Quality Control ★★☆

- **Accuracy** = closeness of a measured value to the **true value**. **Precision** = closeness of repeated readings to **each other** (repeatability). An instrument can be precise but not accurate (consistent bias).
- **Error** = measured value − true value; **correction** = − error. Relative error = error/true value.
- **Sensitivity** = output change / input change. **Resolution** = smallest change that can be detected. **Least count** = smallest value read on the scale. **Range** = max − min measurable values. **Hysteresis** = different readings for loading and unloading. **Drift** = slow change of output with time at constant input. **Dead zone** = largest input change with no output response.
- **Rule of ten (gauge-maker's rule)**: measuring instrument should be about **10 times more accurate** than the tolerance to be measured.

Error type	Cause / examples	Remedy
Systematic (fixed/bias)	Calibration error, zero error, instrument wear, **temperature**, loading effect, **parallax**, Abbe error, cosine error, elastic deformation from contact force	Same sign & magnitude – can be **eliminated** by calibration/correction
Random (accidental)	Small, unpredictable fluctuations – friction, play, operator judgement, vibration	Cannot be eliminated; reduced by **averaging many readings**; treated statistically (σ)

- **Calibration** = comparing an instrument with a standard of higher accuracy and establishing its errors/corrections.
- **Traceability** = unbroken chain of calibrations from the working instrument → working standard → secondary/reference standard → **national standard** → international (SI/BIPM).
- **CSIR-NPL, New Delhi** = National Physical Laboratory, India's **National Metrology Institute** – custodian of national standards (Indian Standard Time also maintained by NPL).
- **NABL** (National Accreditation Board for Testing and Calibration Laboratories, a board of the **Quality Council of India**) accredits labs to **ISO/IEC 17025**. **BIS** = national standards body (IS codes). Legal metrology (weights & measures used in trade, e.g. **weighbridges** at warehouses) – **Legal Metrology Act, 2009**.

Statistical Quality Control (SQC)

- **Control charts** – **Walter A. Shewhart** (Bell Labs, 1924). Distinguish **chance (common/random) causes** from **assignable (special) causes**. Process "in control" = only chance causes present.
- Control limits = **mean ± 3σ** → covers **99.73%** of observations (normal distribution); ±1σ = 68.27%, ±2σ = 95.45%.
- **Control limits ≠ specification limits**: control limits come from process data; specification limits come from the design.
- Out-of-control signals: a point outside limits; **run of 7 (or more) points** on one side of the centre line; trend of 6–7 points continuously rising/falling; cycles.

Chart	Type	Plots	Distribution / limits
$\bar{X}$ **chart**	Variables	Sample mean – central tendency	$\bar{\bar{X}} \pm A_2\bar{R}$ (normal)
R chart	Variables	Sample range – dispersion	UCL = $D_4\bar{R}$, LCL = $D_3\bar{R}$; n = 5: A_2 = 0.577, D_3 = 0, D_4 = 2.114
p chart	Attributes	**Fraction defective** (sample size may vary)	Binomial: $\bar{p} \pm 3\sqrt{[\bar{p}(1-\bar{p})/n]}$
np chart	Attributes	**Number of defectives** (constant n)	$n\bar{p} \pm 3\sqrt{[n\bar{p}(1-\bar{p})]}$
c chart	Attributes	**Number of defects per unit** (e.g. defects per casting, holes per roll of cloth, rivets missing per wing)	**Poisson**: $\bar{c} \pm 3\sqrt{\bar{c}}$

Negative LCL is taken as **zero**. "Defective" = unit fails (p, np); "defect" = individual non-conformity (c, u).

KEY FORMULAE

- Process capability = **6σ** (natural tolerance).
- C_p **= (USL − LSL)/6σ** – potential capability (ignores centring).
- C_{pk} **= min[(USL − μ)/3σ, (μ − LSL)/3σ]** – actual capability (accounts for centring); $C_{pk} \le C_p$; $C_{pk} = C_p$ only when the process is centred.
- $C_p < 1$ → not capable (defectives produced); $C_p = 1$ → just capable (≈ 0.27% outside); $C_p \ge 1.33$ → generally accepted as capable (≈ 63 ppm); six sigma → $C_p = 2$, $C_{pk} = 1.5$.
- Example: USL − LSL = 0.06 mm, σ = 0.01 mm → C_p = 0.06/0.06 = **1.0**.

Acceptance sampling

- Inspect a **random sample** from a lot and accept/reject the whole lot – used when 100% inspection is costly, destructive or causes fatigue errors. Credited to **Dodge and Romig** (Bell Labs).
- **OC (operating characteristic) curve** = probability of acceptance P_a vs lot fraction defective; shows the **discriminating power** of a plan. **Ideal OC curve** = vertical step (accept all lots better than AQL, reject all worse) – possible only with 100% error-free inspection. Larger sample size n → steeper curve.

Nandyal Banking Coaching Center

- **AQL** (Acceptable Quality Level) = worst quality considered acceptable as a process average; associated with **producer's risk α** (probability of **rejecting a good lot**, Type I error), typically **5%**.
- **LTPD** (Lot Tolerance Percent Defective) / RQL = poor quality level; associated with **consumer's risk β** (probability of **accepting a bad lot**, Type II error), typically **10%**.
- **AOQ** = average outgoing quality (after rejected lots are screened); its maximum = **AOQL**.
- Plans: **single, double, multiple, sequential** sampling (sequential needs the smallest average sample). Standards: MIL-STD-105E / ANSI Z1.4 / ISO 2859-1; **IS 2500 (Part 1)** (attributes).

TQM and quality tools

Guru / tool	Associated with
W. Edwards Deming	**PDCA** (Plan–Do–Check–Act; Shewhart cycle), 14 points; Deming Prize (Japan)
Joseph Juran	Quality = "**fitness for use**"; quality trilogy (planning, control, improvement); applied **Pareto** principle ("vital few, trivial many")
Philip Crosby	**Zero defects**; "quality is **free**"; conformance to requirements; cost of quality
Kaoru Ishikawa	**Cause-and-effect (fishbone) diagram** – 6M: Man, Machine, Method, Material, Measurement, Mother-nature (environment); **quality circles**
Genichi Taguchi	**Quality loss function** (quadratic, $L = k(y - m)^2$); robust design, orthogonal arrays, S/N ratio
Shigeo Shingo	**Poka-yoke** (mistake-proofing), **SMED** (single-minute exchange of die)
Pareto chart	Bar chart in descending order + cumulative line; **80/20 rule** (Vilfredo Pareto)
7 QC tools	Check sheet, histogram, Pareto chart, cause-and-effect diagram, scatter diagram, control chart, stratification (or flow chart)

- **Six Sigma** (Motorola, Bill Smith, 1986; popularised by GE – Jack Welch): **3.4 defects per million opportunities (DPMO)** assuming a **1.5σ long-term shift** of the mean. Methodology **DMAIC** (Define, Measure, Analyse, Improve, Control) for existing processes; **DMADV/DFSS** for new designs.
- **Kaizen** = continuous, incremental improvement (Masaaki Imai).
- **5S**: **Seiri** (Sort), **Seiton** (Set in order), **Seiso** (Shine), **Seiketsu** (Standardise), **Shitsuke** (Sustain). Foundation of TPM and lean; directly applicable to warehouse housekeeping.
- **Poka-yoke**: contact, fixed-value and motion-step methods; control-type (stops process) and warning-type.
- **ISO 9001:2015** – QMS requirements; **High-Level Structure (Annex SL) with 10 clauses** (requirements in clauses 4–10); **risk-based thinking**; PDCA; **7 quality management principles** (customer focus, leadership, engagement of people, process approach, improvement, evidence-based decision making, relationship management). **ISO 45001** = OH&S; **ISO 55001** = asset management.

CODE UPDATE

ISO 9001:2015/Amd 1:2024 added **climate-change considerations** to clauses 4.1 (context) and 4.2 (interested parties). ISO 45001:2018 replaced OHSAS 18001.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Control chart for number of defects per casting? → **c chart** (Poisson).
- Percentage of observations within ±3σ? → **99.73%**.
- Producer's risk is the probability of? → **Rejecting a good lot** (Type I, α).
- Six sigma corresponds to how many DPMO? → **3.4**.
- Cp = 1.33 when tolerance = 8σ? → **Yes, 8σ/6σ = 1.33**.
- Seiton in 5S means? → **Set in order**.

12.5 Non-Destructive Testing & Inspection Techniques ★★★

- **NDT/NDE** = inspection without impairing future usefulness of the part; used for welds, castings, forgings, pressure vessels, crane hooks and wire ropes, shafts, storage tanks, ammonia receivers in cold storages.

Method	Principle	Detects	Materials	Li
Visual (VT)	Eye, magnifier, borescope, videoscope	Surface defects, corrosion, misalignment, weld profile	All	Fir **ch** ND on
Liquid / dye penetrant (PT/DPT)	Capillary action of low-viscosity penetrant into open defects, drawn out by developer	**Surface-breaking (open) cracks**, porosity, laps, seams	**Any non-porous** material – ferrous, non-ferrous, plastics, ceramics, glass	De be su for ma ne su
Magnetic particle (MT/MPI)	Defect causes **flux leakage**; iron particles (dry/wet, fluorescent) gather	**Surface and near-surface** (~up to a few mm) cracks	**Ferromagnetic only** (not austenitic SS, Al, Cu)	De de wh **pe to** de aft
Eddy current (ET)	AC coil induces eddy currents; defects change coil impedance	Surface/near-surface cracks, **conductivity, coating thickness**, material sorting, heat-exchanger **tube** inspection	**Electrically conductive** only	Sh eff hig fre sh ref sta ne
Ultrasonic (UT)	High-frequency sound pulses reflected from flaws/back wall	**Internal** flaws – cracks, laminations, lack of fusion; **thickness** (corrosion mapping)	Most metals, plastics, composites	Ne **co** ski op dif co thi co pa su **zo**

Nandyal Banking Coaching Center

Radiography (RT)	X-rays or γ-rays; differential absorption recorded on film/digital detector	**Internal volumetric** defects – **porosity, blowholes, slag inclusions**, shrinkage; permanent record	All	**Radiation hazard** (AERB control); planar cracks seen only if parallel to beam; access to both sides
Acoustic emission (AE)	Listens to stress waves released by **growing** cracks under load	**Active** defects, leaks; monitoring of pressure vessels, tanks during proof test	All	**Passive** – detects only growing defects; Kaiser effect
Infrared thermography (IRT)	IR camera maps surface temperature	Hot spots in electrical panels, bearings, insulation loss, refractory damage, cold-store insulation leaks	All	Surface temperature only; emissivity affects reading
Leak testing (LT)	Bubble (soap/immersion), pressure decay, halogen sniffer, **helium mass spectrometer** (most sensitive)	Leaks in vessels, pipes, refrigeration circuits	Pressure boundaries	—

Penetrant & magnetic particle details

- **PT sequence**: pre-clean → apply penetrant → **dwell time (≈ 5–30 min, typically 10 min)** → remove excess → apply **developer** (white, absorbent) → inspect → post-clean.
- Penetrant types: **visible (red) dye** – inspected in white light; **fluorescent** – under **UV-A (black) light**, more sensitive. Removal: water-washable, post-emulsifiable, solvent-removable.
- **MT**: magnetisation by permanent magnet, **yoke**, prod, coil (longitudinal field), head-shot/central conductor (circular field). **Circular** field detects **longitudinal** cracks; **longitudinal** field detects **transverse** cracks. AC → surface defects (skin effect); half-wave/DC → slightly deeper.

Ultrasonic testing details

- Frequency range for UT: **0.5–25 MHz** typical (most steel work **2–5 MHz**); audible limit is 20 kHz.
- Transducer = **piezoelectric crystal** (quartz, barium titanate, lead zirconate titanate – PZT, lithium sulphate).
- **Couplant** (oil, glycerine, grease, water, gel) excludes air between probe and job – air reflects almost all ultrasound.
- Techniques: **pulse-echo** (single probe, one-side access – most common), **through-transmission** (two probes, both sides), **resonance** (thickness). Immersion testing for automated scanning.
- Displays: **A-scan** (amplitude vs time – flaw depth), **B-scan** (cross-section), **C-scan** (plan view). Advanced: **TOFD** (time of flight diffraction – crack sizing), **PAUT** (phased array).
- Probes: normal (0°, longitudinal waves – plates, laminations, thickness); **angle probes** (45°, 60°, 70°, shear waves – **welds**).
- Velocity in steel: longitudinal ≈ **5900 m/s**, shear ≈ 3200 m/s. Thickness $t = v \cdot t_{transit}/2$.
- UT is best for **planar defects** (cracks, lack of fusion) and thickness measurement of tanks/pipes; RT is best for **volumetric** defects.

Radiography details

Source	Half-life	Typical steel thickness
X-ray tube (up to ~450 kV)	— (switch off = no radiation)	Thin–medium sections; better contrast
Iridium-192	**≈ 74 days**	≈ 10–70 mm (most widely used for site welds)
Cobalt-60	**≈ 5.27 years**	≈ 50–200 mm (thick sections; highest energy)
Selenium-75	≈ 120 days	≈ 5–30 mm

- **Penetrameter / IQI (Image Quality Indicator)**: placed on the source side of the job; checks **radiographic sensitivity/quality** (not defect size). Types: **wire type** (ISO 19232, DIN) and **hole/plaque type** (ASTM). Typical required sensitivity ≈ **2%** of thickness.
- Defects appear **darker** on film (less absorption) – porosity as dark round spots; tungsten inclusions appear **lighter** (denser).
- Indian regulator for radiography sources: **AERB** (Atomic Energy Regulatory Board).

Requirement	Best NDT
Surface crack in aluminium / austenitic SS / non-magnetic casting	**Dye penetrant** (or eddy current)
Surface/near-surface crack in steel crane hook, shaft, gear	**Magnetic particle**
Tube inspection in heat exchangers/condensers; coating thickness; conductivity sorting	**Eddy current**
Internal crack/lamination in plate, forging; wall thickness of tank or pipe from one side	**Ultrasonic**
Porosity, slag in butt welds and castings; permanent record	**Radiography**
Growing crack in pressure vessel during hydro-test	**Acoustic emission**
Loose electrical connection, overheated bearing, insulation failure in cold store	**Thermography**
Refrigerant / ammonia / vacuum leaks	**Leak test** (helium for fine leaks; bubble/sniffer on site)
Broken wires/corrosion inside crane wire ropes	**Magnetic flux leakage (MFL / electromagnetic rope testing)** + visual

MEMORY TRICK

"**Dye for any, Magnet for iron, Eddy for conductors, Ultra for inside cracks, Ray for pores.**" Sources: "**Iridium – ≈ 2½ months (74 d); Cobalt – ≈ 5¼ years**".

Nandyal Banking Coaching Center

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Magnetic particle test cannot be used on? → **Austenitic stainless steel, aluminium, copper (non-ferromagnetic)**.
- Dye penetrant test detects? → **Surface-open defects only**.
- Couplant is used in? → **Ultrasonic testing**.
- Penetrameter (IQI) is used in? → **Radiography** – to check image quality/sensitivity.
- Most suitable NDT for measuring wall thickness of a storage tank from outside? → **Ultrasonic (pulse-echo)**.
- Half-life of Ir-192? → **≈ 74 days**.

12.6 Maintenance Engineering & Tools ★★★

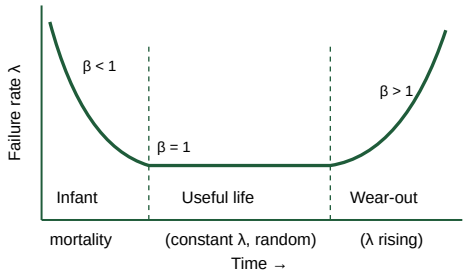
- **Maintenance** = all actions to retain an item in, or restore it to, a state in which it can perform its required function. Objectives: maximum availability, safety, minimum life-cycle cost, longer equipment life.

Type	Basis	Features / when used
Breakdown (corrective / reactive / run-to-failure)	Repair **after failure**	No planning; high downtime and secondary damage; acceptable for cheap, non-critical items with standby (e.g. bulbs, small fans)
Preventive (time-based / planned)	Fixed **calendar or running-hour intervals** – inspection, lubrication, adjustment, replacement	Reduces breakdowns; risk of over-maintenance; e.g. forklift 250-h service, DG set oil change
Predictive / Condition-Based (CBM)	Act when **monitored condition** shows deterioration	Vibration, oil, thermography, ultrasound; uses P–F interval; minimum unnecessary work
Proactive	Eliminate **root causes** of failure	Contamination control, precision alignment and balancing, RCA, design-out
Opportunistic	Done when equipment is down for another reason	Shutdown jobs
Design-out maintenance	Modify design to remove repetitive failure	High-frequency, high-cost failures
RCM (Reliability-Centred Maintenance)	Function- and consequence-based selection of tasks (FMEA-driven)	Originated in civil aviation (Nowlan & Heap, 1978); SAE JA1011
TPM (Total Productive Maintenance)	Involvement of **all employees**, operators do basic maintenance	Japan – **Seiichi Nakajima**, JIPM; first at **Nippon Denso** (1971); goal – zero breakdowns, zero defects, zero accidents

- **TPM 8 pillars** (on a **5S** foundation): (1) **Autonomous maintenance (Jishu Hozen)** – operators clean, inspect, lubricate, tighten; (2) **Planned maintenance**; (3) **Focused improvement (Kobetsu Kaizen)**; (4) **Quality maintenance (Hinshitsu Hozen)**; (5) Early equipment management; (6) Training & education; (7) Safety, health & environment; (8) **Office (administrative) TPM**.
- **Six big losses** (TPM): availability – **breakdowns**, **set-up & adjustment**; performance – **idling & minor stoppages**, **reduced speed**; quality – **process defects/rework**, **start-up (yield) losses**.

KEY FORMULAE

- **OEE = Availability × Performance × Quality**. Availability = operating time / planned production time; Performance = (ideal cycle time × total count)/operating time; Quality = good count / total count.
- World-class OEE ≈ **85%** (≈ 90% A × 95% P × 99.9% Q). Example: 0.90 × 0.80 × 0.95 = **0.684 (68.4%)**.
- Failure rate **λ = number of failures / total operating time**; for constant λ: **MTBF = 1/λ**, **reliability** $R(t) = e^{-\lambda t}$; at t = MTBF, R = e^{-1} = **0.368 (36.8%)**.
- **MTTR** = total repair (down) time / number of repairs; repair rate μ = 1/MTTR.
- Inherent availability **A = MTBF / (MTBF + MTTR)**. Example: MTBF = 450 h, MTTR = 50 h → A = **0.90**.
- Series system: $R_s = R_1 \times R_2 \times \ldots$; $\lambda_s = \Sigma\lambda$. Parallel (redundant): $R_p = 1 - (1 - R_1)(1 - R_2)\ldots$. Two units of R = 0.9: series 0.81, parallel **0.99**.
- **MTTF** is used for non-repairable items; MTBF for repairable items. MTBF = MTTF + MTTR (strict definition); in practice MTBF ≈ MTTF.
- **FMEA: RPN = S × O × D** (Severity × Occurrence × Detection), each rated 1–10 → RPN range **1 to 1000**.



Bathtub curve – failure rate vs time (β = Weibull shape parameter).

Bathtub zone	Failure rate	Causes	Remedy
Infant mortality (early failure / burn-in)	**Decreasing** (Weibull β < 1)	Manufacturing/assembly defects, poor installation, weak parts	**Burn-in / debuggir** run-in, quality cont proper commissior
Useful life (normal operating)	**Constant** (β = 1, exponential)	**Random/chance** failures – overloads, accidents	Redundancy, good operation; prevent replacement does help
Wear-out	**Increasing** (β > 1)	Fatigue, wear, corrosion, ageing	**Preventive replacement/overl** before this zone

Condition monitoring

- **Vibration analysis** – most widely used CM technique for rotating machines. Parameters: **displacement** (μm, low-frequency, sleeve bearings, < ~10 Hz), **velocity** (**mm/s RMS**, 10–1000 Hz – overall machine condition; severity per ISO 10816 / **ISO 20816**), **acceleration** (g, high-frequency > 1 kHz – rolling bearings, gears). Instruments: accelerometer (piezoelectric), FFT analyser, proximity probe.

Nandyal Banking Coaching Center

Fault	Vibration signature
Unbalance	**1× RPM**, predominantly **radial**, steady phase
Misalignment	**2× RPM** (also 1×), **high axial** vibration; 180° phase across coupling
Mechanical looseness	Many harmonics (1×, 2×, 3× …), sub-harmonics (½×)
Rolling bearing defects	**Non-synchronous high frequencies** – BPFO (outer race), BPFI (inner race), BSF (ball spin), FTF (cage)
Oil whirl (sleeve bearing)	**0.42–0.48× RPM**
Gear problems	**Gear mesh frequency** = no. of teeth × shaft speed, with sidebands

- **Shock Pulse Method (SPM)** – for **rolling-element bearing** condition (mechanical shock waves from rolling contact; dBm/dBc values); also used to judge lubrication condition.
- **Oil / wear-debris analysis**: **spectrometric oil analysis (SOAP)** – wear metals in ppm (Fe, Cu, Cr, Al …); **ferrography** – size, shape and type of ferrous wear particles (wear mode); particle count (cleanliness code **ISO 4406**); viscosity, TAN/TBN, water content (Karl Fischer), flash point; **magnetic chip detectors**.
- **Thermography** – electrical panels, bus-bars, motors, bearings, steam traps, refractory, cold-room insulation.

Lubrication

Regime	Feature	Example
Hydrodynamic (thick film)	Full fluid film generated by relative motion (wedge action); no metal contact	Journal bearings at running speed
Hydrostatic	Film by **external pump pressure**; works at zero speed	Heavy slow machines, telescopes, machine-tool slides
Elastohydrodynamic (EHL)	Very thin film with elastic deformation and pressure-viscosity rise	Rolling bearings, gear teeth, cams
Boundary (thin film)	Asperity contact; protected by adsorbed additive layers	Start–stop, slow speed, heavy load

- **Viscosity** – most important property of a lubricating oil. **ISO VG** number = kinematic viscosity in **cSt (mm²/s) at 40 °C** (e.g. ISO VG 68 hydraulic oil). **Viscosity Index (VI)** – change of viscosity with temperature; **higher VI = less change** (better). SAE grades for engine/gear oils ("W" = winter).
- **Grease** = base oil (≈ 70–95%) + **thickener** (soap – lithium, calcium, sodium, aluminium, complex soaps; non-soap – polyurea, bentonite clay) + additives. Used where oil would leak, for sealed/intermittent and difficult-to-access points.
- **Lithium (and lithium-complex) grease** – **multipurpose**, good water resistance and temperature range (most common, NLGI 2). **Calcium** grease – excellent water resistance, low temperature limit (~60–80 °C). **Sodium** grease – higher temperature but **poor water resistance** (washes out).
- **Dropping point** = temperature at which grease passes from semi-solid to liquid – indicates upper temperature limit (operate well below it).

NLGI grade	Worked penetration (0.1 mm, 25 °C, ASTM D217)	Consistency / use
000	445–475	Fluid – enclosed gearboxes, centralised systems
00	400–430	Semi-fluid
0	355–385	Very soft – centralised lubrication
1	310–340	Soft
2	**265–295**	**"Normal" – most common for rolling bearings**
3	220–250	Medium-firm – high-speed / vertical-shaft bearings
4	175–205	Firm
5	130–160	Very firm
6	85–115	Block grease – open plain bearings

Higher NLGI number → **lower penetration → stiffer grease**. 9 grades (000 to 6).

Maintenance tools & alignment

- **Torque wrench** (click, beam/deflecting, dial, digital) – tightens bolts to specified torque; bolt preload F ≈ T/(K d) with nut factor K ≈ 0.2 (dry steel). Re-calibrate periodically (ISO 6789); store click-type at its **lowest setting**.
- **Feeler gauge** – set of thin steel leaves (≈ 0.03–1.0 mm) for gaps: valve tappet clearance, bearing clearance, coupling gap, angular misalignment, soft-foot check.
- **Bearing pullers** (two/three-jaw, hydraulic) for dismounting; **induction bearing heater** for mounting – heat to about **80–90 °C above ambient, max ≈ 110–120 °C** (never use open flame); hydraulic nut and oil-injection method for large tapered-bore bearings. Never hammer on the rolling elements or transmit mounting force through them.
- **Shaft alignment** (motor–pump, motor–gearbox): correct both **parallel (offset)** and **angular** misalignment in vertical and horizontal planes. Methods: **straight-edge & feeler gauge** (rough), **rim-and-face** dial indicator, **reverse dial indicator** (rim readings on both hubs – more accurate, not affected by axial float), **laser alignment** (fastest and most accurate). Correct **soft foot** (< ~0.05 mm) first; check dial-bracket **sag**; move the **movable machine (usually the motor)** with shims.

FMEA, RCA and spare-parts control

- **FMEA** (Failure Mode and Effects Analysis) – **bottom-up**, inductive; lists failure modes, effects, causes and controls. **S** = severity of effect, **O** = likelihood of occurrence, **D** = detectability (**10 = hardest to detect**). Highest RPN (and any high severity) gets action first. **FMECA** adds criticality. AIAG-VDA FMEA (2019) uses **Action Priority (AP)** instead of RPN.
- **FTA (Fault Tree Analysis)** – **top-down**, deductive; starts with an undesired top event and uses AND/OR gates (Boolean logic) – developed at Bell Labs (1962).
- **RCA (Root Cause Analysis)** tools: **5 Whys** (Toyota, Sakichi Toyoda), fishbone diagram, fault tree, Pareto, why-why analysis, change analysis.

Inventory analysis	Basis	Classes

Nandyal Banking Coaching Center

ABC	**Annual consumption value**	A ≈ 10% items / 70% value (tight control); B ≈ 20% / 20%; C ≈ 70% / 10%
VED	**Criticality** of spare (effect of stock-out) – most used for **maintenance spares**	Vital, Essential, Desirable
FSN	Rate of consumption / issue	Fast, Slow, Non-moving
SDE	Procurement difficulty / lead time	Scarce, Difficult, Easy

Warehouse material handling equipment

Equipment	Key facts
Counterbalance forklift (diesel, LPG, battery-electric)	Rated capacity stated at a **load centre** (commonly **500 mm**; 600 mm for larger trucks); capacity **falls** as load centre or lift height increases. **Stability triangle** – three-point suspension; combined CG must stay within it. Mast tilt forward/back; travel with forks low (≈ 150 mm) and mast tilted back. Battery trucks for indoor/cold-store use (no exhaust).
Reach truck	Mast/forks reach forward; narrow-aisle, high racking, indoor
Stacker (manual/electric)	Walk-behind, light-duty stacking with outriggers/straddle legs
Conveyors	Belt (bags, cartons), roller (gravity/powered), chain, slat, **screw** (bulk grain – fully enclosed), **bucket elevator** (vertical lift of grain), pneumatic, truck-loader (telescopic) conveyors
EOT crane (electric overhead travelling)	Three motions: **hoist, cross-travel (trolley), long-travel (bridge)**. IS 3177 (code of practice for EOT cranes) & IS 807 (design of steel cranes); duty classes (M1–M8 per ISO 4301). Limit switches (hoist upper/lower, travel), overload limiter, brakes, anti-collision.

- **Safe Working Load (SWL)/Working Load Limit (WLL)** = maximum load permitted; must be **clearly marked** on every lifting machine and tackle. Breaking load = SWL × factor of safety (wire ropes typically ≈ **5–6**, chains ≈ 4–5, depending on code and duty).
- **Load testing** of cranes (typical IS/ISO practice): **static overload test ≈ 125% of SWL**, **dynamic test ≈ 110% of SWL** (all motions); done after installation, major repair and periodically, by a **competent person**, and certified.

Safety & statutory requirements

- **LOTO (Lockout/Tagout)**: isolate and lock all energy sources (electrical, hydraulic, pneumatic, mechanical, thermal, chemical, gravity/stored energy) before maintenance; each worker applies a **personal lock**; tag identifies who and why. Key step after isolation: **verify zero energy (try-out)**. Origin: OSHA 29 CFR 1910.147.
- **Permit-to-work (PTW)** for hot work, confined space (e.g. grain silos, pits), work at height, electrical work. Confined-space entry needs O_2 **19.5–23.5%** and gas-free certificate (e.g. after phosphine fumigation).
- Hierarchy of controls: **elimination → substitution → engineering controls → administrative controls → PPE** (PPE is the **last** resort).

Factories Act, 1948 – provision	Key value
Factory [Sec 2(m)]	**10 or more** workers with power; **20 or more** without power
Fencing of machinery	**Sec 21**
Hoists and lifts	**Sec 28** – examined by competent person at least once in every **6 months**
Lifting machines, chains, ropes & lifting tackles	**Sec 29** – SWL marked; examined by competent person at least once in every **12 months**; not loaded beyond SWL
Pressure plant	**Sec 31**
Safety officers	**Sec 40-B** – factories with **1000 or more** workers (or hazardous processes as notified)
First-aid box	Sec 45 – one per **150** workers; ambulance room if > **500** workers
Canteen / shelters & lunch room / creche / welfare officer	Sec 46: > **250** workers; Sec 47: > 150; Sec 48: **30** women; Sec 49: **500** or more workers
Weekly hours / daily hours / rest interval / spread-over	Sec 51: **48 h**; Sec 54: **9 h**; Sec 55: ½ h after **5 h**; Sec 56: **10½ h**
Overtime wages	Sec 59: **twice** the ordinary rate
Employment of children	Sec 67: no child who has not completed **14 years**

CODE UPDATE

The **Occupational Safety, Health and Working Conditions Code, 2020 (OSH Code)** subsumes **13 central labour laws** including the Factories Act 1948, Mines Act, Dock Workers Act and Contract Labour Act. Under it a "factory" = **20 or more workers with power, 40 or more without power**; it provides for a single registration, a National OSH Board, and safety committees. The four Labour Codes were brought into force by the Central Government with effect from **21 November 2025**; the final central rules (incl. OSH & Working Conditions (Central) Rules, 2026) were notified on **8 May 2026**, while state rules are being notified in phases – check the latest notification for transitional provisions.

MEMORY TRICK

"**Bathtub = Baby, Adult, Old**": baby failures fall (burn-in), adult failures random (constant λ), old failures rise (replace). Vibration: "**Unbalance = 1×, Misalignment = 2× + axial, Bearing = non-synchronous high**". Lifting: "**Lifts 6, Lifting tackle 12**" (months, Sec 28/29).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- MTBF = 900 h, MTTR = 100 h – availability? → **0.9 (90%)**.
- In the useful-life period of the bathtub curve, failure rate is? → **Constant** (random failures, exponential).
- OEE is the product of? → **Availability × Performance × Quality**.
- RPN in FMEA = ? → **Severity × Occurrence × Detection** (max 1000).
- Most common NLGI grade for rolling-element bearings? → **NLGI 2**.
- Shock pulse method is used to monitor? → **Rolling-element bearings**.

Nandyal Banking Coaching Center

13. Production Planning, Industrial Engineering & Operations Research

> **NOTE**
>
> Industrial engineering and OR usually give **5–8 MCQs**, and most of them are quick numericals: **EOQ**, **line balancing**, **exponential smoothing**, **M/M/1 queue**, **PERT t_e/σ**, **Johnson's rule**, **standard time** and **break-even point**. Theory questions come from layouts, selective inventory control (ABC/VED/FSN), lean terms (5S, kanban, poka-yoke), therbligs and LP/transportation rules (m + n − 1). A warehousing body like CWC especially likes **inventory control, storage and material handling** questions.

13.1 Production Planning & Control ★★★

- **Production Planning & Control (PPC)** = deciding in advance **what, how, where, when and how much** to produce, and then making sure the plan is carried out. Planning is done **before** production; control is exercised **during** production.
- **Routing** = deciding the **path (sequence of operations)** and the machines/work centres for each part. It answers **"what, how and where"**. Document: **route sheet / operation sheet**. Routing is the **first step** of PPC.
- **Loading** = assigning jobs to machines/work centres so that capacity is not exceeded (**machine loading chart**).
- **Scheduling** = fixing the **time** (start and finish dates) for each operation. It answers **"when"**. Types: master schedule, operation schedule; **forward** (start from today) and **backward** (start from due date) scheduling.
- **Dispatching** = the **action/execution phase**: **release of orders** and instructions (job cards, material requisitions, move tickets, inspection orders) to the shop floor, as per the schedule. It is the link between planning and doing.
- **Follow-up / Expediting** = **progress chasing**: checking that work moves as scheduled, finding delays and removing bottlenecks. The person is the **expeditor / progress chaser**.
- Order of PPC functions: **Routing → Loading → Scheduling → Dispatching → Follow-up (Expediting) → Inspection → Corrective action**. Planning phase = routing, loading, scheduling; control phase = dispatching, follow-up, inspection.

Production type	Volume / variety	Machines & layout	Examples
Job (job-shop / unit)	**Low volume, high variety**; often one-off	**General-purpose** machines, **process layout**, highly skilled labour	Ship building, tool room, prototypes
Batch	Medium volume, medium variety; lots made at intervals	General + some special machines, process/cellular layout	Pumps, machine tools, pharma tablets
Mass / Flow	**High volume, low variety** of discrete items	**Special-purpose** machines, **product (line) layout**, semi-skilled labour	Cars, scooters, bulbs
Continuous / Process	Very high volume, non-discrete, 24 × 7	Highly automated, product layout, pipelines	Refinery, cement, fertiliser, sugar
Project	Single large unique item	**Fixed position layout**	Dams, ships, aircraft, buildings

- **Intermittent** production = job + batch (machines change over between products). **Continuous** = mass + process.
- As we move from job → continuous: unit cost **decreases**, flexibility **decreases**, capital investment **increases**, labour skill required **decreases**, work-in-process (WIP) **decreases**.

Layout	Basis / feature	Suited to	Merits / Demerits
Process (functional)	Similar machines grouped together (all lathes in one bay)	**Job & batch**, high variety	Flexible, breakdown of one machine does not stop work; but **high material handling, high WIP**, complex control
Product (line)	Machines in the **sequence of operations**	**Mass/continuous**, standard product	**Lowest handling and WIP**, simple control; but inflexible, **one breakdown stops the line**, high investment
Fixed position	**Product stays still**; men, machines, materials come to it	Very large/heavy items: **ships, aircraft, locomotives, dams**	Minimum movement of the product; high movement of men & equipment
Cellular (Group Technology)	Machines grouped into **cells** that each make a **part family**	Batch production of similar parts	Combines flexibility of process layout with efficiency of product layout; lower set-up time and WIP
Combination (hybrid)	Mix of the above	Most real factories	—

- **Group Technology (GT)**: parts with similar **shape or process** are grouped into families. Coding systems: **Opitz** (German; 5-digit form code + 4-digit supplementary code = 9 digits, plus optional 4-digit secondary code), **MICLASS**, **DCLASS**. Cells are formed using **Production Flow Analysis (PFA)** (Burbidge).
- **Systematic Layout Planning (SLP)** – **Richard Muther**. Its **activity relationship chart** uses closeness codes **A** = absolutely necessary, **E** = especially important, **I** = important, **O** = ordinary, **U** = unimportant, **X** = undesirable.
- Computerised layout: **CRAFT** (Computerised Relative Allocation of Facilities Technique) is an **improvement** type (needs an initial layout, swaps departments to cut cost). **ALDEP, CORELAP, PLANET** are **construction** types (build a layout from scratch).
- Travel chart (from-to chart), string diagram and flow diagram are used to compare layouts by material movement.

Nandyal Banking Coaching Center

Line balancing

- **Line balancing** = assigning tasks to workstations on an assembly line so that each station has nearly equal work, without breaking precedence. Aim: **minimum stations / minimum idle time** for a given cycle time.
- **Cycle time (C)** = time between two successive units leaving the line = **maximum time allowed at each station**. C must be ≥ the **longest single task** time.
- Heuristics: **Largest Candidate Rule**, **Kilbridge & Wester** (column method), **Ranked Positional Weight** (Helgeson & Birnie). COMSOAL is a computer method.

KEY FORMULAE – LINE BALANCING

- Cycle time C = **available production time per period ÷ required output per period**
- Minimum (theoretical) number of stations N_{min} = **Σt / C**, rounded **up** to the next whole number
- Line (balance) efficiency η = **Σt / (N × C)** × 100 (N = actual stations)
- Balance delay = **1 − η** = (N·C − Σt)/(N·C); total idle time = N·C − Σt
- Smoothness index SI = $\sqrt{[\Sigma(S_{max} - S_i)^2]}$ (0 = perfect balance)

- **Example**: 240 units needed in a 480-min shift → C = 480/240 = **2 min**. Sum of task times Σt = 9.5 min → N_{min} = 9.5/2 = 4.75 → **5 stations**. If the line actually needs 6 stations: η = 9.5/(6 × 2) = **79.2 %**, balance delay = **20.8 %**.

Aggregate & capacity planning

- **Aggregate planning** = **medium-range (3–18 months)** plan of total output, workforce and inventory for product **families** (not individual items). It is **disaggregated** into the **Master Production Schedule (MPS)**.
- **Chase strategy**: output **follows demand** each period by hiring/firing or overtime; **low inventory**, high hiring/firing cost.
- **Level strategy**: **constant output and workforce**; inventory (or backlog) absorbs demand changes; **high inventory cost**, stable labour.
- **Mixed strategy**: combination. Capacity options: hire/fire, overtime/idle time, part-time, **subcontracting**. Demand options: pricing, promotion, **back-orders**, counter-seasonal products.
- Methods: trial and error (graphical), **transportation/LP model** (Bowman), **Linear Decision Rule (HMMS – Holt, Modigliani, Muth, Simon)**, management coefficients (Bowman).
- **Capacity**: **design capacity** = maximum possible output; **effective capacity** = design capacity less planned losses (maintenance, set-ups); **actual output** ≤ effective capacity.
- **Utilisation = actual output / design capacity**; **Efficiency = actual output / effective capacity**. E.g. design 100, effective 80, actual 72 → utilisation **72 %**, efficiency **90 %**.
- Capacity strategies: **lead** (add capacity ahead of demand), **lag** (add after demand rises), **match/tracking** (small steps). Short-term checks: **RCCP** (rough-cut, against MPS) and **CRP** (capacity requirements planning, against MRP).

Break-even analysis & plant location

KEY FORMULAE – BREAK-EVEN

- Total cost TC = F + V·Q; revenue R = P·Q (F = fixed cost, V = variable cost per unit, P = selling price per unit)
- Break-even quantity Q_{BEP} **= F / (P − V)**; (P − V) = **contribution per unit**
- P/V ratio = (P − V)/P; BEP in ₹ = **F / (P/V ratio)**
- Profit = (P − V)Q − F; Margin of safety (MOS) = actual sales − BEP sales = **profit / (P/V ratio)** (in ₹)

- **Example**: F = ₹60,000, P = ₹50, V = ₹30 → contribution ₹20/unit → Q_{BEP} **= 3,000 units** (₹1,50,000); P/V ratio = 40 %. At 4,000 units, profit = 4,000 × 20 − 60,000 = **₹20,000**, MOS = **1,000 units**.
- **Angle of incidence** = angle between the sales line and total-cost line at BEP; a **large angle** means profit rises quickly after BEP. Higher fixed cost → higher BEP.
- **Plant location factors**: nearness to **raw material** (weight-losing industries: cement, steel, sugar) or **market** (weight-gaining/perishable: bottling, bakery), labour, power, water, transport, land cost, climate, government incentives, taxes, community attitude, waste disposal, defence aspects.
- Location methods: **factor rating** (weighted score), **locational break-even** (cost–volume), **centre of gravity** method $C_x = \Sigma d_x V / \Sigma V$, $C_y = \Sigma d_y V / \Sigma V$, **transportation model**, **load–distance** method. **Alfred Weber** gave the classical industrial location theory.

MEMORY TRICK

PPC order = "**R**eal **L**adies **S**ee **D**ance **F**reely" → **R**outing (where/how), **L**oading, **S**cheduling (when), **D**ispatching (do), **F**ollow-up (check). Layout ↔ production: **J**ob → **P**rocess, **M**ass → **P**roduct, **Big** → **Fixed**.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Which PPC function decides the sequence of operations? → **Routing**.
- Release of work orders to the shop floor is called? → **Dispatching**.
- Layout used for ship building? → **Fixed position layout**.
- Task times total 30 min, cycle time 7 min – minimum stations? → **5** (30/7 = 4.29 → 5).
- F = ₹40,000, P = ₹25, V = ₹15 – BEP? → **4,000 units**.
- CRAFT is a layout algorithm of which type? → **Improvement** type.

13.2 Forecasting ★★★

- **Forecasting** = estimating future demand; it is the **starting point** of all production planning. Short-range (< 3 months) forecasts are the **most accurate**; accuracy falls as the horizon grows. **Aggregate** (family) forecasts are more accurate than item forecasts.
- **Qualitative (judgemental)** methods – used when **no past data** exists (new product, long range): **Delphi**, **market survey**, **sales-force composite**, **jury of executive opinion**, historical analogy.
- **Delphi method** (developed at **RAND Corporation**) = a panel of experts answers questionnaires **anonymously** in several rounds; a co-ordinator feeds back the summary until **consensus**. Avoids

Nandyal Banking Coaching Center

the "bandwagon" effect of face-to-face meetings. Best for **long-range / technological** forecasts.

- **Quantitative** methods: **time series** (moving average, exponential smoothing, trend projection) and **causal** (regression, econometric).
- Components of a time series: **Trend (T)**, **Seasonal (S)** (repeats within a year), **Cyclical (C)** (business cycles, > 1 year), **Random/irregular (R)**. Multiplicative model: Y = T × S × C × R.
- **Naive forecast**: next forecast = last actual demand.
- **Simple moving average (SMA)**: average of last n periods. **Larger n → smoother** but **slower** to respond. All n values get equal weight. A moving average always **lags** a trend.
- **Weighted moving average (WMA)**: recent data get larger weights (Σweights = 1) → responds faster than SMA.
- **Exponential smoothing**: a weighted average where weights fall **exponentially** with age. Needs only **three data**: last forecast, last actual demand and α. **Smoothing constant α lies between 0 and 1** (usual 0.1–0.3).
- **Higher α** → more weight on recent demand → **more responsive** (less smoothing). **Lower α** → more smoothing, more stable. α = 1 gives the naive forecast.
- An n-period SMA and exponential smoothing have about the same average age of data when **α = 2/(n + 1)**.
- **Trend-adjusted (double) exponential smoothing – Holt's method**: smooths level with α and trend with β; $FIT_t = F_t + T_t$. Seasonal + trend → **Winters' (triple)** method.
- **Linear regression (least squares)**: fits Y = a + bX that minimises the **sum of squared deviations**. Correlation coefficient **r** lies between −1 and +1; **r^2** = coefficient of determination.

KEY FORMULAE – FORECASTING

- SMA: $F_{t+1} = (D_t + D_{t-1} + \dots + D_{t-n+1})/n$ WMA: $F_{t+1} = \Sigma w_i D_i$ ($\Sigma w_i = 1$)
- Exponential smoothing: $\mathbf{F_{t+1} = F_t + \alpha(D_t - F_t) = \alpha D_t + (1 - \alpha)F_t}$
- Least squares: $b = [n\Sigma XY - \Sigma X \Sigma Y] / [n\Sigma X^2 - (\Sigma X)^2]$; $a = \bar{Y} - b\bar{X}$. If X is coded so that ΣX = 0: **$b = \Sigma XY/\Sigma X^2$, $a = \Sigma Y/n$**
- Forecast error $e_t = D_t - F_t$; **MAD = Σ|e|/n**; **$MSE = \Sigma e^2/n$**; **MAPE = (100/n) Σ|e/D|** %
- Bias (mean forecast error) = Σe/n; RSFE = running sum of forecast errors = Σe
- **Tracking signal = RSFE / MAD**; acceptable limits usually **±4** (some texts ±3 to ±8)
- For normally distributed errors **σ ≈ 1.25 MAD** (MAD ≈ 0.8σ)

Error measure	What it shows	Note
MAD	Average size of error	Easy; same units as demand
MSE	Average of squared error	**Penalises large errors** more
MAPE	Error as % of demand	Unit-free; used to compare items
Bias / RSFE	Direction of error	+ve → forecast too **low** (under-forecast); −ve → over-forecast
Tracking signal	Whether forecast is **out of control**	Beyond ±4 MAD → revise model

- **Example (exponential smoothing)**: F_t = 100, D_t = 120, α = 0.2 → F_{t+1} = 100 + 0.2(20) = **104**.
- **Example (least squares)**: demand for 5 years = 10, 12, 15, 16, 22. Code X = −2, −1, 0, 1, 2 → ΣY = 75, ΣXY = 28, ΣX^2 = 10 → a = **15**, b = **2.8**. Forecast for year 6 (X = 3) = 15 + 2.8 × 3 = **23.4**.
- **Seasonal index** = average demand of that season ÷ overall average per season. Quarterly indices add up to **4** (monthly to **12**); an index > 1 means above-average season. **Deseasonalised demand** = actual ÷ seasonal index; seasonal forecast = trend forecast × index.

MEMORY TRICK

"**α = Alert**": high α = alert, jumpy forecast; low α = calm, smooth forecast. Tracking signal "**RSFE over MAD, stay within FOUR**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Forecasting method using anonymous expert rounds? → **Delphi**.
- F_t = 500, D_t = 600, α = 0.3. Next forecast? → **530**.
- To make exponential smoothing more responsive, α should be? → **Increased**.
- Tracking signal is the ratio of? → **RSFE to MAD**.
- Error measure that penalises large errors most? → **MSE**.
- Sum of quarterly seasonal indices? → **4**.

13.3 Inventory Control ★★★

- **Inventory** = idle resource with economic value: raw material, WIP, finished goods, MRO (maintenance, repair & operating) spares. Reasons to hold: decoupling, safety against uncertainty, economy of lot size, anticipation (seasonal), pipeline (in transit).
- **Independent demand** (finished goods, from market) → EOQ/ROP models. **Dependent demand** (components, derived from the end item) → **MRP**.

Inventory cost	Includes	Varies with Q
Ordering / set-up cost (S or C_o)	Paperwork, follow-up, receiving, inspection, machine set-up	Per order; annual = (D/Q)S → **falls** as Q rises
Carrying / holding cost (H or C_c)	Interest on capital (largest part), storage, insurance, obsolescence, deterioration, pilferage; typically **15–30 % of item value per year**	Annual = (Q/2)H → **rises** with Q
Shortage / stock-out cost	Lost sales, goodwill, idle men & machines, back-order cost	Falls with more stock
Purchase (unit) cost	Price × D	Constant (unless discount)

EOQ (Wilson / Harris) model

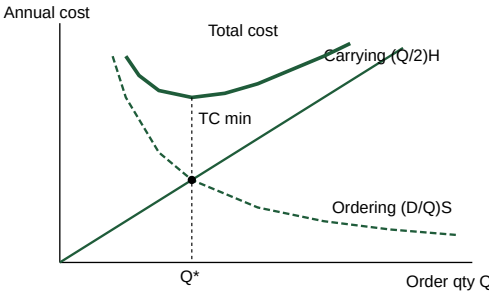
- Assumptions: demand **known and constant**, lead time constant, **instantaneous replenishment**, no shortages, no quantity discount, constant costs. EOQ was given by **F.W. Harris (1913)** and popularised by **R.H. Wilson**.
- At EOQ, **annual ordering cost = annual carrying cost**, and total variable cost is **minimum**. The total-cost curve is **flat near Q***, so small errors in Q change cost very little (EOQ is **insensitive**).
- Average inventory = **Q/2** (no safety stock) or **Q/2 + SS**. Maximum inventory = Q (+ SS).
- EOQ ∝ √D: if demand becomes **4 times**, EOQ **doubles**; if D doubles, EOQ rises by √2 (≈ 41 %). If H = iC (i = carrying rate, C =

Nandyal Banking Coaching Center

unit price), higher price → smaller EOQ.

KEY FORMULAE – EOQ FAMILY

- Annual total variable cost TC(Q) = **(D/Q)S + (Q/2)H**
- **EOQ Q* = √(2DS/H)**; minimum TC* = **√(2DSH)** = H·Q*; ordering cost = carrying cost = √(DSH/2) each
- Number of orders per year N* = D/Q* = √(DH/2S); cycle time T* = Q*/D
- **EPQ / EBQ (finite production rate p, usage d, p > d)**: Q* = **√[2DS / (H(1 – d/p))]**; I_{max} = Q*(1 – d/p); TC* = √[2DSH(1 – d/p)]; production run time = Q*/p
- **EOQ with planned shortages (back-orders, shortage cost B per unit per year)**: Q* = √(2DS/H) × **√[(H + B)/B]**; maximum inventory I_{max} = Q*·B/(H + B). As B → ∞, model reduces to simple EOQ.
- Reorder point **ROP = d × L + SS** (d = demand per day, L = lead time in days)
- Safety stock **SS = z σ_d √L** (σ_d = std. deviation of daily demand, L constant); = z σ_{LT} where σ_{LT} = std. deviation of lead-time demand

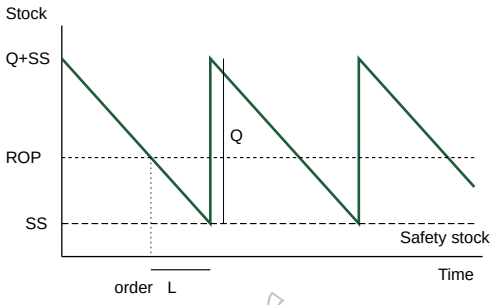


EOQ cost curves: total cost is minimum at Q, where the carrying-cost line crosses the ordering-cost curve.*

- **Example**: D = 1,200 units/yr, S = ₹100/order, H = ₹6/unit/yr → Q* = √(2 × 1200 × 100/6) = √40,000 = **200 units**; orders per year = **6**; ordering cost = carrying cost = **₹600** each; TC* = √(2 × 1200 × 100 × 6) = **₹1,200**.
- **EPQ** is **larger** than EOQ for the same data (factor 1/√(1 – d/p)), because stock builds up slowly while being used. If p → ∞, EPQ → EOQ.
- **Quantity discounts (price breaks)**: (1) find EOQ at each price; (2) if EOQ is below the price-break quantity, raise it to the break quantity; (3) compare **total cost including purchase cost** (DC + (D/Q)S + (Q/2)iC) and pick the lowest. Here total cost includes purchase cost because price changes with Q.

Service level	z value	Service level	z value
84.1 %	1.0	95 %	**1.645**
90 %	1.28	97.7 %	2.0
50 % (no SS)	0	99 %	**2.33**

- **Service level** = probability of **no stock-out** during lead time (cycle service level). Risk of stock-out = 1 – service level. Higher service level → larger z → larger safety stock (rises steeply above ~95 %).
- **Example**: d = 40 units/day, L = 9 days, σ_d = 5, 95 % service → SS = 1.645 × 5 × √9 ≈ **25 units**; ROP = 40 × 9 + 25 = **385 units**.



Saw-tooth inventory model: an order of Q is placed when stock falls to the ROP and arrives after lead time L, just as stock reaches the safety-stock level.

Feature	Q system (fixed order quantity)	P system (fixed period / periodic review)
Order quantity	**Fixed (EOQ)**	**Variable** = target level – stock on hand
Order interval	Variable	**Fixed** (every P days)
Review	**Continuous / perpetual**	Periodic
Trigger	Stock reaches **ROP**	Calendar date
Safety stock	**Smaller** (covers L only)	**Larger** (covers P + L)
Suited to	Costly A items	Many items from one supplier, C items

- **Two-bin system** = a simple **Q (fixed-order quantity)** system: use from bin 1; when it is empty, place an order and use bin 2 (which holds the ROP quantity). Good for cheap **C-class** items (nuts, bolts).
- **(s, S) or min–max system**: when stock falls to s (min), order up to S (max).
- **Inventory turnover ratio** = cost of goods sold ÷ average inventory; higher is better. Days of stock = 365/turnover.

Selective inventory control

Analysis	Basis of classification	Classes
ABC	**Annual consumption value** (usage × unit cost) – Pareto	A ≈ **10 % items, 70 % value**; B ≈ 20 % items, 20 % value; C ≈ **70 % items, 10 % value**
VED	**Criticality** to production (spares)	Vital, Essential, Desirable
FSN	**Movement / issue rate** (consumption rate)	Fast-moving, Slow-moving, Non-moving
SDE	**Availability / procurement difficulty**	Scarce, Difficult, Easy
HML	**Unit price** of the item	High, Medium, Low
XYZ	**Value of stock held** (closing inventory value)	X – high, Y – medium, Z – low stock value
GOLF	**Source** of supply	Government, Ordinary, Local, Foreign
SOS	Seasonality	Seasonal, Off-seasonal

- ABC = "**control by importance and exception**" (also called **Always Better Control**). A items: tight control, frequent review, small lots, accurate records; C items: loose control, bulk ordering, two-bin.
- **ABC–VED matrix** is used for spare parts: an item can be cheap (C) but **vital** (V), so it must still be stocked.

Nandyal Banking Coaching Center

- Note the difference: **ABC uses annual usage value**; **HML uses unit price only**; **XYZ uses value of stock lying in store**.
- **JIT (Just-in-Time)** = get the right item in the right quantity at the right time; aims at **zero inventory**, small lots, reliable nearby suppliers, pull system (details in 13.4).
- Stores records: **bin card** (kept at the bin, quantity only) and **stores ledger** (quantity and value). Issue pricing: FIFO, LIFO, weighted average.

MEMORY TRICK

"**A**BC = **A**nnual value; **V**ED = **V**itality; **F**SN = **F**low (movement); **S**DE = **S**upply; **H**ML = **H**ow much per piece; **X**YZ = e**X**isting stock value". At EOQ: "**ordering = carrying**, cost = √(2DSH)".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- D = 2,400/yr, S = ₹50, H = ₹6 – EOQ? → **200 units**.
- If annual demand becomes four times, EOQ becomes? → **Two times**.
- VED analysis is based on? → **Criticality** of items.
- Classification based on unit price of item? → **HML**.
- Two-bin system belongs to which inventory system? → **Fixed order quantity (Q) system**.
- Safety stock needed is larger in? → **Periodic review (P) system**.

13.4 MRP, ERP & Lean Manufacturing ★★★

- **MRP (Material Requirements Planning)** = computer-based method to plan **dependent-demand** items (components, sub-assemblies, raw material): **what** to order, **how much** and **when**. Developed by **Joseph Orlicky** (IBM, 1960s).
- **Three inputs of MRP**: (1) **Master Production Schedule (MPS)** – what end items, how many, when; (2) **Bill of Materials (BOM)** / product structure file – components and quantities per parent, in levels (end item = level 0); (3) **Inventory status (records) file** – on-hand, on-order, lead times, lot sizes.
- **Outputs**: **planned order releases** (primary), order rescheduling notices, cancellations, exception reports, performance and planning reports.
- MRP logic for each item, level by level: **Gross requirement → minus on-hand and scheduled receipts → Net requirement → Planned order receipt → Planned order release** (offset by **lead time** = time phasing). This is called **explosion** of the BOM.
- **Low-level coding**: an item used at several levels is planned at its **lowest** level so that all gross requirements are added first.
- Planning horizon of MRP ≥ the **cumulative (stacked) lead time** of the product. Time unit = **time bucket** (usually a week).
- **Pegging** = tracing a component requirement back to its parent. **Time fence** = period in which MPS should not be changed (frozen zone).

Lot-sizing rule	Meaning	Effect
Lot-for-Lot (L4L)	Order exactly the net requirement of each period	**Minimum carrying cost, maximum set-ups**; used in JIT
EOQ / fixed order quantity	Order EOQ each time	Can leave remnants (lumpy demand)
POQ (Periodic Order Quantity)	Order to cover a fixed number of periods = EOQ ÷ average period demand (rounded)	Fewer remnants than EOQ
Part-Period Balancing (PPB)	Combine periods until carrying cost ≈ set-up cost	Heuristic
Wagner–Whitin	Dynamic programming	**Optimal** but complex
Silver–Meal, Least Unit Cost	Heuristics using cost per period / per unit	Near-optimal

System	Full form	Scope
MRP	Material Requirements Planning	Materials only (open loop)
Closed-loop MRP	MRP + capacity feedback (CRP)	Materials + capacity
MRP-II	**Manufacturing Resource Planning** (Oliver Wight)	Adds **finance, marketing, capacity, labour**; "what-if" simulation
ERP	**Enterprise Resource Planning**	**Whole enterprise** on one integrated database: production, finance, HR, sales, purchase, supply chain (e.g. **SAP, Oracle**)
DRP	**Distribution Requirements Planning**	Applies MRP logic to **warehouses/distribution network** – finished-goods replenishment

Lean manufacturing / Toyota Production System

- **Toyota Production System (TPS)** – developed by **Taiichi Ohno** (with Eiji Toyoda and **Shigeo Shingo**). Two pillars: **JIT** and **Jidoka**. The term "**lean**" (coined by John Krafcik, 1988, in MIT's vehicle programme) was popularised by the MIT study "The Machine that Changed the World" (Womack, Jones & Roos, 1990).
- Three enemies: **Muda** (waste), **Mura** (unevenness), **Muri** (overburden).
- **JIT**: **pull** system (production triggered by downstream demand), small lots, quick set-ups, few reliable suppliers, zero defects, **zero inventory** as ideal. Inventory is seen as hiding problems ("lowering the water level exposes the rocks").
- **Kanban** = Japanese for "**card / signboard**"; the **information tool of the pull system**. **Withdrawal (conveyance/move) kanban** authorises moving a container; **production kanban** authorises making one. Rule: no part is made or moved without a kanban; defective parts are never passed on.
- **SMED (Single-Minute Exchange of Die)** – **Shigeo Shingo**: reduce set-up/changeover to **under 10 minutes** (single-digit minutes) by converting **internal set-up** (machine stopped) to **external set-up** (machine running). OTED = one-touch exchange of die.
- **Poka-yoke** (Shingo) = **mistake-proofing / fool-proofing** device that prevents or immediately detects an error (e.g. asymmetric connector, limit switch).
- **Jidoka** = **autonomation** ("automation with a human touch"): machine **stops automatically** when a defect occurs. Origin: **Sakichi Toyoda's** automatic loom. Related: **Andon** = visual signal light/board for problems.
- **Heijunka** = **production levelling** (smoothing volume and mix); tool: heijunka box. **Kaizen** = **continuous, small-step improvement** by everyone (**Masaaki Imai**); **Kaikaku** = radical improvement. **Gemba** = the actual place of work; **genchi genbutsu** = "go and see".

Nandyal Banking Coaching Center

- **Value Stream Mapping (VSM)** = drawing all value-adding and non-value-adding steps (material + information flow) from supplier to customer; prepare **current-state** and **future-state** maps. Value-added ratio = value-added time ÷ total lead time (often < 5 %).
- **TPM (Total Productive Maintenance)** – Nakajima, Japan; measure **OEE = Availability × Performance × Quality**; world-class OEE ≈ **85 %**.
- **Six Sigma** – started at **Motorola (Bill Smith, 1986)**, made famous by GE (Jack Welch). Target **3.4 defects per million opportunities (DPMO)** (allowing a 1.5σ long-term shift). **DMAIC** = **Define, Measure, Analyse, Improve, Control** (improve an existing process); **DMADV / DFSS** = Define, Measure, Analyse, Design, Verify (new process). Roles: Champion, Master Black Belt, Black Belt, Green Belt.

5S (Japanese)	English	Meaning
Seiri	**Sort**	Remove unneeded items (red-tag)
Seiton	**Set in order**	"A place for everything, everything in its place"
Seiso	**Shine**	Clean and inspect
Seiketsu	**Standardise**	Make the first three a standard practice
Shitsuke	**Sustain**	Self-discipline, habit

7 wastes (TIMWOOD)	Example
Transportation	Unnecessary movement of material
Inventory	Excess raw material, WIP, finished goods
Motion	Unnecessary movement of **people** (bending, reaching)
Waiting	Idle time for parts, machines, information
Overproduction	Making more/earlier than needed – the **worst waste** (causes the others)
Over-processing	More work or precision than the customer needs
Defects	Scrap, rework, inspection

- An **8th waste** – **unused human talent/skills** – is often added (TIMWOODS, or "DOWNTIME").

KEY FORMULAE – LEAN

- **Takt time = available working time per period ÷ customer demand per period** (sets the pace of production; cycle time should be ≤ takt time)
- **Number of kanbans** $N = D\,L\,(1 + \alpha) / C$ (D = demand rate, L = lead time incl. waiting, α = safety factor, C = container size)
- OEE = Availability × Performance rate × Quality rate

- **Example (takt)**: 450 min available per shift, demand 225 units/shift → takt time = **2 min/unit**.
- **Example (kanban)**: D = 200 units/h, L = 0.5 h, α = 0.2, C = 20 units → N = 200 × 0.5 × 1.2/20 = **6 kanbans**. Maximum inventory ≈ N × C = 120 units.

MEMORY TRICK

Shingo gave two "S"-ideas: **S**MED (quick **S**et-up) and **S**top-mistakes (poka-yoke); **Ohno** gave TPS/kanban. **Jidoka** = machine **J**udges and stops. **Heijunka** = **H**orizontal (levelled) schedule. MRP inputs = "**M-B-I**" (MPS, BOM, Inventory file).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Three inputs of MRP? → **MPS, BOM and inventory status file**.
- Lot-sizing rule giving minimum carrying cost? → **Lot-for-lot**.
- "Seiton" in 5S means? → **Set in order**.
- Mistake-proofing device is called? → **Poka-yoke**.
- Available time 480 min, demand 960 units – takt time? → **0.5 min (30 s)**.
- Six Sigma quality level corresponds to? → **3.4 DPMO**.

13.5 Scheduling & Sequencing ★★★

- **Sequencing** = deciding the **order** in which jobs are processed on machines. **Scheduling** = also fixing their start/finish times.
- Number of possible sequences: n jobs on 1 machine = **n!**; n jobs on m machines (same order not required) = $(n!)^m$.
- **Flow time** = time a job spends in the shop = completion time − arrival (ready) time. If all jobs are ready at time 0, flow time = completion time.
- **Makespan** = total time to finish **all** jobs (completion time of the last job).
- **Lateness** L = completion − due date (can be −ve). **Tardiness** T = max(0, L). **Earliness** = max(0, −L).
- **Average number of jobs in system** = total flow time ÷ makespan. **Utilisation** = total processing time ÷ (makespan × number of machines).

Priority (dispatching) rule	Order jobs by	Property
FCFS (First Come First Served)	Arrival order	Fair; generally poor performance; common in service queues
SPT (Shortest Processing Time)	Increasing processing time	**Minimises mean flow time**, mean waiting time, mean lateness and average WIP (single machine); long jobs may wait too long
EDD (Earliest Due Date)	Increasing due date	**Minimises maximum lateness/tardiness** (single machine)
LPT (Longest Processing Time)	Decreasing processing time	Usually worst for flow time; used for load balancing on parallel machines
CR (Critical Ratio)	Smallest CR first	CR = (due date − today)/work remaining; **CR < 1 → behind schedule**, = 1 on time, > 1 ahead
S/RO (Slack per remaining operation)	Smallest slack/ops first	Slack = time to due date − remaining processing time
Moore–Hodgson algorithm	EDD with removal of longest job when late	**Minimises number of tardy jobs**

- **Worked example (single machine, all jobs ready at 0)**: jobs (processing time, due date) – A(6, 8), B(2, 6), C(8, 18), D(3, 15),

Nandyal Banking Coaching Center

E(9, 23). Σt = 28 = makespan for every sequence.

Rule	Sequence	Completion times	Mean flow time	Total tardiness	Max lateness
FCFS	A-B-C-D-E	6, 8, 16, 19, 28	77/5 = 15.4	0+2+0+4+5 = 11	5
SPT	B-D-A-C-E	2, 5, 11, 19, 28	**65/5 = 13.0** (least)	0+0+3+1+5 = 9	5
EDD	B-A-D-C-E	2, 8, 11, 19, 28	68/5 = 13.6	0+0+0+1+5 = 6	**5** (least possible)

Johnson's rule (n jobs, 2 machines)

- Applies to **n jobs on 2 machines in the same order (M1 → M2)**, i.e. a **flow shop**; it gives the **minimum makespan** (and minimum idle time of M2). Given by **S.M. Johnson (1954)**.
- **Steps**: (1) find the **smallest** processing time among all jobs on both machines; (2) if it is on **M1**, place that job **as early as possible (first)**; if on **M2**, place it **as late as possible (last)**; (3) remove the job and repeat; ties may be broken arbitrarily.
- **3-machine extension** (M1 → M2 → M3): allowed if **min t_{M1} ≥ max t_{M2}** or **min t_{M3} ≥ max t_{M2}**. Then form two fictitious machines G = M1 + M2 and H = M2 + M3 and apply Johnson's rule to G and H.
- **2 jobs on m machines**: graphical method (**Akers**).

Job	J1	J2	J3	J4	J5
M1 (h)	5	1	9	3	10
M2 (h)	2	6	7	8	4

- Smallest = 1 (J2 on M1) → J2 **first**. Next 2 (J1 on M2) → J1 **last**. Next 3 (J4 on M1) → J4 second. Next 4 (J5 on M2) → J5 fourth. J3 in the middle. **Optimal sequence: J2 – J4 – J3 – J5 – J1.**
- M1: J2 0–1, J4 1–4, J3 4–13, J5 13–23, J1 23–28. M2: J2 1–7, J4 7–15, J3 15–22, J5 23–27, J1 28–30. **Makespan = 30 h**; M2 idle = 1 + 1 + 1 = **3 h**; M1 idle after 28 h = 2 h.
- **Gantt chart** (Henry L. Gantt, ~1917) = bar chart of activities/jobs against time. Types: **machine (load) chart**, **progress (layout) chart**, man chart. Drawback: does not show **interdependencies** of activities (overcome by CPM/PERT networks).

KEY FORMULAE – SEQUENCING

- Mean flow time = Σ(completion times)/n (all jobs ready at 0)
- Critical ratio CR = (due date − current date) / remaining processing time
- Utilisation = Σ processing time / (makespan × machines); average jobs in system = Σ flow time / makespan

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Rule that minimises mean flow time on one machine? → **SPT**.
- Rule that minimises maximum lateness? → **EDD**.
- Johnson's rule minimises? → **Makespan** (total elapsed time).
- In Johnson's rule, if the least time is on machine 2, the job is placed? → **Last**.
- Critical ratio 0.8 means the job is? → **Behind schedule**.
- Number of sequences for 4 jobs on 3 machines? → **$(4!)^3$ = 13,824**.

13.6 Work Study & Ergonomics ★★☆

- **Work study** = **method study + work measurement**. Aim: higher productivity with existing resources, at low cost. **F.W. Taylor** = father of scientific management and **time study**; **Frank & Lillian Gilbreth** = fathers of **motion study**.
- **Method study** = systematic recording and critical examination of existing and proposed ways of doing work → to find **easier, better methods** (reduces work content). **Work measurement** = finding the **time** a qualified worker takes to do a job at a defined level of performance (reduces ineffective time; sets standards).
- **Steps of method study – SREDIM**: **S**elect → **R**ecord → **E**xamine → **D**evelop → **I**nstall → **M**aintain. (ILO lists 8 steps: Select, Record, Examine, Develop, Evaluate, Define, Install, Maintain.)
- **Critical examination** uses the **questioning technique**: primary questions – **What (purpose), Where (place), When (sequence), Who (person), How (means)**; secondary questions ask for alternatives. Then: **Eliminate, Combine, Rearrange, Simplify (ECRS)**.

ASME process-chart symbol	Activity	Meaning
○ (circle)	**Operation**	Changes characteristics of the object; the only value-adding activity
□ (square)	**Inspection**	Checks quality/quantity
⇨ (arrow)	**Transport**	Movement from one place to another
D (capital D)	**Delay / temporary storage**	Waiting, not intended
▽ (inverted triangle)	**Permanent storage**	Kept under authorised control (needs requisition to remove)
○ inside □	Combined activity	Operation and inspection together

Recording technique	Use
Outline process chart	Only **operations and inspections** (○ and □) – overall picture
Flow process chart	All **five symbols**; man-type, material-type or equipment-type
Two-handed (operator / left-hand–right-hand) chart	Movements of the two hands at a workplace
Multiple activity (man–machine) chart	Several workers/machines against a common time scale; finds idle time
SIMO chart (simultaneous motion cycle chart)	Micro-motions (therbligs) of both hands from a film, time in **winks** (1 wink = 1/2000 min)
Flow diagram	Path of movement drawn on the floor plan
String diagram	Scale plan with a **thread** showing path; measures distance travelled
Travel chart (from–to)	Tabular record of movements between departments
Cyclegraph / chronocyclegraph	Light on the hand photographed; chronocyclegraph shows **direction and speed** (pear-shaped dots)
Memomotion	Slow-speed filming (≈ 1 frame/s) for long cycles

Nandyal Banking Coaching Center

- **Therbligs** ("Gilbreth" reversed) = basic hand motions. **Gilbreth gave 17**; "**Find**" was added later, making **18** (some texts list 18 directly). Examples: search, select, grasp, transport empty, transport loaded, hold, release load, position, pre-position, inspect, assemble, disassemble, use, avoidable delay, unavoidable delay, plan, rest for overcoming fatigue, find.
- Effective therbligs (advance the work): reach, move, grasp, use, assemble, release; ineffective ones (search, select, hold, delays, plan) should be eliminated.
- **Principles of motion economy** (Gilbreth, Barnes) – three groups: (1) **use of human body** – both hands start and finish together, **never idle at the same time**, symmetric and opposite motions, use lowest classification of movement (finger → wrist → forearm → shoulder), smooth curved motions, momentum; (2) **workplace arrangement** – definite fixed place for tools, gravity-feed bins and drop delivery, tools in the **normal working area**, good lighting, correct chair height; (3) **design of tools and equipment** – use jigs/fixtures/foot pedals to free hands, combine tools, pre-position tools.

Work measurement

- **Techniques**: **stopwatch time study**, **work sampling**, **PMTS** (MTM, Work Factor, MOST), **standard data**, **analytical estimating**, synthesis.
- **Time study**: break the job into **elements**, time several cycles with a stopwatch (fly-back, continuous/cumulative or snap-back methods), **rate** the worker's performance, add **allowances**.
- **Performance rating**: compare the worker's speed with **normal pace** (rating 100 %; e.g. walking at 4.8 km/h ≈ 3 mph, dealing 52 cards in 0.5 min). Systems: speed rating, **Westinghouse (levelling) system** – skill, effort, conditions, consistency; synthetic rating; objective rating.
- **Allowances**: **personal** (≈ 5 %), **fatigue/rest** (≈ 4 % basic), **delay/contingency** (≤ 5 %), policy, special. Total usually **10–20 %** of normal time.
- **Work sampling (activity sampling / ratio-delay study)** – **L.H.C. Tippett (1934)**: many **random, instantaneous** observations to estimate the % of time spent on an activity (e.g. machine idle %). Cheap, no stopwatch, good for long or non-repetitive jobs and groups; based on the **binomial/normal** distribution.
- **PMTS** (predetermined motion time systems) – no rating needed. **MTM (Methods-Time Measurement)** Unit: **1 TMU = 0.00001 h = 0.0006 min = 0.036 s**. **MOST** (Maynard Operation Sequence Technique) – faster, uses index × 10 = TMU. Work Factor (WF) is another PMTS.

KEY FORMULAE – WORK MEASUREMENT

- **Normal (basic) time = observed (average) time × rating factor** (rating as fraction, e.g. 120 % → 1.2)
- **Standard time = normal time × (1 + allowance fraction)** (allowance as % of normal time); if allowance is % of total working time: ST = NT/(1 − A)
- Work sampling sample size, 95 % confidence (z ≈ 2): $\mathbf{n = 4p(1 - p)/L^2}$ (L = absolute accuracy, e.g. 0.05); with relative accuracy S: $n = 4(1 - p)/(S^2p)$
- Number of time-study cycles (95 % confidence, ±5 % accuracy): $N = [40\sqrt{(n\Sigma x^2 - (\Sigma x)^2)} / \Sigma x]^2$
- Labour productivity = output / labour hours; standard hours produced = output × standard time

- **Example (time study)**: observed time 0.8 min, rating 125 % → normal time = **1.0 min**; allowances 10 % → standard time = **1.1 min**. Output in an 8-h day (480 min) = 480/1.1 ≈ **436 pieces**.
- **Example (work sampling)**: pilot study shows machine idle p = 25 %; desired accuracy ±5 % (absolute), 95 % confidence → $n = 4 \times 0.25 \times 0.75/0.05^2$ = **300 observations**.

Wage incentive plans

Plan	Wage (T = time taken, S = standard time, R = rate/h)	Key point
Halsey premium	W = TR + **50 %** (S − T)R (premium 33⅓–50 %)	Time saved bonus shared with employer; guaranteed time wage
Rowan	W = TR + [(S − T)/S] × TR	Bonus % = % time saved; bonus is **maximum at T = S/2**, can never exceed 100 % of time wage; protects employer against loose standards
Taylor's differential piece rate	**Two piece rates**: low rate (≈ 83 % of normal) below standard, high rate (≈ 125 %) at or above standard	No guaranteed minimum; penalises slow workers
Merrick multiple piece rate	**Three** rates (100 %, 110 %, 120 % of basic piece rate as efficiency rises)	Modification of Taylor's plan
Gantt task & bonus	Guaranteed time wage below standard; bonus (20–50 %) on reaching standard	Combines time and piece rate
Emerson efficiency plan	Bonus starts at **66⅔ % efficiency**, rises to 20 % at 100 %	Gradual bonus
Bedaux (point) plan	1 B-point = 1 standard minute; bonus 75 % of points saved	—

- **Example**: S = 10 h, T = 8 h, R = ₹100/h. Halsey: 800 + 0.5 × 2 × 100 = **₹900**. Rowan: 800 + (2/10) × 800 = **₹960**. Rowan gives more than Halsey (50 %) when time saved is less than 50 % of S; both equal at T = S/2.
- **Group incentives**: Scanlon plan (labour-cost savings shared), Rucker plan, Priestman's plan, profit sharing.

Ergonomics

- **Ergonomics (human factors engineering)** = fitting the **job to the worker**: design of workplace, tools, displays, controls and environment to suit human abilities. Term coined by **K.F.H. Murrell (1949)**.
- **Anthropometry** = measurement of human body dimensions. Design range normally covers the **5th to 95th percentile**; clearances (door height) for the **95th** percentile (largest), reach distances for the **5th** percentile (smallest).
- **Normal working area** = arc swept by the forearm with elbow at the side; **maximum working area** = arc of the fully extended arm.
- Environment factors: illumination (lux), noise (dB(A); hearing protection around **85–90 dB(A)** for 8-h exposure),

Nandyal Banking Coaching Center

temperature/humidity, vibration, ventilation. Displays: analogue dial for trend/rate, digital for exact reading.

- Musculoskeletal disorders: carpal tunnel syndrome, low back pain; lifting guide – **NIOSH lifting equation** (load constant 23 kg).

MEMORY TRICK

Method study = "**S**ome **R**eal **E**ngineers **D**o **I**t **M**ethodically" (SREDIM). Process symbols: "**O**ne **I**nspector **T**ook a **D**elayed **S**torage" → Operation ○, Inspection □, Transport ⇨, Delay D, Storage ▽. **R**owan = **R**atio of time saved.

PREVIOUSLY ASKED / LIKELY QUESTIONS

- Symbol "D" in a flow process chart stands for? → **Delay (temporary storage)**.
- Observed time 2 min, rating 90 %, allowance 10 % – standard time? → **1.98 min**.
- 1 TMU equals? → **0.00001 h (0.036 s)**.
- Work sampling was introduced by? → **L.H.C. Tippett**.
- SIMO chart is used in? → **Micro-motion study**.
- In the Halsey plan the worker usually gets what share of time saved? → **50 %**.

13.7 Linear Programming, Transportation & Assignment ★★★

- **Operations Research (OR)** began in Britain during **World War II** (military radar/convoy problems). **Linear programming** – formulated by **Kantorovich (1939)**; **simplex method by George B. Dantzig (1947)**. Karmarkar's interior-point method (1984).
- **LP formulation**: decision variables; **linear objective function** (max profit / min cost); linear constraints; **non-negativity** ($x \geq 0$).
- **Assumptions**: **proportionality (linearity)**, **additivity**, **divisibility (continuity)**, **certainty** (all coefficients known), finiteness. If variables must be whole numbers → **integer programming** (branch and bound, Gomory cutting plane).
- **Graphical method** (only for **2 variables**): plot constraints; the common region satisfying all is the **feasible region**, which is always a **convex set**. The optimum lies at a **corner (extreme) point** – check Z at each corner (or slide the iso-profit line).
- **Example**: Max Z = 3x + 5y; $x \leq 4$; $2y \leq 12$; $3x + 2y \leq 18$; $x, y \geq 0$. Corners: (0,0) Z = 0; (4,0) 12; (4,3) 27; (2,6) 36; (0,6) 30 → **optimum x = 2, y = 6, Z = 36**.
- **Redundant constraint** = one that does not affect the feasible region. **Binding** constraint = satisfied as equality at the optimum (its slack = 0).

Constraint type	Variable added	Coefficient in objective
≤	**+ Slack** (unused resource)	0
≥	**− Surplus + Artificial**	Surplus 0; artificial **−M** (max) / **+M** (min)
=	**+ Artificial**	−M (max) / +M (min)

- **Simplex method** moves from one basic feasible solution (corner) to an adjacent better one. Basic variables = number of constraints (m); with n variables, number of basic solutions ≤ ${}^{n}C_{m}$.
- **Entering variable** (key column): for maximisation the most positive $C_j - Z_j$ (equivalently most negative $Z_j - C_j$). **Leaving variable** (key row): **minimum ratio test** – smallest non-negative ratio of RHS to positive key-column element. The intersection = **pivot (key) element**.
- **Optimality (maximisation)**: all $C_j - Z_j \leq 0$. For minimisation: all $C_j - Z_j \geq 0$. (Or convert: Min Z = −Max(−Z).)
- **Big-M (penalty) method** (Charnes) and **Two-phase method** handle artificial variables. Phase I minimises the sum of artificials; if this minimum is > 0, the problem is **infeasible**.

Special case	Graphical sign	Simplex sign
Degeneracy	More than two constraint lines pass through one corner (redundant constraint at optimum)	**Tie in the minimum ratio**; a **basic variable = 0**; may cause **cycling**
Unbounded solution	Feasible region open in the direction of improvement	All elements of key column ≤ 0 (no ratio can be formed)
Infeasible (no solution)	No common feasible region	**Artificial variable remains in the final basis at a positive value**
Multiple (alternate) optima	Objective line **parallel to a binding constraint**	A **non-basic** variable has $C_j - Z_j$ = **0** in the optimal table

Duality

- Every LP (**primal**) has a **dual**. Primal **max with ≤** constraints ↔ dual **min with ≥** constraints.
- Number of dual variables = number of primal **constraints**; number of dual constraints = number of primal **variables**. RHS of primal ↔ objective coefficients of dual; coefficient matrix is **transposed**.
- Primal **equality** constraint → dual variable **unrestricted in sign**; unrestricted primal variable → dual **equality** constraint. **Dual of the dual = primal**.
- **Weak duality**: any feasible max value ≤ any feasible min value. **Strong duality**: if both have optima, $\mathbf{Z_{max} = W_{min}}$. If primal is unbounded, dual is **infeasible**.
- Optimal dual variable = **shadow price** (marginal value of one more unit of that resource). Complementary slackness: if a resource has slack, its shadow price = 0.
- **Dual simplex** method: starts optimal but infeasible and moves to feasibility. **Sensitivity analysis** = effect of changes in c_j, b_i, a_{ij} on the optimum.

Transportation problem

- Ship goods from **m sources** to **n destinations** at minimum cost. **Balanced** if Σ supply = Σ demand; otherwise add a **dummy row/column** with zero cost.
- Total variables = m × n; constraints = m + n, but only **m + n − 1** are independent → a non-degenerate basic feasible solution has exactly **m + n − 1 occupied cells**.
- **Degeneracy**: occupied cells < m + n − 1 → allocate a tiny quantity **ε (epsilon)** to an independent empty cell (one that does not form a closed loop).
- **Prohibited route** → assign a very large cost **M**. Maximisation (profit) problem → subtract all values from the largest (or put negative signs) and minimise.

Nandyal Banking Coaching Center

- A transportation problem always has a feasible solution (if balanced); with integer supplies/demands the solution is integer.

IBFS method	Rule	Quality
North-West Corner Rule (NWCR)	Start at top-left cell, allocate max, move right/down	Quickest, **ignores cost** – usually worst
Least Cost (Matrix Minima) Method (LCM)	Allocate to the cheapest cell first	Better
Vogel's Approximation Method (VAM)	**Penalty** = difference of the two lowest costs in each row/column; allocate in row/column with **largest penalty** at its least-cost cell	**Best** IBFS; often optimal or near-optimal

- **Optimality test**: **MODI (u-v) method** – for each occupied cell $u_i + v_j = c_{ij}$ (set one u or v = 0); for each empty cell find $\Delta_{ij} = c_{ij} - (u_i + v_j)$. If all $\Delta_{ij} \geq 0$ → **optimal** (for minimisation); any $\Delta_{ij} = 0$ → alternate optimum. Otherwise enter the cell with the most negative Δ.
- **Stepping-stone method**: evaluate each empty cell by a **closed loop** with horizontal and vertical moves through occupied cells; the loop has an **even number of corners (≥ 4)**, signs alternate + / −. The quantity shifted = **smallest allocation in a "−" cell**.
- **Transshipment** problem: goods may pass through intermediate points; converted to a transportation problem by adding buffer stock.

Assignment problem

- Assign **n jobs to n persons/machines one-to-one** at minimum cost (or max profit). It is a special **transportation problem** with every supply and demand = 1; it is **highly degenerate** (only n of the needed 2n − 1 basic cells are positive). Number of possible assignments = **n!**.
- **Hungarian method** (**H.W. Kuhn, 1955**, based on König and Egerváry): (1) **row reduction** – subtract the row minimum from each row; (2) **column reduction**; (3) cover all zeros with the **minimum number of lines**; (4) if lines = **n → optimal**, make assignments on independent zeros; (5) else subtract the smallest uncovered element from all uncovered elements and **add it at intersections** of lines; repeat.
- **Unbalanced** (rows ≠ columns) → add **dummy** row/column with zero cost to make it square.
- **Maximisation** → subtract every element from the **largest element** of the matrix (or multiply by −1), then minimise.
- **Restricted assignment** (person cannot do a job) → cost **M** (or ∞) in that cell.
- **Travelling salesman problem (TSP)**: visit each of n cities **exactly once** and return to start at minimum distance; possible tours = **(n − 1)!** (symmetric: (n − 1)!/2). Solved like an assignment problem with the extra condition of **no sub-tours**; diagonal cells = ∞; methods: branch and bound, nearest-neighbour heuristic.

KEY FORMULAE – LP / TRANSPORTATION

- Occupied cells (non-degenerate) = **m + n − 1**
- MODI: $u_i + v_j = c_{ij}$ (occupied); $\Delta_{ij} = c_{ij} - (u_i + v_j)$ (empty); optimal if all $\Delta_{ij} \geq 0$
- Assignment optimal when minimum covering lines = **n**
- Primal Max $c^T x$, $Ax \leq b$, $x \geq 0$ ↔ Dual Min $b^T y$, $A^T y \geq c$, $y \geq 0$

MEMORY TRICK

Constraint vs variable: "**≤ Slack** (less, so something is **slack**/spare); **≥ Surplus + Artificial**; **= Artificial only**". IBFS quality order "**N**ot **L**ike **V**ogel" → NWCR (worst) < LCM < VAM (best).

PREVIOUSLY ASKED / LIKELY QUESTIONS

- A 4 × 5 transportation problem – number of basic (occupied) cells? → **8**.
- Variable added to a "≥" constraint? → **Surplus (and artificial)**.
- Tie in minimum ratio in simplex indicates? → **Degeneracy**.
- Artificial variable remaining positive in final table means? → **Infeasible solution**.
- Best method for initial solution of transportation problem? → **VAM**.
- Method used to solve assignment problems? → **Hungarian method**.

13.8 Queuing Theory, Network Models, PERT/CPM & Decision Theory ★★★

Queuing (waiting-line) theory

- Pioneered by **A.K. Erlang** (Danish telephone engineer, 1909). Parts of a queuing system: **arrival (input) process**, **queue discipline**, **service mechanism**, **calling population**, system capacity.
- Usual model: arrivals follow a **Poisson distribution** (mean rate λ) ↔ **inter-arrival times are exponential** (mean 1/λ); service times are **exponential** (mean 1/μ). The exponential distribution is **memoryless** (Markovian, "M").
- **Kendall notation (a/b/c)** (D.G. Kendall, 1953): a = arrival distribution, b = service-time distribution, c = number of parallel servers. Extended (Kendall–Lee/Taha): **(a/b/c):(d/e/f)** → d = queue discipline, e = maximum number allowed in system, f = size of calling source. Symbols: **M** = Markovian/exponential (Poisson), **D** = deterministic (constant), $\mathbf{E_k}$ = Erlang-k, **G** = general.
- Example: **M/M/1 : (∞/FCFS)** = Poisson arrivals, exponential service, single server, infinite capacity, first-come-first-served.
- Queue disciplines: **FCFS (FIFO)**, LCFS (LIFO – e.g. stacked items in a store), SIRO (random), priority (pre-emptive / non-pre-emptive).
- Customer behaviour: **balking** (does not join a long queue), **reneging** (leaves after joining, out of impatience), **jockeying** (switches between queues), collusion.
- Steady state needs **ρ = λ/μ < 1**; if λ ≥ μ the queue grows without limit. ρ = **utilisation / traffic intensity** = probability the server is busy.

Nandyal Banking Coaching Center

KEY FORMULAE – M/M/1 : (∞/FCFS)

- Traffic intensity $\rho = \lambda/\mu$; P_0 (system idle) = $1 - \rho$; $P_n = (1 - \rho)\rho^n$; $P(n > k) = \rho^{k+1}$
- Average number in system $L_s = \lambda/(\mu - \lambda) = \rho/(1 - \rho)$
- Average number in queue $L_q = \lambda^2/[\mu(\mu - \lambda)] = \rho^2/(1 - \rho)$; $L_s - L_q = \rho$
- Average time in system $W_s = 1/(\mu - \lambda)$; average waiting time in queue $W_q = \lambda/[\mu(\mu - \lambda)]$; $W_s = W_q + 1/\mu$
- **Little's law:** $L = \lambda W$ ($L_s = \lambda W_s$, $L_q = \lambda W_q$) – valid for any stable queue
- Average length of non-empty queue = $\mu/(\mu - \lambda)$; P(waiting time in system > t) = $e^{-(\mu-\lambda)t}$

- **Example**: λ = 8 per hour, μ = 10 per hour → ρ = **0.8**; P_0 = **0.2**; L_s = 8/2 = **4**; L_q = 64/(10 × 2) = **3.2**; W_s = 1/2 h = **30 min**; W_q = 8/20 h = **24 min**. Check Little: $L_s = \lambda W_s$ = 8 × 0.5 = 4 ✓.
- For **M/D/1** (constant service), $L_q = \rho^2/[2(1 - \rho)]$ = **half** that of M/M/1 – lower variability means shorter queues.

Network models

Problem	Algorithm	Key point
Shortest path (one source to all nodes)	**Dijkstra**	Needs **non-negative** arc lengths; permanent labels. All pairs: **Floyd**; negative arcs: Bellman–Ford
Minimal spanning tree	**Kruskal** (pick cheapest edges, skip those forming a cycle); **Prim** (grow tree from one node by cheapest connecting edge)	Connects all n nodes with **n − 1** edges, no cycle; e.g. laying pipelines/cables
Maximal flow	**Ford–Fulkerson** (augmenting paths)	**Max-flow = min-cut** capacity theorem

PERT / CPM

Feature	PERT	CPM
Nature	**Probabilistic** (uncertain times)	**Deterministic** (known times)
Orientation	**Event**-oriented	**Activity**-oriented
Time estimates	**Three** (a, m, b); **β-distribution**	**One**
Used for	R&D, new projects (Polaris missile, US Navy, 1958)	Construction, maintenance, repetitive projects (Du Pont, 1957); allows **crashing** (time–cost trade-off)

KEY FORMULAE – PERT/CPM

- Expected time $t_e = (a + 4m + b)/6$ (a = optimistic, m = most likely, b = pessimistic)
- Standard deviation $\sigma = (b - a)/6$; variance $\sigma^2 = [(b - a)/6]^2$; project variance = $\Sigma\sigma^2$ **of critical activities**
- Probability: $Z = (T_s - T_e)/\sigma_{cp}$ (T_s = scheduled time); Z = 0 → 50 %
- **Total float** TF = LS − ES = LF − EF = $L_j - E_i - t_{ij}$; **Free float** FF = $E_j - E_i - t_{ij}$; **Independent float** IF = $E_j - L_i - t_{ij}$; **Interfering float** = TF − FF = $L_j - E_j$ (head-event slack)
- **Cost slope = (crash cost − normal cost)/(normal time − crash time)**

- **Example**: a = 2, m = 5, b = 14 days → $t_e = (2 + 20 + 14)/6$ = **6 days**; σ = 12/6 = **2 days**.
- **Critical path** = longest path through the network; activities on it have **zero total float** (when target = earliest finish). Floats: IF ≤ FF ≤ TF. A negative float means the project is behind the target date.
- **Crashing**: shorten **critical** activities, starting with the **lowest cost slope**; watch for new critical paths. Direct cost rises and indirect cost falls as duration is cut; the **optimum duration** is where **total cost is minimum**.
- Network rules: **Fulkerson's rule** for numbering events; **dummy activity** (dashed, zero time and resources) shows dependency only; errors – **looping** (cycle), **dangling** (activity not leading to the end event), redundancy.

Decision theory & game theory

Environment	Criterion	Choose
Uncertainty (probabilities unknown)	**Maximax** (optimistic)	Best of the row maxima
	Maximin (pessimistic, Wald)	Best of the row minima
	Minimax regret (Savage)	Regret = column best − payoff; choose the smallest maximum regret
	Laplace (equally likely)	Highest simple average
	Hurwicz (α = coefficient of optimism)	Highest α(max) + (1 − α)(min)
Risk (probabilities known)	**EMV** (expected monetary value) / EOL	Highest EMV = lowest expected opportunity loss
Certainty	LP, etc.	Single known outcome

- **Worked example** – payoffs (₹ lakh) for states N1, N2, N3: S1 = 40, 30, 10; S2 = 60, 20, −10; S3 = 25, 25, 25.
  Maximax → **S2** (60). Maximin → **S3** (25). Regrets (column best 60, 30, 25): S1 max regret 20, S2 35, S3 35 → minimax regret **S1**. Laplace: S1 26.7, S2 23.3, S3 25 → **S1**. Hurwicz α = 0.6: S1 28, S2 32, S3 25 → **S2**. With P = 0.3, 0.5, 0.2: EMV S1 = 29, S2 = 26, S3 = 25 → **S1**.
- **EVPI** (expected value of perfect information) = expected profit with perfect information − max EMV. Above: EPPI = 0.3 × 60 + 0.5 × 30 + 0.2 × 25 = 38 → **EVPI = 38 − 29 = 9**. EVPI = minimum EOL. Never pay more than EVPI for information.
- **Decision tree**: square node = **decision**, circle node = **chance** event; solved by **rolling back** (right to left) using EMV.
- **Game theory** (**von Neumann & Morgenstern**, 1944): two-person **zero-sum** game – one player's gain = other's loss. Row player uses **maximin**, column player **minimax**. If **maximin = minimax**, a **saddle point** exists → **pure strategies**, and that common value = **value of the game**. Value = 0 → **fair game**. Game with saddle point = strictly determined.
- No saddle point → **mixed strategies**. For a 2 × 2 game [a_{11} a_{12}; a_{21} a_{22}]: $p_1 = (a_{22} - a_{21})/(a_{11} + a_{22} - a_{12} - a_{21})$; $V = (a_{11}a_{22} - a_{12}a_{21})/(a_{11} + a_{22} - a_{12} - a_{21})$. Larger games: **dominance** rule, graphical (2 × n, m × 2), LP.

Nandyal Banking Coaching Center

Simulation & replacement

- **Simulation** = imitating a real system on a model to study its behaviour when an analytical solution is hard. **Monte Carlo simulation** uses **random numbers**: make the cumulative probability distribution, assign random-number ranges to each outcome, draw random numbers and read outcomes. It gives **approximate (not optimal)** answers; accuracy improves with more trials.
- **Replacement of items that deteriorate** (rising running cost, money value ignored): replace at the end of year n when the **average annual total cost is minimum**, i.e. when the **next year's running cost exceeds the current average cost**. Average cost A(n) = [C − S + Σ running costs up to n] / n.
- **Example**: C = ₹10,000, scrap ≈ 0, running costs 1,000, 1,500, 2,500, 4,000, 6,000 in years 1–5 → average cost: 11,000; 6,250; 5,000; **4,750**; 5,000 → **replace after 4 years** (year-5 running cost 6,000 > average 4,750).
- With time value of money: replace when the **weighted (discounted) average cost** is minimum.
- **Items that fail suddenly** (bulbs, resistors): **individual replacement** (replace on failure) vs **group replacement** (replace all at fixed intervals + individual replacement in between). Group policy: replace all at the end of period t if the cost of individual replacements in period (t + 1) exceeds the **average cost per period** up to t.

MEMORY TRICK

M/M/1 = "**one over the gap**": $W_s = 1/(\mu - \lambda)$, $L_s = \lambda \times$ that (Little). Queue values = system values × ρ ($L_q = \rho L_s$, $W_q = \rho W_s$). PERT weights "**1-4-1 over 6**"; σ = "**range over 6**".

PREVIOUSLY ASKED / LIKELY QUESTIONS

- λ = 6/h, μ = 9/h (M/M/1): average number in system? → **2** (L_q = 1.33).
- In Kendall notation M/D/2, "D" means? → **Deterministic (constant) service time**.
- a = 4, m = 6, b = 14 days – expected time? → **7 days** (σ = 1.67 days).
- Algorithm for minimal spanning tree? → **Kruskal / Prim**.
- When maximin equals minimax, the game has a? → **Saddle point**.
- Machine should be replaced when? → **Average annual cost is minimum** (next year's running cost > average cost).

Nandyal Banking Coaching Center

14. Last-Day Revision (Formula & Value Sheet)

Topic	Must-remember facts
Elastic constants & beams	$E = 2G(1 + \nu) = 3K(1 - 2\nu)$; steel $E \approx 200$ GPa, $\nu \approx 0.3$; SS + UDL: $M = wL^2/8$, $\delta = 5wL^4/384EI$; cantilever + end load: $\delta = WL^3/3EI$; τ_{max}/τ_{avg}: rectangle 1.5, circle 4/3
Torsion, springs, columns	$T/J = \tau/r = G\theta/L$; $P = 2\pi NT/60$; $T_e = \sqrt{(M^2 + T^2)}$, $M_e = \frac{1}{2}[M + \sqrt{(M^2 + T^2)}]$; spring $\delta = 8WD^3n/(Gd^4)$; Wahl $K = (4C - 1)/(4C - 4) + 0.615/C$; Euler $P = \pi^2EI/L_e^2$ (L_e: L, L/2, L/√2, 2L)
Theories of failure	Max principal stress (Rankine) → brittle; max shear (Tresca) and distortion energy (von Mises) → ductile; shear yield: Tresca $0.5\sigma_y$, von Mises $0.577\sigma_y$
Mechanisms	DOF = 3(n − 1) − 2j − h; Grashof s + l ≤ p + q; instantaneous centres N = n(n − 1)/2; Coriolis acceleration 2vω; Ackermann cot φ − cot θ = c/b
Gears & belts	m = d/T; addendum 1m, dedendum 1.25m; min teeth to avoid interference ≈ 32 (14.5°), 18 (20°); $T_1/T_2 = e^{\mu\theta}$ (V-belt $e^{\mu\theta \operatorname{cosec} \beta}$); $T_c = mv^2$; max power when $T_c = T_{max}/3$
Flywheel, governors, vibration	$\Delta E = I\omega^2C_s$; Watt governor $h \approx 895/N^2$ m; $\omega_n = \sqrt{(k/m)}$; $f_n = 0.4985/\sqrt{\delta}$ Hz (δ in m); $\zeta = c/2\sqrt{(km)}$; isolation when $\omega/\omega_n > \sqrt{2}$; critical speed = natural frequency
Machine design	$K_f = 1 + q(K_t - 1)$; endurance limit ≈ $0.5\sigma_u$ (steel); Soderberg most conservative, Gerber least; bearing life $L_{10} = (C/P)^k$, k = 3 ball, 10/3 roller; uniform-wear clutch $T = \mu WR_m n$; screw self-locking when φ > α; Lewis $\sigma = F_t/(bmY)$
Fluids	Re < 2000 laminar; $f = 64/Re$; $h_f = fLV^2/2gD$; $V_{max} = 2V$ (pipe); Pelton max η at u = V/2 (ideal); turbine $N_s = N\sqrt{P}/H^{5/4}$; pump $N_s = N\sqrt{Q}/H^{3/4}$; affinity $Q \propto N$, $H \propto N^2$, $P \propto N^3$
Thermodynamics	$c_p - c_v = R$; polytropic $W = (p_1V_1 - p_2V_2)/(n - 1)$; Carnot $\eta = 1 - T_L/T_H$; $COP_{HP} = COP_R + 1$; throttling is isenthalpic; triple point of water 0.01 °C; critical point 374.15 °C, 221.2 bar
Cycles	Otto $\eta = 1 - 1/r^{\gamma-1}$; same r: Otto > Dual > Diesel; same p_{max}: Diesel > Dual > Otto; Brayton $\eta = 1 - 1/r_p^{(\gamma-1)/\gamma}$; reheat mainly raises dryness; regeneration raises η; equivalent evaporation from and at 100 °C uses 2257 kJ/kg
Nozzles & turbines	Critical pressure ratio: air 0.528, superheated steam 0.546, saturated steam 0.577; Parsons = 50% reaction; single-stage impulse max blade $\eta = \cos^2\alpha$ at u/V = cos α/2
IC engines	CR: SI 6–12, CI 14–22; stoichiometric A/F petrol ≈ 14.7; octane: iso-octane 100, n-heptane 0; cetane: cetane 100, α-methylnaphthalene 0; Morse test → SI multi-cylinder FP; Willan's line → CI; 3-way catalyst cuts CO, HC, NOx; EGR cuts NOx; BS-VI from 1 April 2020
Air compressors	Isothermal work minimum; $\eta_v = 1 + C - C(p_2/p_1)^{1/n}$; two-stage optimum $p_2 = \sqrt{(p_1p_3)}$ with equal work per stage; Roots blower has no internal compression; centrifugal/axial: surging
Refrigeration	1 TR ≈ 3.517 kW (211 kJ/min); Carnot $COP_R = T_L/(T_H - T_L)$; VCR COP = $(h_1 - h_4)/(h_2 - h_1)$; R-717 ammonia, R-744 CO_2, R-290 propane, R-600a isobutane; LiBr–water: water is refrigerant; Kigali Amendment 2016 (HFC phase-down)
Psychrometry	$\omega = 0.622p_v/(p - p_v)$; at saturation DBT = WBT = DPT; evaporative cooling at constant WBT; BPF = $(t_2 - t_{ADP})/(t_1 - t_{ADP})$; sensible heating keeps ω constant
Heat transfer	$R_{wall} = L/kA$; $R_{cyl} = \ln(r_2/r_1)/2\pi kL$; critical radius: cylinder k/h, sphere 2k/h; lumped model valid for Bi < 0.1; $\sigma = 5.67 \times 10^{-8}$ W/m²K⁴; $\lambda_{max}T = 2898$ µm·K; Dittus–Boelter $Nu = 0.023Re^{0.8}Pr^n$; laminar tube Nu = 3.66 (constant T), 4.36 (constant q); counter-flow LMTD > parallel; $\varepsilon = 1 - e^{-NTU}$ when C = 0
Materials	APF: BCC 0.68, FCC/HCP 0.74; eutectoid 727 °C, 0.8% C → pearlite; eutectic 1147 °C, 4.3% C → ledeburite; max C in austenite 2.11%, ferrite 0.022%; cementite 6.67% C; Gibbs F = C − P + 2; 18-8 austenitic SS non-magnetic; HSS 18-4-1 (W-Cr-V)
Heat treatment & testing	Annealing – furnace cool; normalising – air cool; hardening – quench → martensite; nitriding needs no quench; Jominy end-quench → hardenability; Brinell 10 mm ball/3000 kg; Vickers 136° pyramid; Charpy & Izod → impact
Casting	Shrinkage, draft, machining allowances positive; shake allowance negative; Chvorinov $t = B(V/A)^2$; riser must solidify last; hot-chamber die casting for Zn/Mg, cold-chamber for Al; investment = lost wax
Welding & forming	DCEN (DCSP) → deep penetration; DCEP (DCRP) → shallow, cleaning action; neutral flame 1:1 (~3200 °C); TIG = non-consumable tungsten; SAW for thick flat plates; resistance heat $H = I^2Rt$; brazing filler > 450 °C, soldering < 450 °C; CE > 0.4 → preheat; blanking: die = blank size; piercing: punch = hole size
Metal cutting	$r = t_1/t_2$; $\tan\varphi = r\cos\alpha/(1 - r\sin\alpha)$; Merchant $2\varphi + \beta - \alpha = 90°$; Taylor $VT^n = C$ (n: HSS ≈ 0.1–0.15, carbide ≈ 0.2–0.25, ceramic ≈ 0.5–0.7); ideal $R_a \approx f^2/(32r)$; flank wear sets tool life; drill point angle 118°
Grinding & NTM	Al_2O_3 for steel, SiC for CI/non-ferrous; wheel code A 46 K 5 V; glazing → too hard a wheel; ECM – no tool wear; EDM – conductive work, dielectric fluid; USM – hard brittle materials; EBM in vacuum
CNC & AM	G00 rapid, G01 linear, G02 CW arc, G03 CCW arc, G90 absolute, G91 incremental; M03 spindle CW, M05 stop, M06 tool change, M30 end; Z-axis along the spindle; seven AM categories (ISO/ASTM 52900); FDM = material extrusion, SLA = vat photopolymerisation
Metrology & QC	Hole-basis: H hole with lower deviation 0; 20 IT grades (IT01–IT18); $i = 0.45\sqrt[3]{D} + 0.001D$ µm; GO gauge checks MMC (Taylor); vernier LC = 1 MSD − 1 VSD; sine bar $\sin\theta = h/L$; best wire $d = (p/2)\sec(\theta/2)$; $C_p = (USL - LSL)/6\sigma$; six sigma = 3.4 DPMO
NDT & maintenance	Magnetic particle → ferromagnetic only; dye penetrant → surface cracks; ultrasonic and radiography → internal flaws; availability = MTBF/(MTBF + MTTR); OEE = A × P × Q; FMEA RPN = S × O × D; bathtub curve: infant mortality, useful life, wear-out

Nandyal Banking Coaching Center

Inventory & forecasting	EOQ = √(2DS/H); at EOQ ordering cost = carrying cost; EPQ = √(2DS/(H(1 − d/p))); ROP = dL + safety stock; ABC = value, VED = criticality, FSN = movement; exponential smoothing $F_{t+1} = F_t + \alpha(D_t - F_t)$; tracking signal = RSFE/MAD
Scheduling, work study, OR	SPT minimises mean flow time; EDD minimises maximum lateness; Johnson's rule for n jobs on 2 machines; standard time = normal time × (1 + allowance); M/M/1: $L_s = \lambda/(\mu - \lambda)$, $W_s = 1/(\mu - \lambda)$, $L = \lambda W$; transportation IBFS: VAM best, m + n − 1 allocations; Hungarian method for assignment; PERT $t_e = (a + 4m + b)/6$

Nandyal Banking Coaching Center

15. 500 Most Expected Questions & Answers

> **NOTE**
> Rapid-fire revision: 500 one-line questions chosen for the CWC AM (Mechanical) paper, in the same order as the chapters. Cover the answer, attempt, then check. Numericals carry a short working hint.

15.1 Engineering Mechanics & Strength of Materials (Q 1–45)

1. Resultant of two equal forces of 10 N each acting at 120° to each other? → **10 N (R = 2P cos 60°)**
2. Resultant of forces of 3 N and 4 N acting at right angles? → **5 N (√(3² + 4²))**
3. Lami's theorem is applicable to how many forces? → **Exactly three coplanar concurrent forces in equilibrium**
4. Number of members in a perfect plane truss having 9 joints? → **15 (m = 2j − 3)**
5. Number of reaction components at a fixed support of a plane structure? → **3 (H, V and moment)**
6. Least force needed to drag a 100 N block on a level floor, μ = 0.75? → **60 N (W sin φ; sin φ = 0.6)**
7. Belt with T_{max} = 3000 N and mass 1 kg/m: speed for maximum power? → **≈ 31.6 m/s (v = √(T_{max}/3m))**
8. For a V-belt with groove half-angle 30°, the effective μ is multiplied by? → **2 (cosec 30°)**
9. MI of a circular section of diameter d about a tangent? → **5πd⁴/64 (πd⁴/64 + A(d/2)²)**
10. Radius of gyration of a 40 mm diameter circular section about a diameter? → **10 mm (k = d/4)**
11. Projectile fired at 20 m/s at 45° (g = 10 m/s²): horizontal range? → **40 m (R = u²/g)**
12. A ball dropped from 4 m rebounds to 1 m. Coefficient of restitution? → **0.5 (e = √(1/4))**
13. Law of machine P = 0.05W + 10, VR = 30. Maximum efficiency? → **66.7% (1/(m × VR) = 1/1.5)**
14. E = 200 GPa and ν = 0.25. Modulus of rigidity G? → **80 GPa (G = E/2(1 + ν))**
15. E = 210 GPa and ν = 0.25. Bulk modulus K? → **140 GPa (K = E/3(1 − 2ν))**
16. Number of independent elastic constants for a general anisotropic material? → **21 (isotropic 2, orthotropic 9)**
17. Steel bar 2 m long, area 100 mm², load 20 kN, E = 200 GPa. Elongation? → **2 mm (δ = PL/AE)**
18. Fully restrained steel bar heated by 50 °C (α = 12 × 10⁻⁶/°C, E = 200 GPa). Stress? → **120 MPa compressive (σ = EαΔT)**
19. Modulus of resilience for σ_y = 250 MPa, E = 200 GPa? → **≈ 0.156 N/mm² = 0.156 MJ/m³ (σ²/2E)**
20. Hardness test using a 136° square-based diamond pyramid indenter? → **Vickers hardness test**
21. Stage of creep with a constant, minimum creep rate? → **Secondary (steady-state) creep**
22. SS beam of span 4 m with a central load of 10 kN. Maximum BM? → **10 kN·m (WL/4)**
23. Cantilever 2 m long carrying a UDL of 5 kN/m. Maximum BM? → **10 kN·m at fixed end (wL²/2)**
24. Section modulus of a rectangle 100 mm wide × 200 mm deep? → **≈ 6.67 × 10⁵ mm³ (bd²/6)**
25. τ_{max}/τ_{avg} for a solid circular section under transverse shear? → **4/3, at the neutral axis**
26. Strongest rectangular beam cut from a circular log: ratio depth/width? → **√2 (b = D/√3, d = √(2/3)D)**
27. Points of contraflexure in a fixed–fixed beam with UDL lie from each end at? → **0.211L**
28. Maximum deflection of a SS beam with central point load W? → **WL³/48EI at mid-span**
29. If the span of a cantilever with an end load is doubled, its deflection becomes? → **8 times (δ ∝ L³)**
30. In the conjugate beam method, the SF of the conjugate beam gives? → **Slope of the real beam**
31. End moments induced in a fixed beam when one support sinks by δ? → **6EIδ/L²**
32. Shaft at 300 rpm transmits a torque of 1 kN·m. Power? → **≈ 31.4 kW (P = 2πNT/60)**
33. If the diameter of a solid shaft is doubled, its torque capacity becomes? → **8 times (T ∝ d³)**
34. Wahl's stress factor for a spring of index C = 6? → **≈ 1.25 ((4C − 1)/(4C − 4) + 0.615/C)**
35. Shaft with M = 3 kN·m and T = 4 kN·m. Equivalent bending moment? → **4 kN·m (½[M + √(M² + T²)])**
36. Column with effective length 2 m and k_{min} = 20 mm. Slenderness ratio? → **100 (L_e/k)**
37. Euler load of a fixed–free column compared with the same column both ends hinged? → **1/4 (L_e = 2L)**
38. Shape of the core (kernel) of a rectangular section b × d? → **Rhombus with diagonals b/3 and d/3**
39. σ_x = 80 MPa, σ_y = 20 MPa, τ_{xy} = 40 MPa. Major principal stress? → **100 MPa (50 + √(30² + 40²))**
40. Mohr's circle for a state of pure shear is centred at? → **The origin (radius = τ)**
41. Von Mises equivalent stress for combined bending σ and torsional shear τ? → **√(σ² + 3τ²)**
42. Maximum difference between Tresca and von Mises predictions occurs in? → **Pure shear (≈ 15.5%; 0.577 vs 0.5)**
43. Thin cylinder d = 1 m, t = 10 mm, p = 2 MPa. Hoop stress? → **100 MPa (pd/2t)**
44. For the same p, d and allowable stress, thin sphere thickness vs thin cylinder? → **Half (pd/4σ vs pd/2σ)**

Nandyal Banking Coaching Center

45. Redundant reactions found from $\partial U/\partial R = 0$ use the? → **Principle of least work (Castigliano's corollary)**

15.2 Theory of Machines & Vibrations (Q 46–85)

46. DOF of a planar mechanism with 6 links and 7 binary lower pairs? → **1 ($3 \times 5 - 2 \times 7$)**

47. Four-bar with links 3, 7, 8, 10 cm; shortest link fixed gives? → **Double crank ($s + l = 13 < p + q = 15$)**

48. Crank & slotted lever: crank 150 mm, pivot distance 300 mm. Quick-return ratio? → **2 : 1 ($\cos(\alpha/2) = 0.5 \rightarrow \alpha = 120°$)**

49. Oscillating cylinder engine is obtained by fixing which link of slider-crank? → **Connecting rod**

50. Number of instantaneous centres in a five-link mechanism? → **10 ($n(n-1)/2$)**

51. Wheel rolls without slip at 10 m/s. Velocity of topmost point? → **20 m/s (I-centre at contact point)**

52. Slider moves at 2 m/s on a link rotating at 5 rad/s. Coriolis acceleration? → **20 m/s² ($2v\omega$)**

53. Hooke's joint, shaft angle 20°, driver 1000 rpm. Max and min driven speed? → **≈ 1064 rpm and ≈ 940 rpm ($N/\cos\alpha$, $N\cos\alpha$)**

54. Number of links in Hart's exact straight-line mechanism? → **6 (Peaucellier has 8)**

55. Scotch yoke converts uniform rotation into? → **Simple harmonic motion of the yoke**

56. SHM follower: lift 30 mm in 90° of cam rotation, ω = 10 rad/s. Max velocity? → **0.3 m/s ($\pi\omega S/2\theta_o$, $\theta_o = \pi/2$)**

57. Base circle radius 30 mm, roller radius 10 mm. Prime circle radius? → **40 mm (base + roller radius)**

58. Recommended maximum pressure angle for a translating (reciprocating) follower? → **≈ 30° (≈ 45° for oscillating)**

59. Follower motion with infinite acceleration at start and end of stroke? → **Uniform velocity**

60. Outside diameter of a 30-tooth gear, module 4 mm (standard addendum)? → **128 mm ($m(T + 2)$)**

61. Circular pitch of a gear with module 5 mm? → **15.71 mm (πm)**

62. Bottom clearance of a 20° full-depth gear of module 4 mm? → **1 mm (0.25m)**

63. Helical gear, 20 teeth, helix angle 30°. Virtual (formative) number of teeth? → **≈ 30.8 ($T/\cos^3\psi$)**

64. Double-start worm with 60-tooth worm wheel. Velocity ratio? → **30 (T_{wheel}/starts)**

65. Epicyclic train: sun 30 teeth, planet 20 teeth (same module). Annulus teeth? → **70 ($T_S + 2T_P$)**

66. Tooth profile with constant pressure angle throughout engagement? → **Involute**

67. Flat belt, μ = 0.3, angle of lap 180°. Tension ratio T_1/T_2? → **≈ 2.57 ($e^{0.3\pi}$)**

68. T_1 = 2000 N, T_2 = 800 N, belt speed 10 m/s. Power transmitted? → **12 kW ($(T_1 - T_2)v$)**

69. Belt T_{max} = 1500 N, mass 0.5 kg/m. Speed for maximum power? → **≈ 31.6 m/s ($\sqrt{(T_{max}/3m)}$)**

70. Sprocket with 20 teeth, chain pitch 25.4 mm. Pitch circle diameter? → **≈ 162.4 mm ($p/\sin(180°/T)$)**

71. Effect of centrifugal tension on power transmitted by a belt? → **None (adds equally to both sides)**

72. Maximum and minimum speeds 202 and 198 rpm. Coefficient of fluctuation of speed? → **0.02 ($2(N_1 - N_2)/(N_1 + N_2)$)**

73. Flywheel I = 100 kg·m², mean speed 300 rpm, C_s = 0.02. Maximum fluctuation of energy? → **≈ 1974 J ($I\omega^2 C_s$, ω = 31.42 rad/s)**

74. Height of a Watt governor at 100 rpm? → **≈ 89.5 mm ($895/N^2$ m)**

75. Hunting of a governor is caused by? → **Too high sensitiveness (over-sensitive / isochronous governor)**

76. Controlling force curve $F_c = ar + b$ indicates a governor that is? → **Unstable ($ar - b$ stable, ar isochronous)**

77. Secondary force of reciprocating mass is treated as a crank of radius ___ at speed ___? → **$r/4n$, rotating at 2ω**

78. Which inline engine is completely balanced for primary and secondary forces and couples? → **Six-cylinder inline (cranks at 120°)**

79. Maximum variation of tractive effort in a two-cylinder locomotive (cranks 90°)? → **$\pm\sqrt{2}(1 - c)m\omega^2 r$**

80. Rotor I = 10 kg·m², spin 100 rad/s, precession 0.5 rad/s. Gyroscopic couple? → **500 N·m ($I\omega\omega_p$)**

81. Propeller clockwise viewed from rear; aeroplane turns left. Effect? → **Nose rises, tail dips**

82. Natural frequency of a spring–mass system, k = 4000 N/m, m = 10 kg? → **20 rad/s (≈ 3.18 Hz) ($\sqrt{(k/m)}$)**

83. Logarithmic decrement for damping ratio ζ = 0.05? → **≈ 0.314 ($2\pi\zeta/\sqrt{(1 - \zeta^2)}$)**

84. Critical damping coefficient for m = 2 kg, k = 800 N/m? → **80 N·s/m ($2\sqrt{(km)}$)**

85. Magnification factor at resonance for ζ = 0.1? → **5 ($1/2\zeta$)**

15.3 Machine Design (Q 86–125)

86. Margin of safety for a component designed with FOS = 2.5? → **1.5 ($MoS = FOS - 1$)**

87. Per distortion-energy (von Mises) theory, yield strength in shear S_{sy} = ? → **0.577 S_{yt}**

88. K_t for an elliptical hole with semi-axis a (⊥ load) = 2b? → **5 ($K_t = 1 + 2a/b$)**

89. K_t = 3 and K_f = 2.5; notch sensitivity q? → **0.75 ($(K_f - 1)/(K_t - 1)$)**

90. Preferred-number (R) series were proposed by? → **Charles Renard (IS 1076)**

91. Cast-iron designation FG 200 means? → **Grey cast iron, minimum tensile strength 200 N/mm²**

Nandyal Banking Coaching Center

92. Chromium content in alloy steel 40Cr4Mo2? → **1% (4 ÷ multiplying factor 4)**

93. Endurance limit of a polished steel specimen with S_{ut} = 600 MPa? → **≈ 300 MPa ($S_e' = 0.5\ S_{ut}$)**

94. σ_{max} = 200 MPa, σ_{min} = −100 MPa; mean and alternating stress? → **σ_m = 50 MPa, σ_a = 150 MPa**

95. Stress ratio R for repeated (zero-to-maximum) stress? → **0 ($\sigma_{min} = 0$)**

96. The Soderberg line joins which two points? → **S_e on σ_a-axis and S_{yt} on σ_m-axis**

97. Reliability factor for 99% reliability? → **0.814**

98. Fatigue strength of steel at 10^3 cycles is about? → **0.9 S_{ut}**

99. Rivet hole diameter for a 16 mm plate by Unwin's formula? → **24 mm (6√16)**

100. Tearing efficiency of a riveted joint with p = 60 mm, d = 20 mm? → **66.7% ((p − d)/p)**

101. Double parallel fillet weld: s = 10 mm, l = 100 mm, τ = 60 MPa; load capacity? → **≈ 84.8 kN (1.414 s l τ)**

102. Throat thickness of a butt weld is taken as? → **Plate thickness (reinforcement neglected)**

103. Pitch of an M16 coarse-series thread? → **2 mm**

104. Tightening torque for an M20 bolt with initial tension 20 kN? → **80 N·m (T = 0.2 P_i d)**

105. Maximum efficiency of a square-threaded screw when friction angle φ = 30°? → **33.3% ((1 − sin φ)/(1 + sin φ))**

106. Power-screw thread used when load acts in one direction only? → **Buttress thread**

107. Lead of a double-start thread with pitch 5 mm? → **10 mm (lead = starts × pitch)**

108. Shaft with M = 3 kN·m and T = 4 kN·m; equivalent twisting moment? → **5 kN·m (√(M² + T²))**

109. Equivalent bending moment for the same shaft (M = 3, T = 4 kN·m)? → **4 kN·m (½[M + √(M² + T²)])**

110. ASME combined shock-fatigue factor K_b for rotating shaft, gradually applied load? → **1.5 (K_t = 1.0)**

111. Length of sunk key (w = d/4) equally strong as shaft in shear? → **πd/2 ≈ 1.571 d**

112. Bolt pitch-circle diameter of a flange coupling for a 50 mm shaft? → **150 mm (3d)**

113. Permissible angle of twist for machine-tool shafts? → **0.25° per metre length**

114. Ball bearing with C = 40 kN under P = 4 kN; rating life? → **1000 million rev ((C/P)³)**

115. L_{10} = 600 million rev at 1000 rpm; life in hours? → **10,000 h (L × 10⁶/(60n))**

116. Rotation factor V when the outer race rotates? → **1.2 (inner race rotating: 1)**

117. Bearing lubrication that maintains a full film even at zero speed? → **Hydrostatic (externally pressurised oil)**

118. Single-plate clutch (n = 2), R_1 = 150, R_2 = 100 mm, μ = 0.3, W = 4 kN; uniform-wear torque? → **300 N·m (n μ W R_m, R_m = 125 mm)**

119. Usual semi-cone angle of a cone clutch? → **≈ 12.5° (12–15°)**

120. Wahl's stress factor for spring index C = 4? → **≈ 1.40 (15/12 + 0.615/4)**

121. Spring: G = 80 GPa, d = 10 mm, D = 80 mm, 10 active coils; stiffness? → **≈ 19.5 N/mm (Gd⁴/8D³n)**

122. Minimum pinion teeth to avoid interference, 14.5° full-depth involute? → **32**

123. Barth velocity factor for ordinary cut gears at v = 3 m/s? → **0.5 (3/(3 + v))**

124. Thin cylinder: p = 2 MPa, D = 1 m, t = 10 mm; hoop stress? → **100 MPa (pD/2t)**

125. Hoop stress in a cast-iron flywheel rim (ρ = 7200 kg/m³) at 25 m/s? → **4.5 MPa (σ = ρv²)**

15.4 Fluid Mechanics & Fluid Machinery (Q 126–165)

126. 1 poise expressed in SI units? → **0.1 Pa·s (1 cP = 10^{-3} Pa·s)**

127. Kinematic viscosity of an oil with μ = 0.1 Pa·s and S = 0.9? → **1.11×10^{-4} m²/s ≈ 1.11 stoke (ν = μ/ρ)**

128. Viscosity of liquids falls on heating mainly because of? → **Reduction in intermolecular cohesion**

129. Fluid whose apparent viscosity decreases with time of shearing? → **Thixotropic (e.g. printer's ink, ketchup)**

130. Capillary rise of water in a 1 mm glass tube (σ = 0.073 N/m, θ = 0°)? → **≈ 29.8 mm (h = 4σ/ρgd)**

131. Excess pressure inside a 1 mm water droplet (σ = 0.073 N/m)? → **292 Pa (Δp = 4σ/d)**

132. Approximate bulk modulus of water? → **2.1–2.2 GPa**

133. Gauge pressure at 10 m depth in oil of specific gravity 0.8? → **≈ 78.5 kPa (p = ρgh)**

134. Total force on a vertical gate 2 m wide, 3 m deep, top edge at free surface? → **≈ 88.3 kN (ρgA $\bar{x}$, $\bar{x}$ = 1.5 m)**

135. Centre of pressure of a vertical triangle (height d), apex at free surface? → **3d/4 below free surface**

136. Vertical component of hydrostatic force on a curved surface equals? → **Weight of liquid (real or imaginary) above the surface**

137. A floating body is in neutral equilibrium when? → **Metacentre coincides with centre of gravity (GM = 0)**

138. Streamlines, pathlines and streaklines coincide in which flow? → **Steady flow**

139. Stream function ψ exists for? → **2-D incompressible flow (rotational or irrotational)**

140. Momentum correction factor β for laminar flow in a circular pipe? → **4/3**

141. Pitot tube shows 0.2 m head of water (C_v = 1); velocity? → **≈ 1.98 m/s (V = √(2gh))**

Nandyal Banking Coaching Center

142. C_d of an external cylindrical mouthpiece? → **0.855**

143. Force of a 50 mm water jet at 20 m/s on a fixed plate normal to it? → **≈ 785 N ($F = \rho A V^2$)**

144. Darcy friction factor for laminar pipe flow at Re = 1600? → **0.04 ($f = 64/Re$)**

145. V_{max}/V_{mean} for laminar flow between fixed parallel plates? → **1.5**

146. Laminar flow, same Δp and length: pipe diameter halved, discharge becomes? → **1/16 ($Q \propto D^4$)**

147. Head loss at sudden enlargement, V_1 = 4 m/s, V_2 = 1 m/s? → **≈ 0.46 m ($(V_1 - V_2)^2/2g$)**

148. Head loss at pipe exit into a reservoir? → **$V^2/2g$ (full velocity head lost)**

149. Water hammer rise, sudden closure, rigid pipe, V = 2 m/s, C = 1400 m/s? → **2.8 MPa ($p = \rho VC$)**

150. Shape factor H = δ^*/θ for a laminar (Blasius) boundary layer? → **≈ 2.59**

151. Drag on a small sphere by Stokes' law? → **$F_D = 3\pi\mu DV$ (valid for Re < ≈ 0.2–1)**

152. Froude model scale 1:25, model velocity 2 m/s; prototype velocity? → **10 m/s ($V_r = \sqrt{L_r}$)**

153. Mach angle for a projectile at Ma = 2? → **30° ($\sin\alpha = 1/Ma$)**

154. Strouhal number is used to study? → **Vortex-shedding frequency (Kármán vortex street)**

155. Number of Pelton buckets for jet ratio m = 12 (Tygun)? → **21 ($Z = 15 + m/2$)**

156. Pelton jet velocity for net head 500 m, C_v = 0.98? → **≈ 97 m/s ($C_v\sqrt{(2gH)}$)**

157. Turbine: N = 300 rpm, P = 10,000 kW, H = 100 m; specific speed and type? → **N_s ≈ 95 → Francis ($N\sqrt{P}/H^{5/4}$)**

158. Same turbine, head made 4 times: speed and power become? → **Speed 2×, power 8× ($N \propto \sqrt{H}$, $P \propto H^{3/2}$)**

159. Turbine with the best part-load efficiency? → **Kaplan (double-regulated: guide vanes + runner blades)**

160. Pump speed increased by 10%; shaft power increases by about? → **33% ($P \propto N^3$; $1.1^3 = 1.331$)**

161. Pump: N = 1450 rpm, Q = 0.1 m^3/s, H = 16 m; specific speed? → **≈ 57, radial flow ($N\sqrt{Q}/H^{3/4}$)**

162. $NPSH_A$: p_{atm} head 10.3 m, p_v head 0.24 m, suction lift 4 m, h_{fs} 0.5 m? → **5.56 m**

163. Single-acting pump: bore 200 mm, stroke 300 mm, 60 rpm; theoretical discharge? → **≈ 9.42 L/s ($Q = ALN/60$)**

164. Hydraulic press: ram 200 mm, plunger 20 mm dia, 100 N on plunger; load lifted? → **10 kN ($W = F \cdot A/a$, ratio 100)**

165. Hydraulic power delivered at 10 MPa and 60 L/min? → **10 kW ($P = pQ$, $Q = 0.001\ m^3/s$)**

15.5 Thermodynamics & Power Cycles (Q 166–210)

166. Efficiency of a Carnot engine working between 500 K and 300 K? → **40% (1 − 300/500)**

167. COP of a Carnot refrigerator between 250 K and 300 K? → **5 ($T_L/(T_H - T_L) = 250/50$)**

168. COP of a reversible heat pump between 27 °C and −3 °C? → **10 ($T_H/\Delta T = 300/30$)**

169. SI unit of entropy? → **kJ/K (specific entropy kJ/kg·K)**

170. Value of the universal gas constant? → **8.314 kJ/kmol·K**

171. Characteristic gas constant R of air? → **0.287 kJ/kg·K ($c_p - c_v$ = 1.005 − 0.718)**

172. Ratio of specific heats γ for a monatomic gas? → **1.67 ($\gamma = 1 + 2/f$, f = 3)**

173. Triple point of water (temperature and pressure)? → **0.01 °C (273.16 K) and 0.611 kPa**

174. Critical temperature of water? → **374.15 °C (≈ 374 °C; critical pressure 221.2 bar)**

175. Latent heat of vaporisation of water at 1 atm (100 °C)? → **2257 kJ/kg**

176. Mollier chart is a plot of? → **Enthalpy (h) versus entropy (s)**

177. Slope of a constant-pressure line on the h–s diagram equals? → **Absolute temperature T ($(\partial h/\partial s)_p = T$)**

178. A PMM of the second kind violates? → **Kelvin–Planck statement of the second law**

179. Clausius inequality for an irreversible cycle? → **$\oint \delta Q/T < 0$**

180. Change in internal energy in free expansion of an ideal gas? → **Zero (Q = 0, W = 0; T unchanged)**

181. In which process is heat transferred equal to change in enthalpy? → **Constant-pressure process ($Q = \Delta H$)**

182. Heat needed to raise 2 kg of air by 10 K at constant volume? → **14.36 kJ (2 × 0.718 × 10)**

183. Air-standard Otto efficiency for r = 4, γ = 1.4? → **≈ 42.6% ($1 - 1/4^{0.4}$)**

184. Nozzle exit velocity for isentropic enthalpy drop 200 kJ/kg (inlet velocity negligible)? → **≈ 632 m/s ($44.72\sqrt{200}$)**

185. Enthalpy of wet steam at 1 atm, x = 0.9 (h_f = 419, h_{fg} = 2257)? → **≈ 2450 kJ/kg (419 + 0.9 × 2257)**

186. Specific steam consumption if net work = 900 kJ/kg? → **4 kg/kWh (3600/900)**

187. Heat rate of a power plant of 40% thermal efficiency? → **9000 kJ/kWh (3600/0.40)**

188. Compression ratio with swept volume 500 cc and clearance volume 50 cc? → **11 ((500 + 50)/50)**

189. For the same compression ratio, Diesel cycle efficiency vs Otto? → **Lower (cut-off ratio factor > 1)**

190. Ideal Brayton cycle efficiency depends on? → **Pressure ratio only (for given γ)**

Nandyal Banking Coaching Center

191. Ericsson cycle consists of? → **Two isothermal and two constant-pressure processes (regenerative)**

192. Distinguishing feature of the Atkinson cycle? → **Expansion ratio greater than compression ratio**

193. Pump work in a Rankine cycle is given by? → **$v_f(p_1 - p_2)$, i.e. $-\int v\,dp$**

194. Effect of lowering condenser pressure on dryness at turbine exhaust? → **Dryness decreases (moisture increases)**

195. An open (direct-contact) feed-water heater also serves as a? → **Deaerator (removes dissolved O_2, CO_2)**

196. Overall efficiency of topping (40%) and bottoming (30%) cycles combined? → **58% ($\eta_1 + \eta_2 - \eta_1\eta_2$)**

197. Cochran boiler is a? → **Vertical, multitubular, internally fired fire-tube boiler**

198. Number of flue tubes in a Lancashire boiler? → **Two (Cornish boiler has one)**

199. Function of a fusible plug? → **Mounting; melts on low water level, extinguishes furnace fire**

200. Last heat-recovery equipment in the flue-gas path of a utility boiler? → **Air preheater (after economiser)**

201. Boiler draught is usually measured in? → **mm of water column (U-tube manometer)**

202. Boiler in which superheated steam evaporates the feed water? → **Loeffler boiler**

203. Critical pressure ratio for air ($\gamma = 1.4$)? → **0.528 ($[2/(\gamma+1)]^{\gamma/(\gamma-1)}$)**

204. Mass flow through a choked convergent nozzle as back pressure is lowered further? → **Remains constant (maximum)**

205. Effect of supersaturated (metastable) flow on nozzle mass flow? → **Increases (about 2–5%)**

206. Optimum blade speed ratio u/V_1 for a 50% reaction (Parsons) stage? → **cos α**

207. Work division between rows of a two-row Curtis stage (optimum)? → **3 : 1 (first : second row)**

208. Propulsive efficiency when jet velocity is twice flight velocity? → **66.7% ($2u/(V_j + u) = 2/3$)**

209. Optimum intermediate pressure for two-stage compression from 1 bar to 16 bar (perfect intercooling)? → **4 bar ($\sqrt{(1 \times 16)}$)**

210. Constant b in van der Waals equation accounts for? → **Co-volume – finite volume of molecules**

15.6 I.C. Engines & Air Compressors (Q 211–245)

211. Air-standard Otto efficiency for $r = 8$, $\gamma = 1.4$? → **≈ 56.5% ($1 - 1/8^{0.4}$)**

212. For the same compression ratio and heat input, the most efficient cycle is? → **Otto (Otto > Dual > Diesel)**

213. Swept volume of a cylinder with 100 mm bore and 120 mm stroke? → **≈ 942 cc ($(\pi/4)D^2L$)**

214. Compression ratio if swept volume = 900 cc and clearance volume = 100 cc? → **10 ((900 + 100)/100)**

215. Mean piston speed for 100 mm stroke at 3000 rpm? → **10 m/s (2LN/60)**

216. An over-square (short-stroke) engine has L/D ratio? → **Less than 1 (high-speed petrol engines)**

217. Firing interval of a 4-stroke, 8-cylinder engine? → **90° (720°/8)**

218. Brake power at torque 200 N·m and 1500 rpm? → **≈ 31.4 kW ($2\pi NT/60000$)**

219. IP = 50 kW, BP = 40 kW. Mechanical efficiency? → **80% (BP/IP); FP = 10 kW**

220. Brake thermal efficiency for BSFC 0.25 kg/kWh and CV 42 MJ/kg? → **≈ 34.3% (3600/(0.25 × 42000))**

221. Morse test: BP with all cylinders 40 kW, with cylinder 1 cut 30 kW. IP of cylinder 1? → **10 kW (40 − 30)**

222. Volumetric efficiency can exceed 100% in? → **Supercharged / turbocharged engines**

223. Equivalence ratio $\varphi = 1.25$ indicates? → **Rich mixture; $\lambda = 1/\varphi = 0.8$**

224. Octane number of a fuel with Performance Number 130? → **110 (100 + (PN − 100)/3)**

225. Which is higher for a given petrol – RON or MON? → **RON (milder test, 600 rpm vs 900 rpm)**

226. A common cetane improver for diesel? → **Amyl nitrate (or ethyl nitrate)**

227. Cold-starting aid needed in indirect-injection (IDI) diesel engines? → **Glow plug**

228. In a simple carburettor, as engine speed increases the mixture becomes? → **Richer; corrected by compensating jet**

229. In a Bosch jerk-type pump, fuel quantity is varied by? → **Rotating plunger (helix) via rack – changes effective stroke**

230. Ignition system whose spark intensity improves with engine speed? → **Magneto ignition**

231. Role of rhodium in a three-way catalytic converter? → **Reduction of NO_x to N_2**

232. Urea concentration in AdBlue/DEF used for SCR? → **32.5% urea in water**

233. BS-VI Phase 2, effective 1 April 2023, added? → **Real Driving Emissions (RDE) testing and stricter OBD-II**

234. Flammability limits of hydrogen in air (by volume)? → **≈ 4–75%**

235. Hit-and-miss governing is used in? → **Small gas engines (old stationary engines)**

236. DG-set engine speed for a 6-pole, 50 Hz alternator? → **1000 rpm (120f/P)**

237. Ideal intermediate pressure for two-stage compression from 1 bar to 25 bar? → **5 bar ($\sqrt{(1 \times 25)}$)**

238. Pressure ratio per stage, three-stage compression from 1 to 27 bar? → **3 ($(27)^{1/3}$)**

Nandyal Banking Coaching Center

239. Volumetric efficiency: clearance ratio 0.04, pressure ratio 6, n = 1? → **80% (1 + C − C × 6)**

240. Limiting pressure ratio (zero delivery) for C = 0.05, isothermal re-expansion? → **21 ((1 + C)/C)**

241. Isentropic delivery temperature: T_1 = 300 K, pressure ratio 8, γ = 1.4? → **≈ 543 K (300 × $8^{0.286}$)**

242. Typical slip factor of a centrifugal compressor? → **≈ 0.9 (Stanitz: 1 − 0.63π/Z)**

243. Leak test: on-load 20 s, unload 80 s. Leakage? → **20% of FAD (T/(T + t))**

244. Pressure dew point of a refrigerated air dryer? → **≈ +3 °C (desiccant dryer ≈ −40 °C)**

245. Power saving per 1 bar reduction in compressor delivery pressure (BEE)? → **≈ 6–10%**

15.7 Refrigeration, Air Conditioning & Psychrometry (Q 246–280)

246. A plant removes heat from a cold space at 35.17 kW. Its capacity in TR? → **10 TR (35.17 ÷ 3.517)**

247. COP of a Carnot refrigerator working between −3 °C and 27 °C? → **9 (270/(300 − 270))**

248. A heat pump has COP 4. COP of a refrigerator on the same cycle and temperatures? → **3 ($COP_R = COP_{HP} − 1$)**

249. Power consumption of a chiller with COP = 5, in kW/TR? → **≈ 0.70 kW/TR (3.517 ÷ 5)**

250. Ideal COP of a Bell-Coleman cycle with pressure ratio 4, γ = 1.4? → **≈ 2.06 (1/($4^{0.286}$ − 1))**

251. Parameter used to compare different aircraft refrigeration systems? → **DART (Dry Air Rated Temperature at turbine exit)**

252. VCR: h_1 = 1450, h_2 = 1650, $h_3 = h_4$ = 350 kJ/kg. COP? → **5.5 (1100/200)**

253. For the cycle above, refrigerant mass flow per TR? → **≈ 0.192 kg/min (211 ÷ 1100)**

254. Volumetric efficiency: clearance 5%, pressure ratio 8, n = 1.5? → **0.85 (1 + 0.05 − 0.05 × $8^{1/1.5}$)**

255. Optimum intercooler pressure for two-stage compression between 2 bar and 18 bar? → **6 bar (√(2 × 18))**

256. Effect of lowering evaporator temperature on capacity of a given compressor? → **Decreases (suction specific volume rises, mass flow falls)**

257. ASHRAE number of $CHCl_2CF_3$? → **R-123 (C − 1 = 1, H + 1 = 2, F = 3)**

258. Refrigerant designated R-718? → **Water (700 + molecular mass 18)**

259. ASHRAE safety group of ammonia? → **B2L (higher toxicity, mildly flammable)**

260. Critical temperature of CO_2, which forces transcritical operation with a gas cooler? → **31.1 °C (at 73.8 bar)**

261. A zeotropic blend such as R-407C must be charged in which phase? → **Liquid phase (avoids fractionation due to glide)**

262. Halide torch flame colour indicating a small halocarbon leak? → **Green (purple for large leak; not for HFCs)**

263. Montreal Protocol was signed in which year? → **1987 (16 Sept, World Ozone Day)**

264. By when must developing (Article 5) countries cut HCFC use by 97.5%? → **2030 (2.5% servicing tail up to 2040)**

265. Reduction in cooling demand targeted by ICAP by 2037-38? → **20–25%**

266. Main operating problem peculiar to LiBr–water absorption chillers? → **Crystallisation of LiBr solution**

267. Maximum COP of absorption system: T_g = 360 K, T_o = 300 K, T_e = 270 K? → **1.5 ((60/360) × (270/30))**

268. Figure of merit of a thermoelectric material? → **$Z = \alpha^2/(\rho k)$**

269. Expansion valve that maintains constant evaporator pressure? → **Automatic expansion valve (AEV)**

270. Cooling tower: water 40 °C → 32 °C, ambient WBT 28 °C. Effectiveness? → **≈ 66.7% (range 8/(range 8 + approach 4))**

271. Valveless, quiet positive-displacement compressor common in split/packaged AC? → **Scroll compressor**

272. Specific humidity when p_v = 3 kPa at 101.325 kPa? → **≈ 0.019 kg/kg d.a. (0.622 × 3/98.325)**

273. Enthalpy of moist air at 30 °C DBT, ω = 0.015 kg/kg d.a.? → **≈ 68.5 kJ/kg d.a. (1.005t + ω(2500 + 1.88t))**

274. Sensible heat to warm 100 m³/min of air by 10 °C? → **≈ 20.4 kW (0.0204 × cmm × Δt)**

275. Bypass factor of a two-row coil, each row having BPF 0.3? → **0.09 ($x^n = 0.3^2$)**

276. Air at 40 °C DBT, 24 °C WBT through an 80%-efficient evaporative cooler. Exit DBT? → **27.2 °C (40 − 0.8 × 16)**

277. Room sensible load 30 kW, latent load 10 kW. RSHF? → **0.75 (30/40)**

278. Value of 1 met (metabolic rate unit)? → **58.2 W/m² of body surface**

279. Fan speed raised by 20% (same fan): increase in power? → **≈ 72.8% (1.2^3 = 1.728)**

280. Equivalent diameter (same velocity) of a 400 mm × 200 mm duct? → **≈ 267 mm (2ab/(a + b))**

15.8 Heat Transfer (Q 281–315)

281. Heat flow through a 0.2 m thick wall, k = 1 W/m·K, A = 1 m², ΔT = 100 K? → **500 W (Q = kAΔT/L)**

282. Thermal diffusivity of a solid with k = 40 W/m·K, ρ = 8000 kg/m³, c = 500 J/kg·K? → **1 × 10^{-5} m²/s (α = k/ρc)**

283. Dimensions of thermal conductivity? → **$MLT^{-3}\theta^{-1}$**

284. Which metal has the highest thermal conductivity? → **Silver (≈ 429 W/m·K), then copper**

Nandyal Banking Coaching Center

285. Effect of absorbed moisture on thermal conductivity of insulation? → **Increases k (water k ≈ 23× air)**

286. Critical radius of insulation on a pipe, k = 0.1 W/m·K, h = 10 W/m²K? → **10 mm ($r_c = k/h$)**

287. Critical radius for a sphere, k = 0.04 W/m·K, h = 8 W/m²K? → **10 mm ($r_c = 2k/h$)**

288. Plane wall (half-thickness 0.1 m, k = 50) generating 10^5 W/m³; $T_{max} - T_s$? → **10 K ($q_g L^2/2k$)**

289. Fin parameter m for a pin fin, h = 100, k = 200 W/m·K, D = 10 mm? → **≈ 14.1 m^{-1} ($m = \sqrt{(4h/kD)} = \sqrt{200}$)**

290. Corrected length of a pin fin with convective tip? → **$L_c = L + D/4$**

291. Efficiency of an insulated-tip fin with mL = 1? → **≈ 76% (tanh 1/1 = 0.762)**

292. Minimum fin effectiveness that justifies using fins (Cengel)? → **$\varepsilon_f \geq 2$**

293. Characteristic length (V/A_s) of a cube of side 6 cm? → **1 cm (a/6)**

294. Steel ball D = 12 mm, k = 50 W/m·K, h = 50 W/m²K: Biot number? → **0.002 ($L_c = D/6$ = 2 mm); lumped valid**

295. In lumped cooling, time for $(T - T_\infty)$ to fall to 13.5% of initial? → **2τ ($e^{-2} = 0.135$)**

296. Error function solution erf[x/2√(αt)] applies to which body? → **Semi-infinite solid (sudden surface temperature change)**

297. Why does metal feel colder than wood at the same temperature? → **Higher thermal effusivity √(kρc)**

298. In Nusselt number hL/k, k refers to? → **Fluid (k of solid used in Biot number)**

299. Dittus–Boelter exponent of Pr when the fluid is being heated? → **0.4 (0.3 for cooling)**

300. Turbulent tube flow: velocity doubled, h increases by factor? → **≈ 1.74 ($h \propto V^{0.8}$)**

301. Laminar fully developed tube flow, q = const, k = 0.6 W/m·K, D = 20 mm; h? → **≈ 131 W/m²K (4.36 × 0.6/0.02)**

302. Hydraulic diameter of a 4 cm × 2 cm rectangular duct? → **≈ 2.67 cm (4A/P = 32/12)**

303. Critical Reynolds number for flow over a flat plate? → **5×10^5**

304. Laminar flat plate, Pr = 8: ratio δ_t/δ? → **0.5 ($Pr^{-1/3}$)**

305. Volume expansion coefficient β of an ideal gas? → **1/T (T absolute, K)**

306. In turbulent free convection ($Nu \propto Ra^{1/3}$), h is independent of? → **Characteristic length**

307. In nucleate boiling (Rohsenow), heat flux varies with excess temperature as? → **$q \propto \Delta T_e^{\ 3}$**

308. Condensation h on horizontal and vertical tubes is equal when L = ? → **≈ 2.87 D (horizontal preferred otherwise)**

309. Wavelength of maximum emission of a black body at 1000 K? → **≈ 2.9 μm (2898/1000)**

310. Emissive power of a black body at 1000 K? → **56.7 kW/m² (σT^4)**

311. Concentric spheres r_1 = 1 m (inner), r_2 = 2 m: view factor F_{21}? → **0.25 ($A_1/A_2 = (r_1/r_2)^2$)**

312. Number of equal-ε radiation shields to cut heat exchange to 1/4? → **3 (Q/(n + 1))**

313. LMTD when terminal temperature differences are 60 °C and 20 °C? → **≈ 36.4 °C (40/ln 3)**

314. Counter-flow exchanger, C_{min}/C_{max} = 1, NTU = 1: effectiveness? → **0.5 (NTU/(1 + NTU))**

315. Minimum area density for a compact heat exchanger? → **≈ 700 m²/m³ (car radiator ≈ 1000)**

15.9 Engineering Materials & Heat Treatment (Q 316–350)

316. Relation between lattice parameter a and atomic radius r in FCC? → **$a = 2\sqrt{2}\,r$ (atoms touch along face diagonal, $4r = \sqrt{2}\,a$)**

317. Atomic packing factor of a simple cubic structure? → **0.52 (π/6)**

318. Number of octahedral and tetrahedral voids per FCC unit cell? → **4 octahedral + 8 tetrahedral**

319. Miller indices of a plane with intercepts 1, 2, ∞? → **(210) – reciprocals 1, ½, 0, clear fractions**

320. Interplanar spacing d_{111} of a cubic crystal with a = 0.36 nm? → **≈ 0.208 nm (a/√3)**

321. Resolved shear stress for σ = 100 MPa, φ = 60°, λ = 45°? → **≈ 35.4 MPa ($\sigma \cos\varphi \cos\lambda = 100 \times 0.5 \times 0.707$)**

322. Effect of a Frenkel defect on density of the crystal? → **No change (vacancy–interstitial pair, no atom lost)**

323. Theoretical density of Cu (FCC, a = 0.3615 nm, A = 63.5)? → **≈ 8.93 g/cm³ ($\rho = nA/(a^3 N_A)$, n = 4)**

324. True strain corresponding to an engineering strain of 0.2? → **≈ 0.182 (ln 1.2)**

325. Approximate UTS of a steel with BHN 200? → **≈ 690 MPa (3.45 × BHN)**

326. Vickers hardness for P = 30 kgf and mean diagonal 0.5 mm? → **≈ 222 HV ($1.854 \times 30/0.5^2$)**

327. Microhardness test using an elongated diamond pyramid with 7 : 1 diagonals? → **Knoop hardness test**

328. Standard Charpy V-notch specimen size? → **10 mm × 10 mm × 55 mm**

329. Which crystal structure shows no ductile–brittle transition? → **FCC (Al, Cu, austenitic SS) – used for cryogenic service**

330. Most conservative mean-stress fatigue criterion? → **Soderberg line (uses yield strength)**

331. Usual value of constant C in the Larson–Miller parameter? → **≈ 20 (T in K, rupture time in hours)**

332. Degrees of freedom in a two-phase region of a binary alloy at constant pressure? → **1 (F = C − P + 1 = 2 − 2 + 1)**

Nandyal Banking Coaching Center

333. Eutectic temperature and composition of the Pb–Sn system? → **183 °C, 61.9 % Sn**

334. Peritectic reaction in the Fe–Fe_3C diagram? → **δ + L → γ at 1495 °C, 0.17 % C**

335. Pro-eutectoid cementite in a 1.2 % C steel (lever rule)? → **≈ 6.8 % ((1.2 − 0.8)/(6.67 − 0.8))**

336. Curie temperature (A_2) of ferrite? → **768 °C – magnetic change, no structural change**

337. Maximum solubility of carbon in ferrite? → **0.022 % at 727 °C**

338. Approximate M_s temperature of eutectoid plain carbon steel? → **≈ 220 °C**

339. Product of medium tempering (350–500 °C) of hardened steel? → **Troostite – used for springs**

340. Standard Jominy end-quench specimen size? → **25 mm dia × 100 mm long; one end water-quenched**

341. Composition of Hadfield manganese steel? → **11–14 % Mn, 1–1.4 % C; austenitic, work-hardening**

342. Austenitic stainless steel grade stabilised with titanium? → **AISI 321 (Nb-stabilised: 347)**

343. Nickel content of IS steel 40Ni14? → **3.5 % Ni (14 ÷ 4 multiplying factor)**

344. Composition of gun metal? → **88 % Cu, 10 % Sn, 2 % Zn**

345. Approximate composition of duralumin? → **Al + ~4 % Cu, 0.5 % Mg, 0.5 % Mn**

346. Element used to vulcanise natural rubber? → **Sulphur (cross-links cis-polyisoprene chains)**

347. Longitudinal modulus: E_f = 70 GPa, E_m = 3 GPa, V_f = 0.6? → **43.2 GPa ($E_fV_f + E_mV_m$)**

348. Pilling–Bedworth ratio range for a protective oxide film? → **1 to 2 (e.g. Al, Cr, Ni, Ti)**

349. Coating on food-can steel that is cathodic to steel? → **Tin – if scratched, steel corrodes faster**

350. Grossman's ideal critical diameter D_I is the bar diameter giving? → **50 % martensite at centre in an ideal quench (H = ∞)**

15.10 Casting, Welding & Fabrication (Q 351–385)

351. Pattern length for a 500 mm grey CI casting, shrinkage allowance 10 mm/m? → **505 mm (500 + 0.01 × 500)**

352. Colour used on a pattern to mark core prints? → **Yellow**

353. Permeability number of standard AFS specimen: p = 10 g/cm², t = 1 min? → **≈ 50 (P_N = 501.2/(p·t))**

354. Casting modulus (V/A) of a sphere of 60 mm diameter? → **10 mm (V/A = D/6)**

355. A 100 mm cube solidifies in 10 min; time for a 200 mm cube (same conditions)? → **40 min (t ∝ (V/A)², V/A doubles)**

356. Velocity of metal at sprue base for a 20 cm effective head? → **≈ 1.98 m/s (v = √(2gh))**

357. Sprue: head 4 cm at top, 16 cm at bottom, bottom area 3 cm²; top area? → **6 cm² ($A_t/A_b = \sqrt{(h_b/h_t)} = 2$)**

358. In a pressurised gating system, the choke (minimum area) is at? → **The ingates**

359. Freezing ratio X in Caine's riser method is defined as? → **$(A/V)_{casting} \div (A/V)_{riser}$**

360. Metal inserts in the mould wall used to promote directional solidification? → **External chills**

361. Zone of fine equiaxed grains adjacent to the mould wall in an ingot? → **Chill zone**

362. Die-casting machine using a gooseneck immersed in molten metal? → **Hot-chamber die casting (Zn, Sn, Pb, Mg)**

363. Speed of a 400 mm dia horizontal centrifugal mould for G-factor 60? → **≈ 518 rpm ($\omega = \sqrt{(60g/R)} \approx 54.2$ rad/s)**

364. Smooth rounded cavities near casting surface from excess moisture/low permeability? → **Blow holes**

365. Pattern material in full-mould (lost-foam) casting? → **Expanded polystyrene (EPS) – vaporised by metal**

366. Ideal extrusion pressure: σ_f = 100 MPa, extrusion ratio 7.39? → **200 MPa ($p = \sigma_f \ln R$, ln 7.39 = 2)**

367. Heat input: η = 0.8, 20 V, 200 A, travel speed 5 mm/s? → **640 J/mm ($H = \eta VI/v$)**

368. Welding set rated 300 A at 60% duty cycle; current at 100% duty cycle? → **≈ 232 A ($I_2 = I_1\sqrt{(D_1/D_2)}$)**

369. Tungsten electrode commonly used for DC TIG welding of steel? → **2% thoriated tungsten (pure W for AC)**

370. Helium vs argon as TIG shielding gas – effect of helium? → **Hotter arc, higher voltage, deeper penetration**

371. Heat source in electroslag welding after start-up? → **Resistance (I^2R) heating of molten slag**

372. Coating type of an E7018 electrode? → **Basic low-hydrogen, iron powder (bake before use)**

373. Fraction of DC arc heat liberated at the anode (textbook value)? → **About two-thirds**

374. Minimum bend radius for sheet of thickness t with 25% reduction of area? → **R = t (R/t = 50/r − 1 = 1)**

375. Total O_2 volume for complete combustion of 1 volume C_2H_2? → **2.5 volumes (1 cylinder + 1.5 from air)**

376. Approximate ignition (kindling) temperature of steel in oxy-fuel cutting? → **≈ 870 °C (800–900 °C)**

377. Spot weld: I = 10 kA, R = 100 μΩ, t = 0.1 s; heat generated? → **1000 J ($H = I^2Rt$)**

378. Friction stir welding was invented by? → **TWI (The Welding Institute), UK, 1991**

379. Steel with 0.20% C, 0.90% Mn (no other alloys); IIW CE and preheat? → **CE = 0.35; no preheat needed (< 0.4)**

380. Weld symbol placed below the reference line indicates weld on? → **Arrow side**

381. Typical joint clearance for capillary flow in brazing? → **≈ 0.025–0.125 mm**

Nandyal Banking Coaching Center

382. Composition and melting range of SAC305 lead-free solder? → **Sn–3.0Ag–0.5Cu; ≈ 217–220 °C**

383. Angle of bite: roll radius 250 mm, draft 5 mm? → **≈ 8.1° (cos α = 1 − Δh/2R = 0.99)**

384. Punch diameter to blank a 50 mm disc, clearance 0.1 mm per side? → **49.8 mm (die = 50 mm, punch = blank − 2c)**

385. Blank diameter for a cylindrical cup d = 50 mm, h = 30 mm (sharp corner)? → **≈ 92.2 mm (D = √(d² + 4dh))**

15.11 Machining, Metal Cutting & CAM (Q 386–430)

386. Orthogonal cutting: chip thickness ratio r = 0.5, rake angle α = 10°. Shear angle? → **≈ 28.3° (tan φ = r cos α/(1 − r sin α) = 0.539)**

387. Uncut chip thickness 0.2 mm, chip thickness 0.5 mm. Chip reduction coefficient? → **2.5 (ζ = t_2/t_1 = 1/r)**

388. Merchant's theory: α = 10°, friction angle β = 40°. Shear angle? → **30° (φ = 45° + α/2 − β/2)**

389. Lee & Shaffer theory: α = 10°, β = 40°. Shear angle? → **15° (φ = 45° + α − β)**

390. Shear strain when φ = 45° and α = 0°? → **2 (γ = cot φ + tan(φ − α) = 1 + 1)**

391. Cutting force F_c = 1200 N, cutting speed 120 m/min. Cutting power? → **2.4 kW (P = F_cV = 1200 × 2 m/s)**

392. F_c = 1000 N, width of cut 2 mm, uncut thickness 0.2 mm. Specific cutting energy? → **2.5 J/mm³ (F_c/bt_1 = 2500 N/mm²)**

393. Cutting speed 60 m/min, chip thickness ratio 0.4. Chip velocity? → **24 m/min (V_c = rV)**

394. Turning with feed 0.3 mm/rev and approach (principal cutting-edge) angle 60°. Uncut chip thickness? → **≈ 0.26 mm (t_1 = f sin κ)**

395. Location of maximum temperature on a cutting tool? → **Rake face, slightly behind the cutting edge (crater zone)**

396. Type of chip formed when machining grey cast iron? → **Discontinuous (segmental) chip**

397. Tool material preferred for hard turning of steels above 45 HRC? → **CBN (cubic boron nitride)**

398. Composition of 18-4-1 high-speed steel? → **18% W, 4% Cr, 1% V**

399. ISO carbide application group (colour) for cast iron? → **K group (red)**

400. ASA side cutting-edge angle is 15°. ORS principal cutting-edge (approach) angle? → **75° (φ = 90° − C_s)**

401. Taylor's equation $V T^{0.25}$ = 200. Tool life at V = 100 m/min? → **16 min ($T = (200/100)^4$)**

402. Tool life 81 min at 60 m/min (n = 0.5). Tool life at 90 m/min? → **36 min ($T_2 = 81 \times (60/90)^2$)**

403. Tool life 10 min at 100 m/min and 40 min at 50 m/min. Taylor exponent n? → **0.5 ($2 = 4^n$)**

404. ISO 3685 tool-life criterion for average (uniform) flank wear? → **VB = 0.3 mm**

405. Feed 0.4 mm/rev, nose radius 1.0 mm. Ideal R_a? → **5 µm ($R_a \approx f^2/32r$ = 0.16/32 mm)**

406. n = 0.25, tool-change time 2 min, tool cost ₹6/edge, machine cost ₹2/min. Minimum-cost tool life? → **15 min ($T = (1/n - 1)(C_m t_c + C_t)/C_m$)**

407. n = 0.2, tool-change time 1.5 min. Tool life for maximum production rate? → **6 min ($T = (1/n - 1)t_c = 4 \times 1.5$)**

408. Bar of 100 mm diameter turned at 300 rpm. Cutting speed? → **≈ 94.2 m/min (V = πDN/1000)**

409. Turning: feed 0.2 mm/rev, depth 2 mm, V = 100 m/min. MRR? → **40,000 mm³/min (MRR = f d V)**

410. Length 200 mm turned at 400 rpm with feed 0.25 mm/rev. Time per pass? → **2 min (L/fN = 200/100)**

411. Taper D = 50 mm, d = 40 mm, taper length 100 mm, job length 200 mm. Tailstock set-over? → **10 mm (s = L(D − d)/2l)**

412. Cut 2 mm pitch thread with 6 mm lead screw. Change gears (driver/driven)? → **20T/60T (ratio = 2/6 = 1/3)**

413. Drilling a 10 mm hole at feed 0.2 mm/rev, 500 rpm. MRR? → **≈ 7854 mm³/min ($(\pi D^2/4)$ f N)**

414. Milling cutter with 10 teeth, 200 rpm, feed 0.1 mm/tooth. Table feed? → **200 mm/min ($f_m = f_z$ Z N)**

415. Slab milling: f_z = 0.1 mm, depth 4 mm, cutter Ø 100 mm. Max undeformed chip thickness? → **0.04 mm ($2f_z\sqrt{(d/D)} = 2 \times 0.1 \times 0.2$)**

416. Simple indexing for 24 divisions? → **1⅔ turns: 1 turn + 10 holes on 15-hole circle (40/N)**

417. Crank movement for angular indexing of 27°? → **3 turns (θ/9°)**

418. Shaper crank turns 120° during the return stroke. Quick-return ratio? → **2 ((360° − 120°)/120°)**

419. In grinding-wheel marking A 46 K 5 V, the number 46 denotes? → **Grain (grit) size – medium**

420. Glazing of a grinding wheel is mainly caused by? → **Wheel grade too hard (dull grains not shed)**

421. Centreless through-feed: regulating wheel Ø 250 mm, 30 rpm, inclined 5°. Axial feed? → **≈ 2.05 m/min (πdN sin θ)**

422. Roughness grade N7 corresponds to R_a of? → **1.6 µm**

423. ECM of iron (A = 56, Z = 2, ρ = 7.8 g/cm³) at 1000 A. Volumetric MRR? → **≈ 37 mm³/s (IA/(ZFρ), F = 96,500 C)**

424. Typical spark gap in EDM? → **≈ 0.025 mm (0.01–0.05 mm)**

425. Non-traditional process with the highest MRR? → **Plasma arc machining (≈ 75,000 mm³/min)**

426. In 3-2-1 location, DOF restricted by the three locators on the primary surface? → **5 DOF**

427. Stepper motor 1.8°/step drives a 5 mm lead screw directly. BLU? → **0.025 mm (5/200 steps)**

428. Degree of a Bezier curve defined by 4 control points? → **3 – cubic (degree = n − 1)**

Nandyal Banking Coaching Center

429. Minimum DOF a robot needs to reach any position with any orientation? → **6 (3 arm + 3 wrist: roll, pitch, yaw)**

430. AM category used to print sand moulds and cores for casting? → **Binder jetting**

15.12 Metrology, Inspection & Maintenance (Q 431–455)

431. The fundamental deviation (letter symbol) fixes what of the tolerance zone? → **Its position relative to the zero line**

432. Ø25 H7 (0/+21 µm) with g6 (−7/−20 µm) shaft – maximum clearance? → **0.041 mm (UL hole − LL shaft = 21 + 20 µm)**

433. Tolerance of grade IT7 in terms of standard tolerance unit i? → **16i**

434. The GO end of a plug gauge is made to which limit of the hole? → **Lower limit (maximum material condition)**

435. Vernier: 19 main-scale divisions of 1 mm = 20 vernier divisions. Least count? → **0.05 mm (1 MSD − 1 VSD = 1/20)**

436. Which instrument obeys Abbe's principle – micrometer or vernier caliper? → **Micrometer (vernier caliper violates it)**

437. 200 mm sine bar set with 34.73 mm slip gauges – angle? → **10° (sin θ = 34.73/200 = 0.1736)**

438. Least count of a vernier universal bevel protractor? → **5 minutes (1/12°)**

439. In an autocollimator, a reflector tilt of θ deflects the returning beam by? → **2θ**

440. Best wire size for an M10 × 1.5 metric thread? → **≈ 0.866 mm (0.577 × 1.5)**

441. Constant chord of a 20° involute gear of module 4 mm? → **≈ 5.55 mm (1.387 m)**

442. Roughness grade N9 corresponds to Ra of? → **6.3 µm**

443. Lay symbol "M" on a surface texture symbol means? → **Multidirectional lay**

444. Optical flat under sodium light (λ = 0.589 µm) shows 4 fringes – height difference? → **≈ 1.18 µm (4 × λ/2)**

445. Mechanical comparator based on a twisted thin metal strip? → **Johansson Mikrokator**

446. USL 50.04, LSL 49.96 mm, mean 50.01, σ = 0.01 mm – C_{pk}? → **1.0 (min of 0.03/0.03, 0.05/0.03); C_p = 1.33**

447. Control chart for fraction defective with varying sample size? → **p chart (binomial)**

448. Consumer's risk β is the probability of? → **Accepting a bad lot (Type II error), typically 10%**

449. UT pulse-echo in steel (v = 5900 m/s), back-wall echo after 10 µs – thickness? → **29.5 mm (t = v·t/2)**

450. In magnetic particle testing, a circular magnetic field best detects? → **Longitudinal cracks (defects perpendicular to flux)**

451. λ = 0.001 per hour (constant) – reliability for 100 h? → **≈ 0.905 ($e^{-0.1}$)**

452. Availability 85%, performance 90%, quality 95% – OEE? → **≈ 72.7% (0.85 × 0.90 × 0.95)**

453. Vibration dominant at 2× RPM with high axial component indicates? → **Misalignment**

454. ISO VG 68 oil means? → **Kinematic viscosity ≈ 68 cSt (mm^2/s) at 40 °C**

455. Factories Act Sec 29: lifting machines and tackle examined by competent person at least once every? → **12 months**

15.13 Production Planning, Industrial Engineering & OR (Q 456–500)

456. Which PPC function fixes the start and finish times of operations? → **Scheduling (answers "when")**

457. Progress chasing to keep work on schedule is called? → **Follow-up / expediting**

458. Layout best suited to mass production of a standard product? → **Product (line) layout**

459. Closeness code "X" in Muther's activity relationship chart means? → **Undesirable**

460. 400 units needed in a 480-min shift – cycle time? → **1.2 min (480/400)**

461. Σt = 20 min, 5 stations, cycle time 5 min – line efficiency? → **80 % (20/(5 × 5))**

462. F = ₹1,00,000, P = ₹60, V = ₹40 – break-even quantity? → **5,000 units (F/(P − V))**

463. Selling price ₹80, variable cost ₹60 – P/V ratio? → **25 % ((80 − 60)/80)**

464. Design capacity 120 units, actual output 90 units – utilisation? → **75 % (actual/design)**

465. Aggregate planning strategy with constant output and workforce? → **Level strategy (inventory absorbs demand changes)**

466. F_t = 200, D_t = 180, α = 0.1 – next forecast? → **198 (200 + 0.1 × (−20))**

467. Forecast errors 4, −2, 6, −4 – MAD? → **4 (Σ|e|/n = 16/4)**

468. RSFE = 20, MAD = 4 – tracking signal and inference? → **5; beyond ±4, forecast out of control**

469. D = 8,000 units/yr, S = ₹40/order, H = ₹4/unit/yr – EOQ? → **400 units (√(2 × 8000 × 40/4))**

470. Annual demand 6,000 units, EOQ 300 units – orders per year? → **20 (D/Q*)**

471. If carrying cost per unit becomes four times, EOQ becomes? → **Half (EOQ ∝ 1/√H)**

472. Usage 20 units/day, lead time 5 days, safety stock 30 – reorder point? → **130 units (dL + SS)**

473. z = 2, $σ_d$ = 4 units/day, lead time 4 days – safety stock? → **16 units ($zσ_d\sqrt{L}$ = 2 × 4 × 2)**

474. ABC analysis classifies items on the basis of? → **Annual consumption value (usage × unit cost)**

Nandyal Banking Coaching Center

475. "A" class items typically form? → **≈ 10 % of items, ≈ 70 % of annual value**

476. FSN analysis is based on? → **Movement / issue rate (fast, slow, non-moving)**

477. For the same data, EPQ (finite production rate) compared with EOQ is? → **Larger, by factor $1/\sqrt{(1 - d/p)}$**

478. MRP is used for which type of demand items? → **Dependent demand (components, sub-assemblies)**

479. Primary output of an MRP system? → **Planned order releases**

480. MRP-II stands for? → **Manufacturing Resource Planning**

481. Jidoka in the Toyota Production System means? → **Autonomation – machine stops automatically on a defect**

482. Which of the seven wastes is considered the worst? → **Overproduction**

483. Availability 90 %, performance 95 %, quality 98 % – OEE? → **≈ 83.8 % (0.9 × 0.95 × 0.98)**

484. D = 100 units/h, L = 2 h, α = 0.1, container 25 units – kanbans? → **9 (100 × 2 × 1.1/25 = 8.8, round up)**

485. Jobs of 5, 3, 8, 2 h on one machine by SPT – mean flow time? → **8.75 h (completions 2, 5, 10, 18)**

486. Due in 12 days, 16 days of work remaining – critical ratio? → **0.75; job is behind schedule**

487. Number of possible sequences of 5 jobs on one machine? → **120 (5!)**

488. Number of therbligs originally given by Gilbreth? → **17 ("Find" added later makes 18)**

489. Observed time 1.5 min, rating 80 %, allowances 15 % – standard time? → **1.38 min (1.5 × 0.8 × 1.15)**

490. Work sampling: p = 0.2, ±5 % absolute accuracy, 95 % confidence – observations? → **256 ($4 \times 0.2 \times 0.8/0.05^2$)**

491. Rowan plan: S = 8 h, T = 6 h, R = ₹50/h – total wage? → **₹375 (300 + (2/8) × 300)**

492. Feasible region of a linear programming problem is always a ___ set? → **Convex set**

493. Number of dual variables equals the number of primal ___? → **Constraints**

494. Expected value of perfect information (EVPI) equals? → **Expected profit with perfect information – max EMV (= min EOL)**

495. 3 sources × 4 destinations – occupied cells in a non-degenerate basic solution? → **6 (m + n − 1)**

496. Hungarian method: solution is optimal when minimum covering lines equal? → **n, the order of the square matrix**

497. M/M/1 with λ = 4/h, μ = 5/h – average waiting time in queue? → **48 min ($\lambda/[\mu(\mu - \lambda)] = 0.8$ h)**

498. Little's law relates average number in system to? → **L = λW (arrival rate × time in system)**

499. PERT: a = 1, m = 4, b = 13 days – expected time and σ? → **t_e = 5 days, σ = 2 days**

500. Normal 10 days/₹5,000; crash 6 days/₹7,000 – cost slope? → **₹500 per day (2000/4)**

Nandyal Banking Coaching Center